LAKSHYA ADVANCED UNIT TEST (LAUT)

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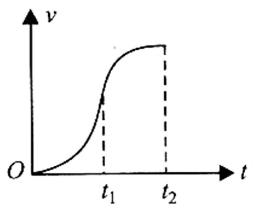
Test No : 2103	3 Hrs.		

Hints & Solutions

PART A - PHYSICS

SECTION I - SINGLE ANSWER CORRECT TYPE

1. a)



From 0 to t_1, acceleration is increasing linearly with time, hence v–t graph should be parabolic upwards. From t_1 to t_2, acceleration is dereasing linearly with time, hence v–t graph should be parabolic downwards.

2. d) $\dfrac{\cos\theta_2}{\cos\theta_1}$

If the particles collide in mid-air,they same displacement in horizontal direction. So their velocity components along horizontal should be same:

3. a) $V_0 = \sqrt{\dfrac{2gH}{5}}$

$0 = u\cos 30^0 - g\sin 30^0 t$

$t = \dfrac{u\cos 30^0}{g\sin 30^0}$

$-H\cos 30^0 = -u\sin 30^0 t - \dfrac{1}{2}g\cos 30^0 t^2$

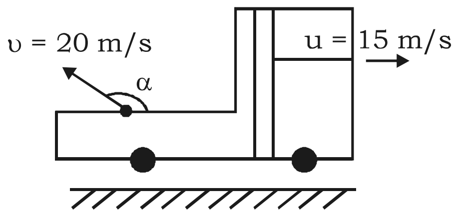
By equation (1) and (2), we get

$H = \dfrac{u^2}{g}\left[1+\dfrac{\cot^2\alpha}{2}\right] u = \sqrt{\dfrac{2gH}{5}}$.

$\{\alpha = 30^0\}$

4. a) $\cos^{-1}\left(-\dfrac{3}{4}\right)$

Let the stone be projected at an angle α to the direction of motion of truck with a speed of v = 20 m/s.



Since the resultant displacement along horizontal is zero, the velocity along horizontal = 0

$15 + 20\cos\alpha = 0 \quad \Rightarrow \cos\alpha = -\dfrac{3}{4}$

$\Rightarrow \alpha = \cos^{-1}\left(-\dfrac{3}{4}\right) = 138^0 35$

5. a) 17400 N

Acceleration at t = 1 s, $a_1 = \dfrac{3.6}{2} = 1.8$ m/s.

6. c) 1 m

Given $t = \sqrt{x} + 3$. Squaring, we have

$x = t^2 - 6t + 9 \qquad \ldots \text{(i)}$

velocity $v = \dfrac{dx}{dt} = \dfrac{d}{dt}(t^2 - 6t + 9)$

$= 2t - 6 \qquad \ldots \text{(ii)}$

Find t from Eq. (ii) when v = 0. Use this value of t is Eq. (i).

7. d) $\mathbf{\frac{p^2}{4m}}$

For maximum horizontal range, $\theta = 45^0$. Also

p = mu which gives u = p/m.

The kinetic energy is minimum when the body is at the highest point of its trajectory. At this point the velocity is

$u_x = u\cos\theta = u\cos 45^0 = \frac{u}{\sqrt{2}}$

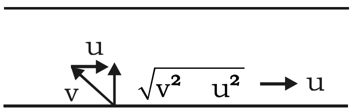
∴ Minimum K.E.

$= \frac{1}{2}m \times \left(\frac{u}{\sqrt{2}}\right)^2 = \frac{(mu)^2}{4m} = \frac{p^2}{4m}$.

8. c) $\mathbf{\frac{t^2+T^2}{t^2-T^2}}$

Let velocity of man in still water be v and that of water with respect to ground be u.

Velocity of man perpendicular to river flow with respect to ground = $\sqrt{v^2-u^2}$



Velocity of man downstream = v + u

As given, $\sqrt{v^2-u^2}\ t = (v+u)\,T$

$\Rightarrow (v^2-u^2)\,t^2 = (v+u)^2\,T^2$

$\Rightarrow (v-u)\,t^2 = (v+u)\,T^2$

$\therefore \frac{v}{u} = \frac{t^2+T^2}{t^2-T^2}$

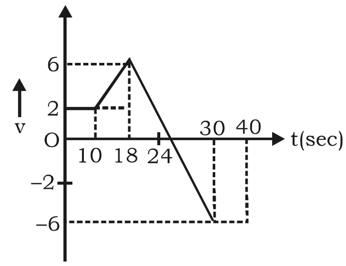
SECTION II - MULTIPLE ANSWER CORRECT TYPE

9. a) the maximum value of the position coodinate of the particle is 54 m

c) the particle is at the position of 36 m at t = 18 s

d) the particle is at the position of 36 m at t = 30 s

Maximum value of position coordinate = initial coordinate + area under the graph up to t = 24 s (as up to t = 24 s the displacement of the particle will be positive (see figure)).



$$\text{Value} = -16 + \left[(2\times10)+\left(\frac{2+6}{2}\right)\times(18-10) + \frac{1\times6}{2}\times(24-18)\right]$$

= – 16 + [20 + 32 + 18] = 54 m

At time t = 18 s

Position = – 16 + Area of graph up to t = 18 s

= – 16 + [20 + 32] = 36 m

At time t = 30 s

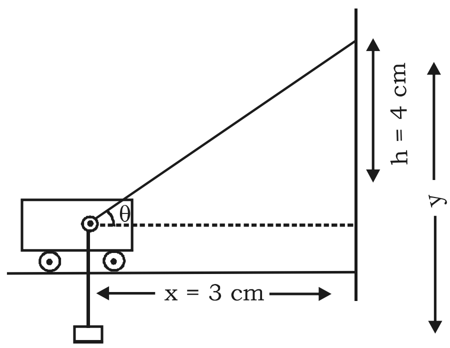
Position = – 16 + Area of graph up to t = 30 s

$= -16 + \left[70 - \frac{1}{2}\times 6\times 6\right] = 36$ m

10. b) $\mathbf{a_B = 0}$

c) $\mathbf{v_B = \frac{3}{5}\,v_A}$

$(y-h) + \sqrt{x^2+h^2} = l$



$$\frac{dy}{dt} + \frac{x}{\sqrt{x^2+h^2}}\frac{dx}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{\sqrt{x^2+h^2}}\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{3}{5}v_A$$

$|u_B| = \frac{3}{5} v_A$

$$\frac{d^2y}{dt^2} = v_A \frac{h^2}{(x^2 + h^2)3/2} \quad \ldots \text{(i)}$$

$$a_B = v_A \frac{16}{(5)^3}$$

$$a_B = \frac{16}{125} v_A \quad \ldots \text{(ii)}$$

11. a) $\mathbf{a_1 \neq a_2 \neq a_3}$

c) $\mathbf{a_1 > a_2 > a_3}$

$$a_1 = \frac{2mg - mg}{m} = g$$

$$a_2 = \frac{mg + mg - mg}{2m} = g/2$$

$$a_3 = \frac{2mg - mg}{3m} = g/3$$

clearly $a_1 > a_2 > a_3$

12. a) $\mathbf{\frac{T}{t} = 2}$

c) $\mathbf{x = 40\sqrt{2}\,m}$

$$t = \frac{2u\sin\theta}{g} = \frac{2 \times 20 \times \sin 30^0}{10} = 2 \text{ s}$$

Initial downward velocity

$= u \sin \theta = 20 \times \sin 30^0 = 10 \text{ ms}^{-1}$.

The time taken to fall through a height of 40 m is given by

$40 = 10t_1 + \frac{1}{2} \times 10 \times t_1^2$ which gives $t_1 = 2$s.

Hence, the total time taken to hit the ground is T = 2 + 2 = 4 s. Therefore T/t = 2. Also, the horizontal distance travelled in 4 s = $(u \cos \theta) \times T = 20 \times \cos 30^0 \times 4$

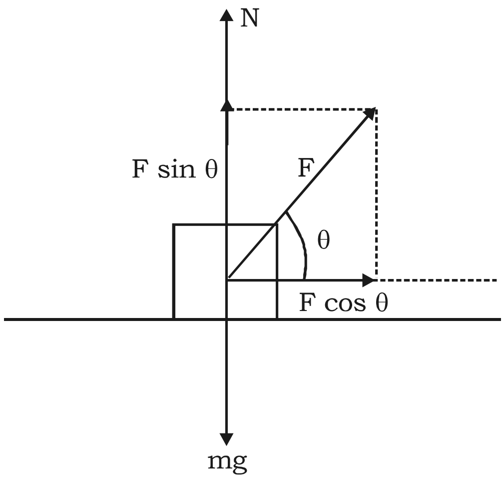
$= 40\sqrt{3}$ m.

Hence the correct choices are (a) and (c).

13. b) the block will move on the surface with a variable acceleration.

c) the block will lose contact with the surface after travelling a distance $\mathbf{x_0 = \frac{mg}{k\sin\theta}}$.

The horizontal and vertical components of F are F cos θ and F sin θ (see Fig. 5.102).



$N + F \sin \theta = mg$

$\Rightarrow \quad N = mg - F \sin \theta$

$= mg - kx \sin \theta$

Where N is the normal reaction.

The block will lose contact with surface at $x = x_0$ for which N = 0. Putting N = 0 and $x = x_0$, we have $0 = mg - kx_0$

$\sin \theta \Rightarrow x_0 = \frac{mg}{k\sin\theta}$

The acceleration a of block along the horizontal surface is given by

$ma = F \cos \theta = kx \cos \theta$

$$\Rightarrow a = \frac{kx\cos\theta}{m}$$

which depends upon x. Hence the correct choices are (b) and (c).

14. a) The force on the 7th coin (counted from the bottom) due to all the coins above it is 3mg vertically downwards.

b) The force on the 7th coin by the 8th coin (both counted from the bottom) is 3mg vertically downwards.

d) The reaction force of the 6th coin from the bottom on the 7th coin from the bottom is 4mg vertically upwards.

The 7^{th} coin from the bottom has 3 coins above it. Hence, the force on the 7^{th} coin = weight of 3 coins = 3 mg, vertically downwards.

Since the 8^{th} coin has 2 coins above, it supports the weight of two coins. Hence the force on the 7^{th} coin by the 8^{th} coin = weight of 8^{th} coin + weight of two coins above it = weight of three coins = 3 mg vertically downwards.

From Newton's third law, the reaction force exerted by the 6^{th} coin on the 7^{th} coin is equal and opposite to the action force exerted by the 7^{th} coin on the 6^{th} coin. Now, the force exerted by the 7^{th} coin on the 6^{th} coin = weight of 7^{th} coin + weight of 3 coins above it = weight of 4 coins = 4 mg vertically downwards. Hence, the reaction of the 6^{th} coin on the 7^{th} coin = 4 mg vertically upwards.

Hence the correct choices are (a), (b) and (d).

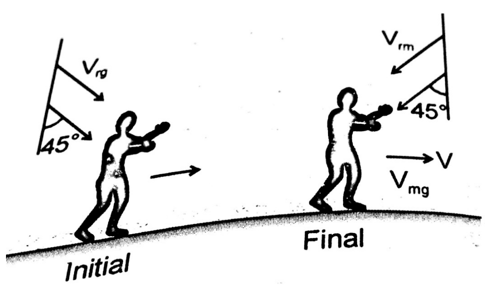
15. c) Speed of the man when he finds rain to be fallling at angle 45° with the vertical, is 4m/s.

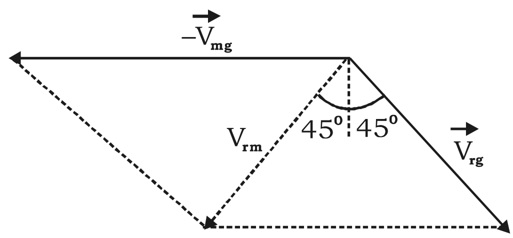
d) The man has travelled a distance 16m on the road by the time he again finds rain to be falling at angle 45°.

$\vec{V}_{rg} = \vec{V}_{rm} + \vec{V}_{mg}$

$\vec{V}_{rm} = \vec{V}_{rg} - \vec{V}_{mg}$

$V_{rm} \cos 45^o = V_{rg} \cos 45^o$





$V_{rm} = 2\sqrt{2}$ m/s = V_{rg}

$V_{rm} \cos 45^o = V_{mg} - V_{rg} \cos 45^o$

$V_{mg} = 2\sqrt{2}\frac{1}{\sqrt{2}} + 2\sqrt{2}\frac{1}{\sqrt{2}} = 4$ m/s

using $v^2 = u^2 + 2as$ for the motion of man, s = 16 m.

16. a) If m ≤ 1 he can not reach a point on other bank directly opposite to his starting point

d) He can reach the other bank on other point, whatever be the value of m.

He can only reach the opposite point if he can cancel up the velocity of river by his component of velocity.

Paragraph - 1

17. d) $\frac{900}{11}$N

18. d) 120/77 m/s²

19. c) 180/77 m/s²

From the constraint relations we can see that

$3T X_B = 2T X_A$

$X_A = \frac{3}{2} X_B \quad \Rightarrow \quad a_A = \frac{3}{2} a_B$

So let a_B = A then $a_A = 1.5a$

Writing equation of motion:

From block A,

$2T = 70a_A = 105_a = 3 \times 35a$

$35a = \frac{2}{3}T$ ... (i)

From block B.

$300 - 3T = 35a_B = 35a$ ... (ii)

Solving eqs. (i) and (ii) we get

$300 - 3T = \frac{2T}{3}$

$\Rightarrow$ 900 – 9T = 2T $\Rightarrow$ 900 = 11T

$T = \frac{900}{11} N$

$a_A = \frac{180}{77}$ m/s² and $a_B = \frac{120}{77}$ m/s²

Paragraph - 2

20. c) $\frac{3}{4}mg$

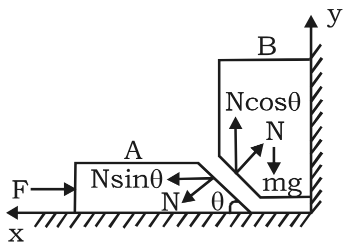
For equilibrium of block (A)

F = N sin θ

N = F/sin θ

To lift block B from ground

N cos θ ≤ mg



$\frac{F}{\sin\theta} \cos\theta \geq mg$

$F \geq mg \tan\theta = mg\left(\frac{3}{4}\right)$

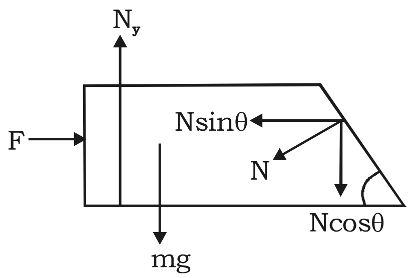
So $F_{min} = \frac{3}{4}$ mg

21. d) $\mathbf{mg - \frac{4F}{3}}$

If both the blocks are stationary,
Balancing forces along x-direction

F = N sin θ $\Rightarrow$ N = F/sin θ

Balancing forces along y-direction



$N_y = mg + N\cos\theta = mg + \left(\frac{F}{\sin\theta}\right)$

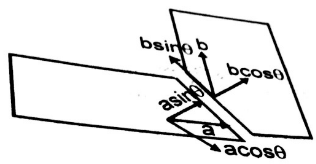
cos θ = mg + F cot θ

$N_y = mg + \frac{4F}{3}$

22. a) $\frac{3a}{4}$ **upwards**

To keep regular contact

a sin θ = b cos θ



$b = a\tan\theta = \frac{3}{4}a$