LAKSHYA ADVANCED UNIT TEST (LAUT)

00 – 00		Q. Booklet Serial No: 300815	
Test No : 2104	3 Hrs.		

Hints & Solutions

PART A - PHYSICS

SECTION I - MULTIPLE ANSWER CORRECT TYPE

1. **b) the average speed is $\frac{2\upsilon_1\upsilon_2}{(\upsilon_1+\upsilon_2)}$**

   c) the average velocity is zero

   Let s = distance between points A and B. It t_1 is the time taken to go from A to B and t_2 the time taken to return from B to A, then

   $t_1 + t_2 = \frac{s}{\upsilon_1} + \frac{s}{\upsilon_2}$

   $\therefore$ Average speed $= \frac{\text{Total distance travelled}}{\text{Total time taken}}$

   $= \frac{s+s}{\frac{s}{\upsilon_1}+\frac{s}{\upsilon_2}} = \frac{2\upsilon_1\upsilon_2}{(\upsilon_1+\upsilon_2)}$

   Since the net displacement is zero as the body returns to the strating point, the average velocity = 0.

2. **a) The net displacement of the bullet in 10 s is zero**

   b) The total distance travelled by the bullet in 10 s is 250 m

   c) The rate of change of velocity with time is constant throughout the motion of the bullet

   d) The bullet is fired at an initial velocity of 50ms^{-1} directed vertically upwards

   Since the bullet returns to its point of projection, its net displacement is zero, which is choice (a). The bullet taken 5 s to reach the maximum height. Therefore, initial speed (u) of the bullet is ($\because$ final velocity = 0)

   $u = gt = 10 \times 5 = 50$ ms^{-1} directed upwards which is choice (d). The maximum height (h) attained by the bullet is $h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (5)^2 = 125$ m.

   Therefore, the total distance travelled by the bullet in 10 s = 125 + 125 = 250 m, which is choice (b). For heights h << radius of the earth, the magnitude of g is constant, i.e. the rate of change (c). Thus, all four choices are correct.

3. **a) the velocity with which the missile was projected is $\sqrt{gx}$**

   b) you have a warning time of $\sqrt{\frac{x}{2g}}$

   c) the speed of the missile when it was detected is $\sqrt{\frac{gx}{2}}$

   d) the maximum height attained by the missile is $\frac{x}{4}$.

   For maximum range θ = 45° for which $R_{max} = \upsilon_0^2/g$ Hence $\upsilon_0 = \sqrt{g \times R_{max}} = \sqrt{gx}$ which is choice (a). The warning time is half the time of flight (Since the missile is first detected at half-way point. Hence, warning time is

   $t = \frac{t_f}{2} = \frac{\upsilon_0 \sin\theta}{g} = \frac{\sqrt{gx}\sin 45^0}{g} = \sqrt{\frac{x}{2g}}$

   Hence choice (b) is also correct. At half-way point, the missile is at its maximum height.

Therefore, the vertical component of velocity is zero at this point. Hence the velocity is given only by the horizontal component which is υ_x = $u_0 \cos\theta = \sqrt{gx}\ \cos 45^o = \sqrt{\frac{gx}{2}}$ which is choice (c). The maximum height attained is h_{max} = $\frac{\upsilon_0^2 \sin^2\theta}{2g} = \frac{gx \times \sin^2 45^0}{2g} = \frac{x}{4}$. Thus all the four choices are correct.

4. **b) horizontal range will be 1%**
   c) time of flight will be 0.5%

We know that h = $\frac{\upsilon_0^2 \sin^2\theta}{2g}$. The increase δh in h when υ_0 changes by $\delta\upsilon_0$ can be obtained by partially differenting this expression. Thus

$$\delta h = \frac{2\upsilon_0 \delta\upsilon_0 \sin^2\theta}{2g},\ \frac{\delta h}{h} = \frac{2\delta\upsilon_0}{\upsilon_0}$$

$$\therefore\ \frac{\delta\upsilon_0}{\upsilon_0} = \frac{1}{2}\frac{\delta h}{h} = \frac{1}{2} \times 0.01 = 0.005$$

Now range R = $\frac{\upsilon_0^2 \sin^2\theta}{g}$

$$\therefore\ \frac{\delta R}{R} = \frac{2\delta\upsilon_0}{\upsilon_0} = 2 \times 0.005 = 0.01 = 1\%$$

Time of flight is T = $\frac{2\upsilon_0 \sin\theta}{g}$

$$\therefore\ \frac{\delta T}{T} = \frac{\delta\upsilon_0}{\upsilon_0} = 0.005 = 0.5\%$$

5. b) Their relative acceleration may have any value between 0 and 4m/s²
   c) Both of the frames may be stationary with respect to earth
   d) The frames may be moving with same acceleration in same direction

Acceleration of particle w.r.t. frame S_1 :

$$\vec{a}_p - \vec{a}_{s_1} = 2\hat{n}$$

Acceleration of particle w.r.t. frame S_2 :

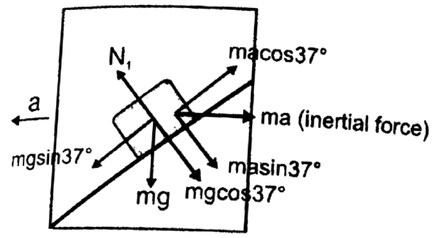
$$\vec{a}_p - \vec{a}_{s_2} = 2\hat{m}$$

where $\hat{m}$ and $\hat{n}$ are unit vectors in any directions. Now the relative acceleration of frames :

$$\vec{a}_{s_2} - \vec{a}_{s_1} = 2(\hat{n} - \hat{m}).$$

Its magnitude can have any value between 0 to 4 m/s², depending upon the directions of $\hat{m}$ and $\hat{n}$.

6. a) $N_3 > N_1 > N_2 > N_4$
   c) $b_2 > b_3 > b_4 > b_1$

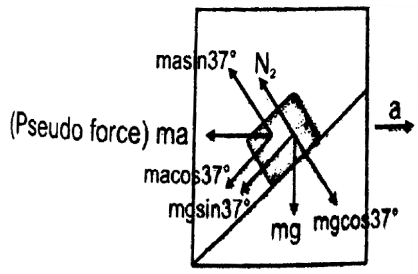
1) 

Balancing forces perpendicular to incline
N_1 = mg cos 37° + ma sin 37°

$$N_1 = \frac{4}{5}mg + \frac{3}{5}ma$$

and along incline mg sin 37° – ma cos 37° = mb_1

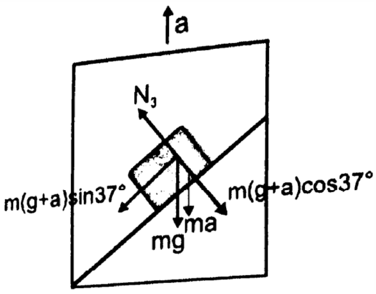
$$b_1 = \frac{3}{5}g - \frac{4}{5}a$$

2) 

Similarly for this case get $N_2 = \frac{4}{5}mg - \frac{3}{5}ma$

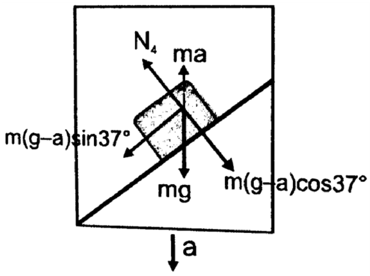
and $b_2 = \frac{3}{5}g + \frac{4}{5}a$

$$N_2 = \frac{4}{5}mg - \frac{3}{5}ma$$

3) 

Similarly for this case get $N_3 = \frac{4}{5}mg - \frac{4}{5}ma$

and $b_3 = \frac{3}{5}g + \frac{3}{5}a$

4) 

Similarly for this case get $N_4 = \frac{4}{5}mg - \frac{4}{5}ma$

and $b_4 = \frac{3}{5}g - \frac{3}{5}a$

7. **a)** The force exerted on block of mass m by the right spring is 6 N to the left.

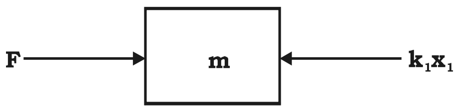
c) The net force on block of mass m is zero

d) The normal force exerted by block of mass m on block of mass M is 6 N.

Let F be the force exerted by mass m on mass M. FBD of mass M

$F = k_1x_1 = 2 \times 3 = 6N$

FBD of mass m



$k_2x_2 = F = 6N$ to the left

Hence the force exerted on block of mass m be the right spring (k_2x_2) is 6 N to the left. From FBD, the normal reaction (F) between blocks is 6 N. As system is at rest, net force on block of mass m is zero.

8. **b)** $N_B = \sqrt{3}N_A$

c) $N_A = \frac{Mg}{2}$

For equilibrium $N_A \cos 60^o + N_B \cos 30^o = mg$ and $Na \sin 60^o = NB \sin 30^o$

On solving $NB = \sqrt{3}$ NA ; $NA = \frac{Mg}{2}$

SECTION II - MATRIX MATCH TYPE

1. **A-P, B-R, C-R, D-R**

Just after spring 2 is cut, the net force acting on the block D changes and it is acting in upward direction and hence D accelerates. As there is no change in the elongation of spring 1, the equilibrium of A and B wouldn't be disturbed. Similarly, we can find reasons for D.

2. **A - S, B - P, C - R, D - Q**

The initial velocity of A relative to B is

$\vec{u}_{AB} = \vec{u}_A - \vec{u}_B = \left(8\hat{i} - 8\hat{j}\right)$

$\therefore$ $u_{AB} = 8\sqrt{2}\ m/s$

Acceleration of A relative to B is -

$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B = \left(-2\hat{i} + 2\hat{j}\right) m/s^2$

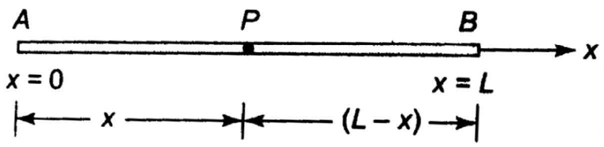
$\therefore$ $a_{AB} = 2\sqrt{2}\ m/s^2$

since B observes initial velocity and constant acceleration of A in opposite directions, Hence B observe A moving along a straight line. from frame of B Hence time when $v_{AB} = 0$ is $t = \frac{u_{AB}}{a_{AB}} = 4$ sec.

SECTION III - INTEGER TYPE

1. **8**

Mass per unit length of the rope is $m = \frac{M}{L}$. Let us find the tension at point P at a distance x from the end x = 0. Let T be the tension in the rope at point P.



For part AP the tension is towards the positive x-direction and for part BP the tension is towards the negative x-direction. If a is the acceleration produced in the rope by the constant force F, then for part AP,

T = (mass of AP) × a or T = mxa ...i

For part BP, we have

F – T = (mass of BP) × a = m(L – x)a ...ii

From (i), we have $a = \frac{T}{mx}$

Using this in (ii), we get

$$F - T = m(L - x) \times \frac{T}{mx} = \frac{(L-x)T}{x}$$

$$\text{or } F = T\left[\frac{(L-x)}{x} + 1\right] = T\frac{L}{x} \text{ or } T = \frac{Fx}{L}$$

$$\text{At } x = L - \frac{L}{5} = \frac{4L}{5},$$

$$T = \frac{F}{L} \times \frac{4L}{5} = \frac{4F}{5} = \frac{4 \times 10}{5} = 8 \text{ N}$$

2. 5

Impulse from t = 4 ms to t = 16 ms = area under the F – t graph = area of EBCD

= area of trapezium EBCF + area of ΔCDF

$$= \frac{1}{2} \times (200 + 800) \text{ N} \times (2 \times 10^{-3} \text{ s}) + \frac{1}{2} \times 800 \text{ N} \times (10 \times 10^{-3} \text{ s})$$

= 1 + 4 = 5 Ns.

3. 5

$$s = u + \frac{a}{2}(2n - 1)$$

u = 100 m/sec, a = –10 m/sec², and s = 5 m

5 = 100 – 5(2n – 1) gives n = 10 sec

When thrown up with velocity 200 m/sec, the body will take 20 sec to reach the highest point. The distance travelled in 20th second is

200 – 5(20 × 2 – 1) = 5m

In the last second of upward journey, the body will travel the same distance.

4. 5

U = 16 m/s (initial), υ = 0, S = 0.4 m

$$\text{Deceleration } a = \frac{\upsilon^2 + u^2}{2s} = \frac{0 - 16^2}{2 \times 0.4} = 320 \text{ m/s}^2$$

$$\text{time} = \frac{\upsilon - u}{a} = \frac{0 - 16}{-320} = 0.05 \text{ sec}$$

5. 5

$$\upsilon_x = \frac{dx}{dt} = 3 \text{ and}$$

$$\upsilon_y = \frac{dx}{dt} = 4 - 10t = 4 - 10(0) = 4$$

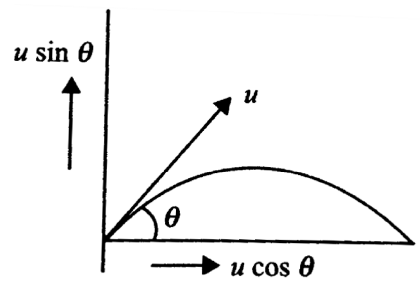
$$\upsilon = \left[\upsilon_x^2 + \upsilon_y^2\right]^{1/2} = [3^2 + 4^2]^{1/2} = 5 \text{ m/sec}$$

6. 4

Initial velocity $u = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

Velocity after time t is

$$\upsilon = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}$$



Since u and υ are perpendicular to each other,

$\vec{u}.\vec{\upsilon} = 0$

$(u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}) \times [u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}] = 0$

$U^2\cos^2\theta + u^2\sin^2\theta - u\sin\theta\, gt = 0$

$$t = \frac{u}{g\sin\theta} \qquad (\because u = 20 \text{ m/sec and } \theta = 30^0)$$

$$t = \frac{20}{10 \times \frac{1}{2}} = 4 \text{ sec}$$

7. 8

Mass m_1 moves with a constant velocity of tension in the lower string

$T_1 = m_1 g = (1)(10) = 10N$ ...(i)

Therefore, tension in the upper string is

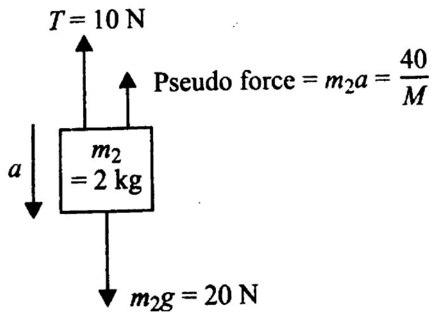
$T_2 = 2T_1 = 20N$ ...(ii)

Therefore, acceleration of block M is

A = er ...(iii)

This is also the acceleration of pulley 2.

Absolute acceleration of mass m_1 is zero. Thus, acceleration of m_1 relative to pulley 2 is upwards or acceleration of m_2 with respect to pulley 2 is downwards. Refer to free body diagram of m_2 with respect ot pulley 2.



Equation of motion gives

$$20 - \frac{40}{M} - 10 = 2a = \frac{40}{M}$$

Solving this, we get M = 8 kg.

8. 8

Let a be the absolute upward acceleration of the monkey and a' be the absolute downward acceleration of the rope. Obviously a' is also the rightward acceleration of M. Then B = a – (–a') (since relative acceleration is the vector difference between the absolute accelerations). or b = a = a'.

Considering the upward motion of the monkey

$T - mg = ma$ ... (i)

Considering the rightward motion of M

$T = M a' = M(b - a)$ ... (ii)

Eliminating a between (i) and (ii), we get T in terms of b.

$$T = \frac{mM}{m+M}(g + b)$$

$$= \left(\frac{m \times 2m}{m + 2m}\right)(10 + 2) = \frac{2m}{3} \times 12 = 8 \text{ N}$$

PART C - MATHS

SECTION I - MULTIPLE ANSWER CORRECT TYPE

1. a) $\frac{5\pi}{24}, \frac{\pi}{24}$

b) $-\frac{7\pi}{24}, \frac{11\pi}{24}$

c) $-\frac{115\pi}{24} - \frac{119\pi}{24}$

We have been given $\frac{1-\tan x}{1+\tan x}$ = tan y

$= \tan\left(x - \frac{\pi}{6}\right)$

$\Rightarrow \tan\left(\frac{\pi}{6} - x\right) = \tan\left(x - \frac{\pi}{6}\right)$

$\therefore \frac{\pi}{4} - x = n\pi + x - \frac{\pi}{6} \quad \therefore \left(\frac{5\pi}{12} - n\pi\right) = 2x$

Or $x = (5 - 12n)\frac{\pi}{24}$ and $y = x - \frac{\pi}{6}$

For n = 0, 1 and 10.

2. c) $\mathbf{n\pi + (-1)^n \frac{\pi}{6}, n \in I}$

d) $\mathbf{n\pi, n \in I}$

3. a) **$f(x)$ is an even function**

b) **$f(x)$ is even if $f(0) = 1$**

c) **$f(x)$ is odd if $f(0) = 0$**

d) **$f(x)$ is even if $f(0) = 0$**

$f(x + y) + f(x - y) = 2f(x).f(y)$

$\Rightarrow f(0 + 0) + f(0 - 0) = 2f(0).f(0)$

$\Rightarrow 2f(0) = 2[f(0)]^2 \Rightarrow f(0) = 1$ or $f(0) = 0$

If $f(0) = 0$, then $f(0 + y) + f(0 - y) = 2f(0).f(y)$

$\Rightarrow f(y) + f(-y) = 0$

$\Rightarrow$ f is an odd function.

Also replacing $y = 0$ in the given expression,

$f(x) = 0, \ \forall \ x \in R$

$\Rightarrow f(x)$ would be odd and even simultaneously.

If $f(0) = 1$, then $f(0 + y) + f(0 - y) = 2f(0).f(y)$

$f(y) + f(-y) = 2f(y)$

$\Rightarrow f$ is an even function.

4. a) **one - one and onto**

lET $h(x) = f(x) - g(x) = \begin{cases} x; & x \in \text{irrational} \\ -x; & x \in \text{rational} \end{cases}$

Þ then function $h(x)$ is one-one and onto.

5. c) $(-\infty, -3) \cup (-2, -1) \cup (2, \infty)$

Case I :

$0 < |x| - 1 < 1$ i.e. $1 < |x| < 2$, then

$x^2 + 4x + 4 \le 1 \Rightarrow x^2 + 4x + 3 \le 0$

$\Rightarrow -3 \le x \le -1$

So $x \in (-2, -1)$

Case II :

$|x| - 1 > 1$ i.e. $|x| > 2$, then

$x^2 + 4x + 4 \ge 1$

$\Rightarrow x^2 + 4x + 3 \ge 0$

$x \ge -1$ or $x \le -3$.

So, $x \in (-\infty, -3] \cup (2, \infty)$.

6. c) **domain of $f = (3, \infty)$**

d) **range of $f = \{0\}$**

$f(x) = \log_{[x-1]} \text{sgn } x$

Domain of $f = [3, \infty)$

Range of $f = \{0\}$

7. a) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ **are in HP**

c) $\mathbf{2dbf = aef + cde}$

8. a) $a^{100} + c^{100} > 2b^{100}$

b) $a^3 + c^3 > 2b^3$

c) $a^5 + c^5 > 2b^5$

d) $a^2 + c^2 > 2b^2$

b is the H.M. of a and c and their G.M. $= \sqrt{ac}$

G.M > H.M $\Rightarrow \sqrt{ac} > b$

And A.M. of an and c^n > G.M. of a^n and cn

$\frac{a^n + c^n}{2} > \sqrt{a^n c^n}$

$a^n + c^n > 2\left(\sqrt{ac}\right)^n > 2b^n$

Put n = 100, 3, 5, 2

SECTION II - MATRIX MATCH TYPE

1. A-P, B-Q, C-Q, D-P

2. A-S, B-R, C-R, D-S

A) $S_p = \frac{P}{2}[2A + (P-1)d] = a$

$\frac{2a}{p} = 2A + (p-1)d$ ... (1)

$\frac{2b}{q} = 2A + (q-1)d$ ... (2)

$\frac{2c}{r} = 2A + (x-1)d$ ... (3)

Multiply (1), (2) and (3) by $(q-r)$, $(r-p)$ and $(p-q)$ and add

$\therefore \sum \frac{a}{p}(q-r) = 0$.

B) Let common ratio be taken as k and a be the first term

$R = T_r = ak^{r-1}$

$R^{s-t} = a^{s-t}\ k^{(r-1)(s-1)}$

Similarly, $S^{t-r} = a^{t-r}\ k^{(s-1)(t-r)}$

$T^{r-s} = a^{r-s}\ k^{(t-1)(r-s)}$

Multiplying the above three and knowing that

$A^m A^n A^p = A^{m+n+p}$

$R^{s-t} S^{t-r} T^{r-s} = a^0 k^0 = 1$.

C) $x = a + (m-1)d = AR^{m-1}$

$y = a + (n-1)d = AR^{n-1}$

$z = a + (p-1)d = AR^{p-1}$

$y - z = (n-p)d$, $z - x = (p-m)d$,

$x - y = (m-n)d$

$x^{y-z}.\ y^{z-x}.\ z^{x-y} = (AR^{m-1})^{(n-p)d}.\ (AR^{n-1})^{(p-m)d}.\ (AR^{p-1})^{(m-n)d}$

$= A^0R^0 = 1$.

D) $a = AR^{p-1}$, $b = AR^{q-1}$, $c = AR^{r-1}$ for G.P.

$a = \frac{1}{A_1 + (p-1)D}$, $b = \frac{1}{A_1 + (q-1)D}$,

$c = \frac{1}{A_1 + (r-1)D}$ for H.P.

$\frac{1}{b} - \frac{1}{c} = (q-r)D$ ... (1)

$\log a = \log A = (p-1)\log R$

$= (\log A - \log R) + (\log R)P$ ... (2)

$\sum\left(\frac{1}{b} - \frac{1}{c}\right)\log a = D(\log A - \log R)$

$\sum(q-r) + D\log R \sum p(q-r) = 0$

$\therefore \sum \frac{c-b}{bc}\log a = 0$

multiplying by – abc

$\sum a(b-c)\log a = 0$

SECTION III - INTEGER TYPE

1. 1

We evaluate each term, $(81)\frac{1}{\log_5 9}$

$= (81)\log_9 5 \left(\log_a b = \frac{1}{\log_b a}\right)$

$= 9^{2\log_9 5} = 9^{2\log_9 25} = 25$ (using fundamental log property)

$3^{\frac{3}{\log_{\sqrt{6}} 3}} = 3^{3\log_3\sqrt{6}} = 3^{\log_3(\sqrt{6})^3}$

$\left(\text{Using } \log_b a = \frac{1}{\log_a b}\right)$

$= (\sqrt{6})^3$ (Using fundamental log property)

Similarly, $\left[(\sqrt{7})^{\frac{2}{\log_{25} 7}} - (125)^{\log_{25} 6}\right]$

$= 7^{\frac{1}{2}\frac{2}{\log_{25} 7}} - 5^{3\log_{25} 6} = 7^{\log_7 25} - 5^{\frac{3}{2}\log_5 6}$

$= 25 - 6\sqrt{6}$

The expression becomes

$\frac{(25+6\sqrt{6})(25-6\sqrt{6})}{409} = \frac{625-216}{409} = 1$

2. **2**

Here $4[1 + \cot^2 \pi(a+x)] + a^2 - 4a = 0$

$\Rightarrow 4\cot^2\pi(a+x) + (a-2)^2 = 0$

$\Rightarrow a - 2 = 0$ and $\cot^2\pi(a+x) = 0 \Rightarrow a = 2$.

3. **3**

Let $a_1, a_2, a_3, a_4, a_5, a_6$ be six arithmetic means

$\Rightarrow 1, a_1, a_2, \ldots\ldots, a_6, 9/2$ will be in A.P.
Now, $9/2 = 1 + 7d \Rightarrow 7/2 = 7d \Rightarrow d = 1/2$

Hence $a_4 = 1 + \left(\frac{1}{2}\right) = 3$.

4. **9**

Using A.M. ≥ G.M., we have

$\frac{1}{3}(a + b + c) \geq (abc)^{1/3}$ and

$\frac{1}{3}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq \left(\frac{1}{abc}\right)^{1/3}$

$\Rightarrow \frac{1}{3}(a + b + c).\ \frac{1}{3}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 1$

$\Rightarrow (a + b + c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 9.$

5. **1**

$2^{\frac{1}{\cos^2 x}}\sqrt{y^2 - y + \frac{1}{2}} \leq 1$

$2^{\frac{1}{\cos^2 x}}\sqrt{\left(y-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} \leq 1$

Minimum value of $2^{\frac{1}{\cos^2 x}} = 2$

Minimum value of $\sqrt{\left(y-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{2}$

$\Rightarrow$ Minimum value of $2^{\frac{1}{\cos^2 x}}\sqrt{y^2 - y + \frac{1}{2}}$ is 1

$\Rightarrow$ (1) is possible when

$$2^{\frac{1}{\cos^2 x}}\sqrt{\left(y-\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = 1$$

$\Rightarrow \cos^2 x = 1$ and $y = 1/2 \Rightarrow \cos x = \pm 1$
$\Rightarrow x = n\pi$, where $n \in 1$.
Hence $x = n\pi$ and $y = 1/2$.

6. **4**

The given equation may be written as
$8^{(1+|\cos x| + |\cos^2 x| + |\cos^3 x| + \ldots + \infty)} = 8^2$

$\Rightarrow 1 + |\cos x| + |\cos^2 x| + |\cos^3 x| + \ldots + \infty = 2$
As we have $-\pi < x < \pi$, $x \neq 0$, we have $|\cos x| < 1$

Hence $\frac{1}{1-|\cos x|} = 2$

$\Rightarrow |\cos x| = \frac{1}{2}$

$\therefore$ Values of x in (– p, p) for which $\cos x = \pm\frac{1}{2}$

are $\pm\frac{\pi}{3}, \frac{2\pi}{3}$

7. **2**

Here A = φ ⇒ P(A) = (φ) = {φ} so that P(P(φ))
= {φ, {φ}}.

8. **7**