XI - MATHS - SOLUTIONS

Section I - Multiple Choice Type

1. **b)** $t_3\, t_4 = 3$
   c) $t_1\, t_2\, t_3\, t_4 = -3$

2. **a)** **cannot represent a real pair of straight lines**
   b) **represents an ellipse, if $c > 0$**
   c) **represents empty set, if $c > 0$**
   d) **a point, if $c > 0$**
   $\left(\sqrt{3}x - 3\sqrt{3}\right)^2 + \left(2y+4\right)^2 = c$
   Comparing this with
   $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
   we get $h^2 < ab$
   so no locus for $c < 0$
   Ellipse for $c > 0$ and point for $c = 0$

3. **a)** **always passes through a fixed point**
   d) **passes through the point (–2, 1)**
   Solving equation of parabola with x-axis (i.e. $y = 0$), we get
   $(a - b)x^2 + (b - c)\, x + (c - a) = 0$ which should have two equal values of x, as x–axis touches the parabola
   $\Rightarrow (b - c)^2 - 4(a - b)(c - a) = 0$
   $\Rightarrow (b + c - 2a)^2 - 0$
   $\Rightarrow b + c = 2a$
   i.e. $ax + by + c = 0$ always pass through (–2, 1)

4. **a)** $a_1 = 2$
   b) $a_2 = 3$
   d) $a_8 = 55$
   The cases for a_1 : {H, T} i.e., $a_1 = 2$
   The case for a_2 : {HT, TH, TT}, $a_2 = 3$
   For $n \geq 3$, If the first outcome is H then next just T and then a_{n-2}. If the first outcome is T then a_{n-1} should follow.
   So, $a_n = 1 \times 1 \times a_{n-2} + 1 \times a_{n-1}$
   $\Rightarrow a_n = a_{n-2} + a_{n-1}$
   So, $a_3 = a_1 + a_2 = 5$, $a_4 = 3 + 5 = 8$ and so on.

5. **a)** **There are exactly 3 Indian classic songs in top 5 is $(5!)^3$.**
   b) **Top rank goes to Indian classic song is 6(9!).**
   c) **The ranks of all western songs are consecutive is 4! 7!.**
   d) **The 6 Indian classic songs are in a specifies order is $^{10}P_4$.**
   a) $^6C_3 \,.\, ^4C_2 \,.\, 5!\, 5! = (5!)^3$
   b) $^6C_1 \,.\, 9!$
   c) $(6 + 1)!\, 4!$
   d) $^{10}P_4$

6. **a)** $\lambda \in (0, 1)$
   b) $\lambda \in \left(-\frac{3}{5}, 1\right)$
   c) $\lambda \in \left(\frac{1}{2}, \frac{3}{5}\right)$

d) $\lambda \in \left(-\frac{3}{5}, 0\right)$

$R = \left(\frac{\lambda+1}{\lambda+1}, \frac{3\lambda+1}{\lambda+1}\right) = \left(1, 1+\frac{2\lambda}{\lambda+1}\right)$

$\Rightarrow y_1^2 - 4x_1 < 0$

$\Rightarrow \left(1+\frac{2\lambda}{\lambda+1}\right)^2 - 4 < 0$

$\Rightarrow \left(1+\frac{2\lambda}{\lambda+1}+2\right)\left(1+\frac{2\lambda}{\lambda+1}-2\right) < 0$

$\Rightarrow \frac{2\lambda}{\lambda+1} > -3$ and $\frac{2\lambda}{\lambda+1} < 1$

$\Rightarrow 5\lambda > -3$ and $\lambda < 1$

$\Rightarrow \lambda > -\frac{3}{5}$ and $\lambda < 1$

$\Rightarrow \lambda \in \left(-\frac{3}{5}, 1\right)$

7. c) $^{20}C_{9-1}$

d) Number of different ways of exchanging 11 books of A with the 9 books of B

$(1 + x)^{11} = {}^{11}C_0 + {}^{11}C_1 x + {}^{11}C_2 x^2 + \ldots + {}^{11}C_{11} x^{11}$ ... (i)

$(1 + x)^9 = {}^{9}C_0 + {}^{9}C_1 x + {}^{9}C_2 x^2 + \ldots + {}^{9}C_9 x^9$ ... (ii)

On multiply (i) and (ii) and compare coefficient of x^{11} on both sides and put $x = 1$

$^{20}C_{11} = {}^{11}C_{11}\,{}^{9}C_0 + {}^{11}C_{10}\,{}^{9}C_1 + \ldots + {}^{11}C_2\,{}^{9}C_9$

$\therefore {}^{20}C_9 - 1 = {}^{11}C_{10}\,{}^{9}C_1 + \ldots + {}^{11}C_2\,{}^{9}C_9$

8. a) $(1004)^2$ if equal sides do not exceed 1004.

b) $2(1004)^2$ if equal sides exceed 1004.

c) $3(1004)^2$ if equal sides have any length ≤ 2008.

If the sides are a, a, b, then the triangle forms only when $2a > b$, so for any $a \in N$, b can change from 1 to $2a - 1$. When $a \leq 1004$ then number of triangles

$= 1 + 3 + 5 + \ldots + (2(1004) - 1) = (1004)^2$ and if $1005 \leq a \leq 2008$, b can take any value from 1 to 2008.

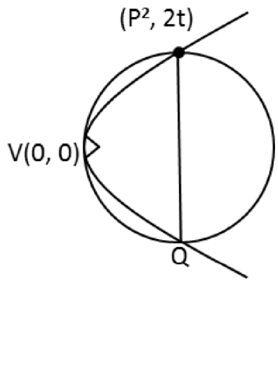
But a has 1004 possibilities, hence number of triangles $= 1004 \times 2008 = 2(1004)^2$

$\therefore$ Total number of isosceles triangles $= 3(1004)^2$

9. a) (16, 8)

b) (16, –8)

Slope of PV $= \frac{2t-0}{t^2-0} = \frac{2}{t}$



Equation of QV is $y = -\frac{t}{2}(x)$

On solving with $y^2 = 4x$, Q = $\left(\frac{16}{t^2}, \frac{-8}{t}\right)$

Area of ΔPVQ is $\frac{1}{2}$. PV.VQ = 20

$\Rightarrow$ PV.VQ = 40

By solving above equation $t = \pm 4, \pm 1$

10. a) **Length of latus rectum of parabola is $8\sqrt{2}$**
b) **Equation of tangent at the vertex is $x + y - 5 = 0$**
c) **Equation of axis of the parabola is $x - y - 3 = 0$**

Section II – Matrix Matching Choice Type

1. **A- Q, S ; B – Q, S ; C – P ; D - R**

A) Non-negative integral solutions of
R + B + G + Y = 8 is ${}^{8+4-1}C_{4-1} = {}^{11}C_3$

B) $D_1 + D_2 + D_3 + D_4 = 9$, $D_1 \neq 0$
$D_1 - 1 + D_2 + D_3 + D_4 = 8$
${}^{8+4-1}C_{4-1} = {}^{11}C_3$

C) Select '3' place out of 21 places = ${}^{21}C_3$

D) $\left[\frac{17}{3}\right] + \left[\frac{17}{3^2}\right] = 5 + 1 = 6$

2. **A- S ; B – R ; C – P ; D - Q**

A) ${}^2C_1 . {}^5P_2 = 2.20 = 40$

B) $\frac{6!}{3!3!} = \frac{720}{6.6} = 20$

C) ${}^{22-1}C_{4-1} = {}^{21}C_3$

D) ${}^{23-7-2}C_{10-7} = {}^{14}C_3$

Section III - Integer Type

1. **0**

${}^{100}C_{50} = \frac{100!}{50!50!}$

Exponent of 7 in 100! = 16

Exponent of 7 in 50! $\left[\frac{50}{7}\right] + \left[\frac{50}{7^2}\right] = 8$

Exponent of 7 in $(50!)^2 = 16$

$\therefore$ Exponent of 7 in ${}^{100}C_{50} = 16 - 16 = 0$

2. **5**

Since 2009 = 7 × 7 × 41. Therefore, 41 must divide $n^2(n - 1)$, which implies that 41 is a factor of either n or $n - 1$. In particular, $n \geq 41$. For $n = 41$, neither n nor $(n - 1)$ is divisible by 7. For $n = 42$, n^2 is divisible by 7^2, since n is divisible by 7. Therefore, $n = 42$ is the smallest integer.

3. **2**

Let the point P be (h, k)

$\therefore$ Equation of the normal is
$K = mh - 2m - m^3$

$\Rightarrow m^3 + m(2 - h) + k = 0$

$\Rightarrow m_1m_2m_3 = -k \Rightarrow m_3 = -\dfrac{k}{\alpha}$

$\Rightarrow \left(-\dfrac{k}{\alpha}\right)^3 - \dfrac{k}{\alpha}(2-h) + k = 0$

$\Rightarrow k^2 = \alpha^2 h - 2\alpha^2 + \alpha^3$

Thus the locus of (h, k) is

$y^2 = \alpha^2 x - 2\alpha^2 + \alpha^3$

On comparing it with $y^2 = 4x$, we get $\alpha^2 = 4$

and $-2\alpha^2 + \alpha^3 = \alpha = 2$

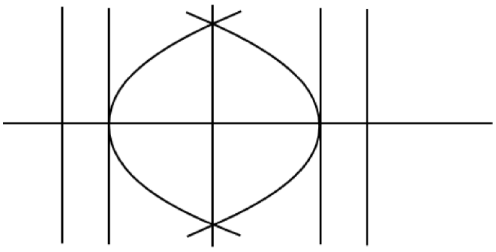
4. **8**

Length of latus rectum is $4\sqrt{2} = 4a$

$\Rightarrow a = \sqrt{2}$

Equation of directrix is $x + y = \lambda$

Whose distance from (4, 4) is $2\sqrt{2}$



$\therefore \left|\dfrac{4+4-\lambda}{\sqrt{2}}\right| = 2\sqrt{2} \Rightarrow 8 - \lambda = \pm 4$

$\Rightarrow \lambda = 4$ or 12 $\therefore \dfrac{\lambda_1 + \lambda_2}{2} = 8$

5. **8**

The curve is $(y + 1)^2 = 4(x - 1)$

Equation of the normal to the given curve is

$y + 1 = m(x - 1) - 2m - m^3$

which passes through (h, k)

$m^3 + m(3 - h) + 1 + k = 0$

$m_1m_2m_3 = -(1 + k) \Rightarrow m_3 = (1 + k)$

$(\because m_1m_2 = -1)$

$(1 + k)^3 + (1 + k)(3 - h) + (1 + k) = 0$

$(1 + k)^2 + 3 - h + 1 = 0$

$(y + 1)^2 = (x - 4)$

$\therefore \dfrac{\mu}{2} = -1 \Rightarrow \mu = -2, \lambda = 4$

$\therefore \lambda - 2\mu = 8$

6. **4**

Centre of circle $(a, 0)$ and radius $2a$

Equation of circle $(x - a)^2 + y^2 = 4a^2$

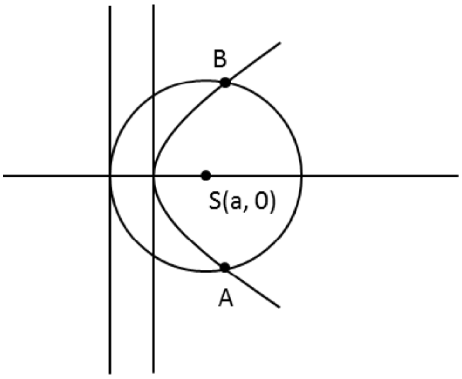
$x^2 + y^2 - 2ax - 3a^2 = 0$ and $y^2 = 4ax$

Solving, $x^2 + 4ax - 2ax - 3a^2 = 0$

$x^2 + 2ax - 3a^2 = 0$

$x = -3a, a$ and $y = \pm 2a$

$\therefore$ Length of AB = $4a$



7. **4**

Number of the form $(2n + 1)$, $n \in I$ are all odd natural numbers.
Thus we have to find all odd numbers dividing 120.
These numbers are 1, 3, 5, 15.
Hence, number of divisors = 4.

8. **3**

$x_1 + x_2 + x_3 = 15$
$0 \le x_1 \le 5$, $0 \le x_2 \le 10$, $0 \le x_3 \le 15$
n = coeff. of x^{15} in $(1 - x^6)(1 - x^{11})(1 - x^{16})(1 - x)^{-3}$
$n = 66$
$\Rightarrow \frac{n}{22} = 3$