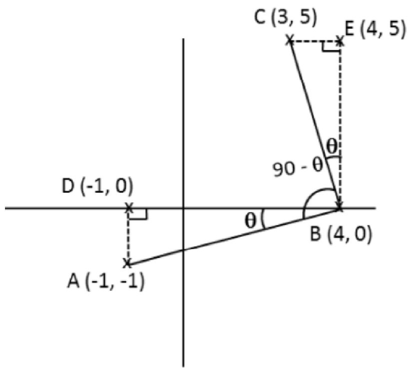
XI - MATHS - SOLUTIONS

61. **a)** isosceles and right angled



By observation

BE = BD = 5

EC = DA = 1

& $\angle$BDA = $\angle$BEC

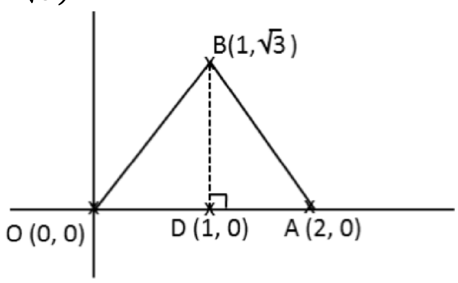
$\Rightarrow \Delta$BEC $\cong \Delta$BDA

$\Rightarrow$ BC = BA

& $\angle$CBA = 90^0.

$\Rightarrow \Delta$ABC is isosceles and right angled

62. **d)** $\left(1, \frac{1}{\sqrt{3}}\right)$



By observation

D is the mid point of AB & BD = $\sqrt{3}$

= $\frac{\sqrt{3}}{2}(\text{OA})$.

$\Rightarrow \Delta$OAB is equilateral

$\Rightarrow$ Incenter is at centroid.

i.e. at $\left(\frac{0+1+2}{3}, \frac{0+0+\sqrt{3}}{3}\right) = \left(1, \frac{1}{\sqrt{3}}\right)$.

63. **b)** $(3x-1)^2 + (3y)^2 = a^2 + b^2$

Let (h, k) be the centroid.

$(a \cos t, a \sin t)$, $(b \sin t, - b \cos t)$

$h = \dfrac{a\cos t + b\sin t + 1}{3}$

$\Rightarrow 3h - 1 = a \cos t + b \sin t$

$k = \dfrac{a\sin t - b\cos t + 0}{3}$

$\Rightarrow 3k = a \sin t - b \cos t$

Squaring and adding,

$(3h - 1)^2\ (3k)^2 =$

$a^2(\cos^2 t + \sin^2 t) + b^2(\sin^2 t + \cos^2 t)$

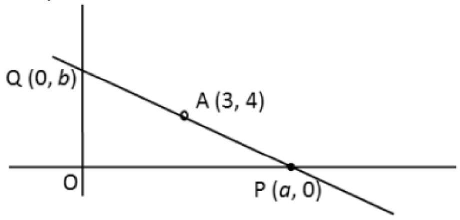
$+ 2ab \sin t \cos t - 2ab \sin^2 \cos^2 t$

$a^2 + b^2$.

$\therefore$ Locus is,

$(3x-1)^2 + (3y)^2 = a^2 + b^2$.

64. **b)** $4x + 3y = 24$



Let the line meet the axes at P(a, 0), Q (0, b)

Since A is mid point. of PQ

$\Rightarrow a = 2(3) = 6$

$b = 2(4) = 8$.

By double intercept form of a line,

$\frac{x}{6} + \frac{y}{8} = 1$

$\Rightarrow 8x + 6y = 48 \Rightarrow 4x + 3y = 24$

65. **b)** $(1, -2)$

a, b, c are in H.P $\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$\Rightarrow 2\left(\frac{1}{b}\right) = \frac{1}{a} + \frac{1}{c} \Rightarrow \frac{1}{a} - \frac{2}{b} + \frac{1}{c} = 0$

$\Rightarrow \frac{1}{a} + \frac{(-2)}{b} + \frac{1}{c} = 0$

Match this with $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$.

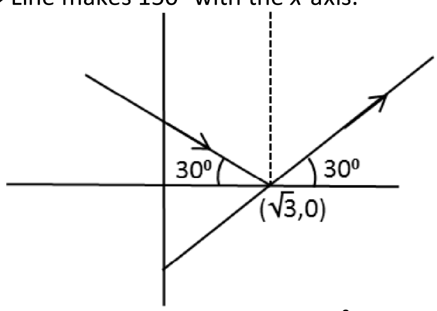
The line must pass through $(1, -2)$

66. **b)** $\sqrt{3}y = x - \sqrt{3}$

given equation can be written as,

$y = \frac{-1}{\sqrt{3}}\ x + 1$, slope $= \frac{-1}{\sqrt{3}}$

$\Rightarrow$ Line makes 150^0 with the x-axis.



$\Rightarrow$ Reflected line will make 30^0 with the

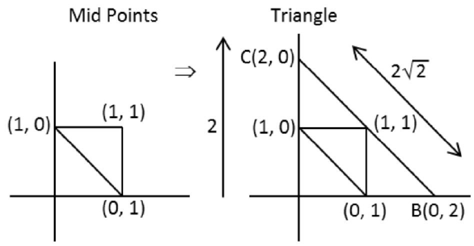
x-axis
By symmetry incident ray has y-intercept = 1 then reflected ray has y-intercept = – 1
$\Rightarrow$ Equation of reflected ray is,
$y = \frac{1}{\sqrt{3}}x - 1$ i.e. $\sqrt{3}y = x - \sqrt{3}$

67. **c)** ± 1
$\Rightarrow my^2 - m^2xy + xy - mx^2 = 0$
$\Rightarrow my(y - mx) + x(y - mx) = 0$
$\Rightarrow (my + x)(y - mx) = 0$
$y = mx$ or $y = \frac{-1}{m}x$
Now $xy = 0$
$\Rightarrow x = 0$ or $y = 0$ i.e. x-axis or y-axis.
$\Rightarrow y = mx$ & $y = \frac{-1}{m}x$
are bisectors of angle between x-axis & y-axis.
$\Rightarrow m = 1$ or -1

68. **a)** Exactly one value of p
Since the lines are $\perp$ to same line, they are $||^{el}$ to each other.
$\Rightarrow$ slopes are same
$\Rightarrow p(p^2+1) = \frac{-(p^2+1)^2}{(p^2+1)}$
$\Rightarrow p = -1$

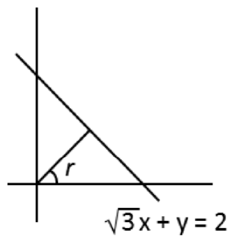
69. **c)** $2x + 3y = 1$
Let the centroid be (h, k) and the third vertex be (α, β)
Then, $(h, k) = \left(\frac{2-2+\alpha}{3}, \frac{-3+1+\beta}{3}\right)$.
$\Rightarrow h = \frac{\alpha}{3}$, $k = \frac{-2+\beta}{3}$.
i.e. $\alpha = 3h$, $\beta = 3k + 2$.
Also, $2\alpha + 3\beta = 9$
(since (α, β) lies on $2x + 3y = 9$)
$\Rightarrow 2(3h) + 3(3k + 2) = 9$
$\Rightarrow 2h + 3k + 2 = 3 \Rightarrow 2h + 3k = 1$.
Locus is $2x + 3y = 1$

70. **b)** $2 - \sqrt{2}$



If vertices are A (0, 0), B (2, 0), C (0, 2)
Then side are $a = 2\sqrt{2}$, $b = 2$, $c = 2$
$\therefore$ x-coordinate of incenter is,
$$\frac{2\sqrt{2}(0)+2(2)+2(0)}{2\sqrt{2}+2+2}$$
$$= \frac{4}{4+2\sqrt{2}} = \frac{2}{2+\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}+1} = \frac{\sqrt{2}(\sqrt{2}-1)}{2-1}$$
$= 2 - \sqrt{2}$

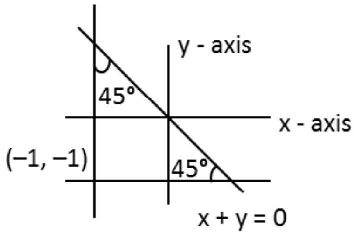
71. **a)** 30^0



The line can be re written as,
$x\left(\frac{\sqrt{3}}{2}\right) + y\left(\frac{1}{2}\right) = 1$
i.e. $x \cos 30° + y \sin 30° = 1$ which is the normal form of the line.
$\Rightarrow \alpha = 30°$ is the inclination of the $\perp$ from x-axis

72. **d)** A straight line parallel to Y-axis
Let the point be S(h, k).
Then, $(h - (-1))^2 + (k - 0)^2 + (h - 2)^2 + (k - 0)^2$
$= 2((h - 1)^2 + (k - 0)^2)$
$\Rightarrow h^2 + 2h + 1 + k^2 + h^2 - 4h + 4 + k^2$
$= 2(h^2 - 2h + 1 + k^2)$
$\Rightarrow -2h + 5 = -4h + 2 \Rightarrow h = \frac{3}{2}$
$\therefore$ Locus is $x = \frac{3}{2}$, when is straight line parallel to y-axis

73. **d)** $x + 1 = 0, y + 1 = 0$
$x + y = 0 \Rightarrow y = -x$



As can be seen from the figure,
The required lines are $x = -1$, $y = -1$.
i.e. $x + 1 = 0$ and $y + 1 = 0$.

74. **c)** $a(x - c) + b(y - d) = 0$

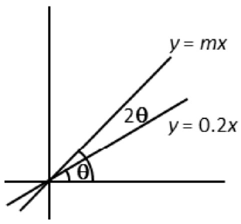
Slope of required line = $\left(-\dfrac{a}{b}\right)$.

It passes through $(c, d) \Rightarrow$ equation is,

$y - d = \left(-\dfrac{a}{b}\right)(x - c)$

$b(y - d) + a(x - c) = 0$

75. **b)** $y = \left(\dfrac{5}{12}\right)x$

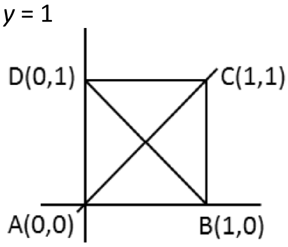


Let required line by y = mx
Then m = tan 2θ

Where $\tan\theta = 0.2 = \dfrac{1}{5}$

$\therefore\ m = \dfrac{2\left(\frac{1}{5}\right)}{1 - \left(\frac{1}{5}\right)^2} = \dfrac{5}{12} \Rightarrow y = \dfrac{5}{12}x$

76. **a)** $y = x$, $x + y = 1$

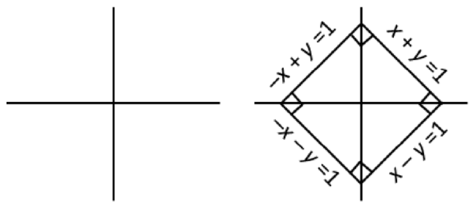


Equation of AC is $y = x$

Equation of BD is $\dfrac{y - 0}{x - 1} = -1$

i.e. $y + x = 1$

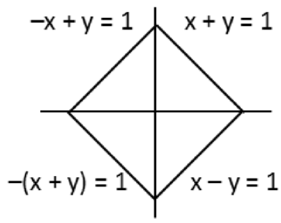
77. **b)** 2



Here,
$|x| = x$ when $x \geq 0$ and $|x| = -x$ when $x < 0$.
III[ly] $|y| = y$ when $y \geq 0$ and $|y| = -y$,
when $y < 0$.
$\Rightarrow$ The curve represents a square of vertices (1, 0), (0, 1), (–1, 0), (0, –1).

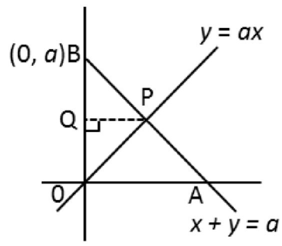
Area = $\left(\sqrt{2}\right)^2 = 2$

78. **a)** a square



Given $|x| + |y| = 1$
$\therefore$ Equation is
$x + y = 1$ in I quadrant
$-x + y = 1$ in II quadrant
$-x - y = 1$ in III quadrant
$x - y = 1$ in iv quadrant
$\therefore$ Locus is a square

79. **d)** $\dfrac{a^2}{2|1 + a|}$



Area of Δ OBP

$= \dfrac{1}{2}\text{OB} \times \text{P}a$

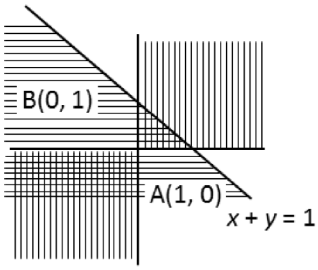
OB = a

Solve the two Given to get P

$y = \dfrac{a^2}{a + 1} \Rightarrow \text{PQ} = \dfrac{a}{1 + a}$

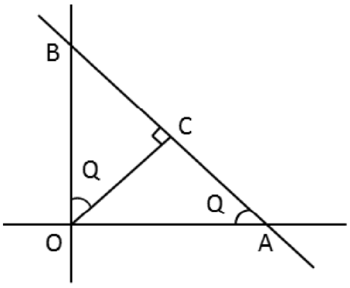
Area = $\left|\dfrac{1}{2}a.\dfrac{a}{(1 + a)}\right| = \dfrac{a^2}{2|1 + a|}$

80. **a)** P lies either inside the triangle OAB or in the third quadrant



$x + y < 1$ is represented by shaded region with horizontal lines.
Also, $xy > 0$
$\Rightarrow$ x and y are both + ve or – ve.
$\Rightarrow$ I quadrant or III quadrant.
$\Rightarrow$ Intersection is given by Δ OAB and III quadrant.

81. **d)** 9 : 1



Slope of AB = –3
$\Rightarrow$ tan Q = + 3
Now, BC = OC tan Q

$$AC = \frac{OC}{\tan Q}$$

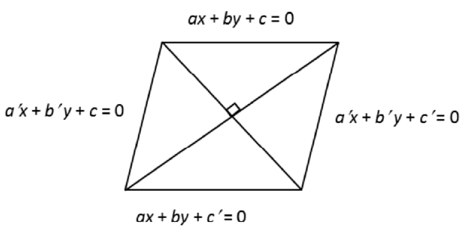
$$\Rightarrow \frac{BC}{AC} = \tan^2 Q = \frac{9}{1}$$

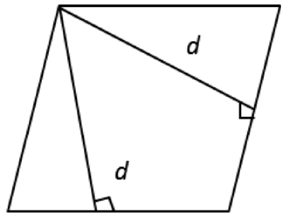
82. **d)** ± 1
Form the equation,

$$m = \frac{2a-3 \pm \sqrt{(2a-3)^2 - 4(a^2-1)a}}{2(a^2-1)}$$

For a line parallel to y-axis
Slope is undefined
Since at least one root, $m = \infty$ (undefined)
$\Rightarrow$ $a^2 - 1 = 0$ $\Rightarrow a = \pm 1$

83. **c)** $a^2 + b^2 = (a')^2 + (b')^2$



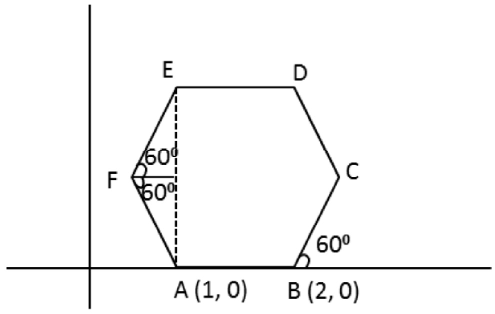


Since the diagonals are $\perp$, the given quadrilateral is a rhombus. So, the distances between sides are equal.

$$\therefore \left|\frac{c'-c}{\sqrt{a^2+b^2}}\right| = \left|\frac{c'-c}{\sqrt{(a')^2+(b')^2}}\right|$$

$\Rightarrow$ $a^2 + b^2 = (a')^2 + (b')^2$

84. **c)** $x + \sqrt{3}y = 4$



AB = 1
C = (2 + 1cos60⁰, 0 + sin 60⁰)

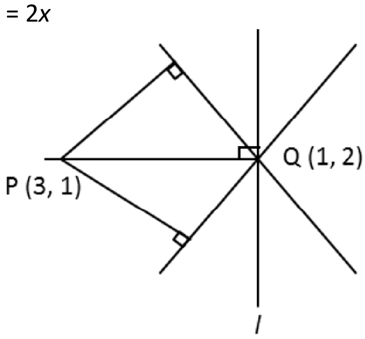
$$= \left(\frac{5}{2}, \frac{\sqrt{3}}{2}\right)$$

E = (1, 1 × sin 60 + 1 × sin 60)
 = (1, $\sqrt{3}$)
Equation of CE is,

$$y - \sqrt{3} = \left(\frac{\frac{\sqrt{3}}{2} - 3}{\frac{5}{2} - 1}\right)(x-1)$$

i.e. $\sqrt{3}\,y + x = 4$

85. **a)** $y = 2x$



Distance of P from line l will be maximum when PQ $\perp$ l.

Slope q PQ = $\left(\frac{1}{-2}\right)$

$\Rightarrow$ Slope of l = 1

Equation l is $y - 2 = 2(x - 1)$

i.e. $y = 2x$

86. a) two value of a

Condition for perpendicularity is,

$3a + a^2 - 2 = 0 \Rightarrow a^2 + 3a - 2 = 0$

$a = \frac{-3 \pm \sqrt{17}}{2}$

$\therefore$ 2 values of a

87. a) $8/5$

$9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$ ... 1

$9x^2 - 12xy + 16y^2 = (3x - 4y)^2$

Consider the lines

$3x - 4y + C_1$ and $3x - 4y + C_2$

$\therefore (3x - 4y + C_1)(3x - 4y + C_2) = 9x^2 - 24xy + 16y^2 - 12x + 16y - 12$

Comparing the constant, $C_1C_2 = -12$

Comparing coefficient of x,

$3(C_1 + C_2) = -12 \Rightarrow C_1 + C_2 = -4$

$\Rightarrow C_1 = -6, C_2 = +2$ or vice versa.

$\therefore$ Lines are $3x - 4y - 6 = 0$ and

$3x - 4y + 2 = 0$

Distance between the lines is, $\left|\frac{6-(-2)}{5}\right| = \frac{8}{5}$

88. **b)** $g(a_1 + b_1) = g_1(a + b)$

The points of intersection satisfy both the curves

$\Rightarrow ax^2 + 2hxy + by^2 = -2gx$

$a_1x^2 + 2h_1xy + b_1y^2 = -2g_1x$

or $\frac{ax^2 + 2hxy + by^2}{a_1x^2 + 2h_1xy + b_1y^2} = \frac{g}{g_1}$

$\Rightarrow (ag_1 - a_1g)x^2 + 2(g_1h - h_1g)xy + (g_1b - gb_1)y^2 = 0$

This a homogeneous equation of degree 2.

$\Rightarrow$ It represents a pair of lines through origin.

The lines are $\perp$ if

$ag_1 - a_1g + g_1b - gb_1 = 0$

$g_1(a + b) = g(a_1 + b_1)$

89. **c)** a set of four lines forming a square

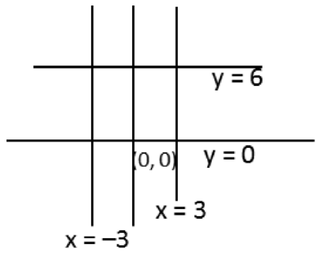
We have,

$x^2y^2 - 9y^2 - 6x^2y + 54y = 0$

$\Rightarrow y^2(x^2 - 9) - 6y(x^2 - 9) = 0$

$\Rightarrow y(y - 6)(x - 3)(x + 3) = 0$

$\Rightarrow y = 0, y = 6, x = 3, x = -3$



So, the given equation represents set of four lines that form a square.

90. **d)** $ax^2 - 2hxy + by^2 = 0$

y = 0 is the X-axis.

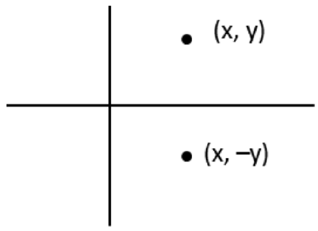


Image of every point (x, y) w.r.t mirror $y = 0$ is $(x, -y)$

So, replace y with $(-y)$ to get image of pair of lines.

$ax^2 + 2hx(-y) + by^2 = 0$

i.e. $ax^2 - 2hxy + by^2 = 0$