LAKSHYA ADVANCED UNIT TEST (LAUT)

00 – 00		Q. Booklet Serial No: 170116	
Test No : 2110	3 Hrs.		

Hints & Solutions

PART A - PHYSICS

SECTION I - MULTIPLE ANSWER CORRECT TYPE

1. **b) greater in process A than in B**

2. **c) 40 J**

Since fraction of total energy taken up by the body as rotational kinetic energy is $\frac{K^2}{R^2 + K^2}$

$\Rightarrow \frac{\frac{1}{2}I\omega^2}{\text{Total Energy}} = \frac{K^2}{R^2 + K^2}$

$\Rightarrow \frac{1}{2}I\omega^2 = \left(\frac{K^2}{R^2 + K^2}\right)mgh$

$\Rightarrow \frac{1}{2}I\omega^2 = \frac{2}{7}(2 \times 10 \times 7)$

$\Rightarrow \frac{1}{2}I\omega^2 = 40$ J

3. **a) $\frac{L}{4}$**

Since L = Iω

$\Rightarrow$ L = $(mr^2)\omega$

when r becomes $\frac{r}{2}$, L becomes $\frac{L}{4}$.

4. **d) None of these**

$a = \frac{g}{1 + \frac{M}{2m}}$

So, $v^2 = 2gh$

$\Rightarrow$ v is independent of R.

5. **c) $4\pi^2 mr^2 n^2$**

R.K.E. = $\frac{1}{2}I\omega^2$

$\Rightarrow$ R.K.E = $\frac{1}{2}(2mr^2)(2\pi n)^2$

$\Rightarrow$ R.K.E = $4\pi^2 mr^2 n^2$

6. **d) $\sqrt{\frac{4}{3}gh}$**

7. **d) $\frac{5r}{2}$**

For looping the loop v = $\sqrt{5gr}$.

By Law of Conservation of Energy

Loss in potential energy = Gain in Kinetic energy

$\Rightarrow mgh = \frac{1}{2}mv^2$

$\Rightarrow h = \frac{5r}{2}$

8. **c)** 5 : 1

$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$

$\Rightarrow \frac{2}{3} = \frac{m_1 - m_2}{m_1 + m_2}$

$\Rightarrow 2m_1 + 2m_2 = 3m_1 - 3m_2$

$\Rightarrow 5m_2 = m_1$

$\Rightarrow \frac{m_1}{m_2} = \frac{5}{1}$

SECTION II - MATRIX MATCH TYPE

1. **A-Q, B-S, C-Q, D-S**

$$I_1 = 2\left(\frac{ml^2}{12}\right) + 2\,(m)\left(\frac{l}{2}\right)^2 = \frac{2}{3}ml^2$$

$$I_2 = 0 + 2\left(\frac{ml^2}{3}\right) + ml^2 = \left(\frac{5}{3}\right)ml^2$$

$$I_3 = 4\left[\frac{ml^2}{3}\sin^2 45^0\right] = \frac{2}{3}\ ml^2 = I_1$$

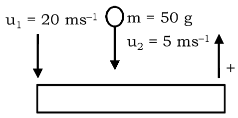
2. **A-Q, B-P, C-S, D-R**

In general as $v_p = 2v\sin\left(\frac{\theta}{2}\right)$

SECTION III - INTEGER TYPE

1. **2**

The situation is analysed in the figure. We can consider mass of platform to be very large compared to that of ball, so we have



$$\vec{v}_2 = \vec{u}_1\ e = \left(\frac{v_2 - v_1}{u_2 - (-u_1)}\right) = \left(\frac{v_2 - v_1}{u_1 - (-u_2)}\right)$$

$$\Rightarrow \vec{v}_2 = 2\vec{u}_1 - \vec{u}_2$$

Thus rebound velocity of ball is

$\vec{v}_2 = 2 \times 20 - (-5) = 45\ ms^{-1}$ upward

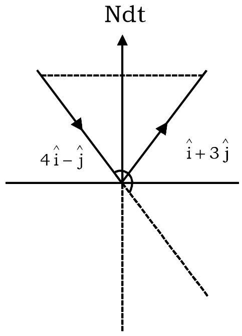
$= \frac{90}{2}$

2. **9**

$$\int N dt = 1[\hat{i} + 3\hat{j}) - (4\hat{i} - \hat{j})] = -3\hat{i} + 4\hat{j}$$

Component of $4\hat{i} - \hat{j}$ along $-3\hat{i} + 4\hat{j}$

$$= \frac{-12-4}{25}(-3\hat{i} + 4\hat{j}) = -\frac{16}{25}(-3\hat{i} + 4\hat{j})$$



Speed of approach $\frac{16}{25}\sqrt{25} = \frac{16}{5}$

Component of $\hat{i} + 3\hat{j}$ along $-3\hat{i} + 4\hat{j}$ is

$$\frac{-3+12}{25}(-3\hat{i} + 4\hat{j})$$

Speed of separation = 9/5

Speed of separation = e × speed of apporach

$e = \frac{6}{16}$ n = 9

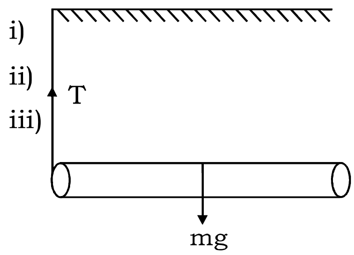
3. **5**

$Mg - T = Ma_y$

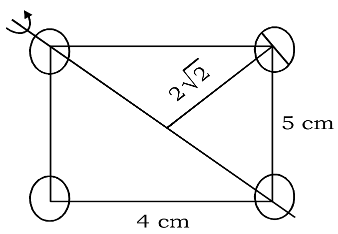
$T\left(\frac{L}{2}\right) = \frac{ML^2}{12}\alpha$ and $a_y = \frac{L}{2}\alpha$

On solving, we get

$T = \frac{Mg}{4} = 5N$



4. **9**



$$I = \left(\frac{2}{5}MR^2\right) 2 + \left(\frac{2}{5}MR^2 + Mx^2\right)2$$

$$= \left(\frac{2}{5}MR^2\right) 2 + \left(\frac{2}{5}MR^2\right) 2 = (Mx^2)\ 2$$

$$= 4\left(\frac{2}{5}MR^2\right) + 2Mx^2 = \frac{8}{5}MR^2 + 2Mx^2$$

$$= \left[\frac{8}{5}\times 0.5\times\left(\frac{\sqrt{5}}{2}\right)^2 + 2\times(0.5)\times(4\times 2)\right]10^{-4}$$

$$= \left[\frac{5}{5}+8\right]\times 10^{-4} = 9\times 10^{-4} = N\times 10^{-4}$$

So, N = 9.

5. **0**

Let f be the friction force acting on the disc in the backward direction. Then 2F – f = ma and (F + f)r = I

$$\Rightarrow (F + f)r = \frac{1}{2}mr^2\frac{a}{r}$$

∴ solving the above equations, we get f = 0.

6. **2**

F + f = ma

Torque equation about point of contact

$F2R = I_p\alpha$

$$F2R = \left(\frac{2}{5}MR^2 + mR^2\right)\alpha$$

$$F = \frac{7}{10}mR\alpha \qquad ...(ii)$$

For no sliding case a = Rα ...(iii)

From (ii) and (iii), $a = \frac{10F}{7m}$ ...(iv)

From (i) and (iv), $F + f = m.\frac{10}{7}\frac{F}{m}$

$$\Rightarrow f = \frac{3}{7}F$$

i.e., friction will act in backward direction. For no sliding $f \le \mu N$

$$\frac{3}{7}F \le \mu mg \qquad \Rightarrow F \le \frac{7}{3}\mu mg$$

$$F \le \frac{7}{3}\times\frac{2}{7}\times 3\times 10 \qquad \Rightarrow F \le 20\ N$$

= 10(2)

7. **6**

Total kinetic energy of loop $= \frac{1}{2}I\omega + \frac{1}{2}Mv^2$

$$= \frac{1}{2}(MR^2\omega^2) + \frac{1}{2}Mv^2 = Mv^2 = 8J \text{ (given)}$$

Total kinetic energy of disc $= \frac{1}{2}I\omega^2 + Mv^2$

$$= \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v^2}{R^2}\right) + \frac{1}{2}Mv^2 = \frac{3}{4}Mv^2 = \frac{3}{4}(8) = 6J$$

8. **5**

Initial momentum

$$\vec{p}_i = -mv\hat{j} + mv\left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right) + mv\left(-\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right)$$

Final momentum

$$\vec{p}_f = -mv\left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right) + m\vec{v}_c$$

Now $\vec{p}_f = \vec{p}_i$

$$\Rightarrow \vec{v}_c = -v\hat{j} + v(\sqrt{3}\hat{i} + \hat{j}) + v\left(-\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right)$$

$$\Rightarrow \vec{v}_c = v\left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right)$$

$\Rightarrow v_c = 5ms^{-1}$