XI - MATHS – PAPER II
SOLUTIONS

SECTION I (Multiple Answer Correct)

1. **a)** $2n\pi$

 b) $2n\pi + \frac{\pi}{2}$

$(\sin^3\theta + \cos^3\theta) - (1 - \sin\theta \cdot \cos\theta) = 0$

$\Rightarrow (\sin\theta + \cos\theta)(1 - \sin\theta \cdot \cos\theta) - (1 - \sin\theta \cdot \cos\theta) = 0$

$\Rightarrow (1 - \sin\theta \cdot \cos\theta)(\sin\theta + \cos\theta - 1) = 0$

If $\sin\theta \cdot \cos\theta = 1 \Rightarrow \sin 2\theta = 2$ ... (Not possible)

$\Rightarrow \sin\theta + \cos\theta = 1$

$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$

$\Rightarrow \theta - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$. $(n \in z)$

$\Rightarrow \theta = 2n\pi$ or $2n\pi + \frac{\pi}{2}$

2. **a)** $f\left(\frac{\pi}{2}\right) = 1$

 b) $f(\pi) = 0$

 c) $f\left(\frac{-3\pi}{2}\right) = 1$

 d) $f\left(\frac{\pi}{4}\right) = \frac{1-\sqrt{2}}{\sqrt{2}}$

$[\pi^2] = [9.8596] = 9$

$\therefore f(x) = \sin 9x + \sin(-10x)$

$= \sin 9x - \sin 10x$

$f\left(\frac{\pi}{2}\right) = \sin\left(\frac{9\pi}{2}\right) - \sin(5\pi) = 1$

$f(\pi) = \sin 9\pi - \sin 10\pi = 0$

$f\left(-\frac{3\pi}{2}\right) = \sin\left(-\frac{27\pi}{2}\right) - \sin(-15\pi)$

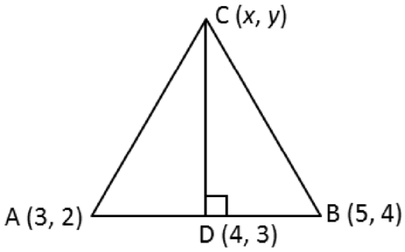
$= 1$

$f\left(\frac{\pi}{4}\right) = \sin\left(\frac{9\pi}{4}\right) - \sin\left(\frac{5\pi}{2}\right)$

$= \frac{1}{\sqrt{2}} - 1 = \frac{1-\sqrt{2}}{\sqrt{2}}$

3. **a)** $\left(4-\sqrt{3}, 3+\sqrt{3}\right)$

 b) $\left(4+\sqrt{3}, 3-\sqrt{3}\right)$



$AB = \sqrt{(5-3)^2 + (4-2)^2}$

$= 2\sqrt{2}$

$\therefore CD = \frac{\sqrt{3}}{2}\left(2\sqrt{2}\right)$

$= \sqrt{6}$

$m_{AB} = 1$

$m_{CD} = -1$

$\therefore \tan\theta = -1 \Rightarrow \sin\theta = \frac{1}{\sqrt{2}}$ and

$\cos\theta = -\frac{1}{\sqrt{2}}$

Point c is $\sqrt{6}$ units from point D

$x - 4 = \pm\frac{\sqrt{6}}{\sqrt{2}} = \pm\sqrt{3}$

$\therefore x = 4 \pm \sqrt{3}$

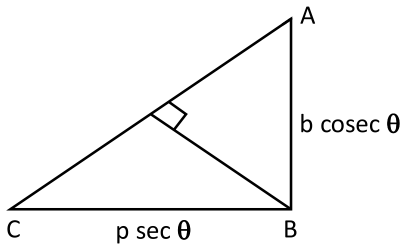
$y - 3 = \pm\frac{\sqrt{6}}{\left(-\sqrt{2}\right)} = \mp\sqrt{3}$

$\therefore y = 3 \mp \sqrt{3}$

$\therefore \left(4+\sqrt{3}, 3-\sqrt{3}\right)$ and $\left(4-\sqrt{3}, 3+\sqrt{3}\right)$

4. **b)** $\frac{\pi}{8}$

 c) $\frac{3\pi}{8}$



$p^2 \sec^2\theta + p^2 \operatorname{cosec}^2\theta = \left(2\sqrt{2}\right)^2 p^2$

$\Rightarrow \frac{1}{\sin^2\theta . \cos^2\theta} = 8$

$\Rightarrow \sin^2\theta = \frac{1}{2} \Rightarrow (\sin 2\theta)^2 = \left(\frac{1}{\sqrt{2}}\right)^2$

$\Rightarrow 2\theta = n\pi + \frac{\pi}{4}, n \in z$

$\Rightarrow \theta = \frac{2\pi}{2} + \frac{\pi}{8}$

for $n = 0, \theta = \frac{\pi}{8}$

for $n = 1, \theta = \frac{3\pi}{8}$

5. **b)** –5/3

Let m_1 and m_2 are slopes of altitudes through B and C respectively.

$m_{AC} = \frac{3-2}{2+3} = \frac{1}{5}$

$m_{AB} = \frac{3+3}{2-4} = -3$

$m_1.m_{AC} = -1$

$\therefore m_1 = -\frac{1}{\left(\frac{1}{5}\right)} = -5$

$m_2.m_{AB} = -1$

$\therefore m_2 = \frac{-1}{-3} = \frac{1}{3}$

$\therefore m_1.m_2 = \frac{-5}{3}$

6. **a)** 3, 5, 7

b) 15, 5, –5

$a - d + a + a + d = 15 \Rightarrow a = 5$

$(a - d + 1), (a + 3), (a + d + 9)$ are in G.P.

$\therefore (a + 3)^2 = (a - d + 1)(a + d + 9)$

$8^2 = (5 - d + 1)(5 + d + 9)$

$64 = (6 - d)(14 - d)$

$d^2 + 8d - 20 = 0$

$d = -10, 2$

$\therefore$ 15, 5, –5 and 3, 5, 7 are those numbers.

7. **b)** $\frac{1}{7}$

c) 7

$\frac{\log_7 x}{\log_7 a} \cdot \frac{\log_7 a}{\log_7 7} = \frac{\log_7 7}{\log_7 x}$

$\log_7 x = \frac{1}{\log_7 x}$

$\therefore (\log_7 x)^2 = 1$

$\therefore \log_7 x = 1$ or $\log_7 x = -1$

$\therefore x = 7$ or $x = \frac{1}{7}$

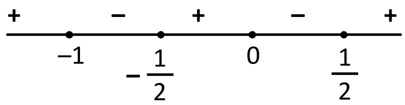
8. **a)** $\left(-\infty, \frac{-3}{2}\right)$

d) $\left(\frac{1}{2}, 3\right)$

$S = \frac{2x-1}{x(2x^2+3x+1)} = \frac{2x-1}{(2x+1)(x+1)x}$

for s to be positive

$\frac{2x-1}{(2x+1)(x+1)x} > 0$



$\Rightarrow x \in (-\infty, -1) \cup \left(-\frac{1}{2}, 0\right) \cup \left(\frac{1}{2}, \infty\right)$

SECTION II - Paragraph Type

9. **a)** $\pm 5\sqrt{5}$

10. **d)** 6

Adding equations.

$x(\cos\theta + \sin\theta)^3 = 27$

$\Rightarrow x^{1/3}(\cos\theta + \sin\theta) = 3$ ....(i)

Subtracting equations

$x(\cos\theta - \sin\theta)^3 = 1$

$\Rightarrow x^{1/3}(\cos\theta - \sin\theta) = 1$ ....(ii)

Dividing equations (i) and (ii)

$\cos\theta + \sin\theta = 3\cos\theta - 3\sin\theta$

$\Rightarrow \tan\theta = \frac{1}{2}$

Case I :

$\sin\theta = \frac{1}{\sqrt{5}}$ and $\cos\theta = \frac{2}{\sqrt{5}}$

$\theta = 2n\pi + \alpha$, where $0 < \alpha < \frac{\pi}{2}$ and

$\sin\alpha = \frac{1}{\sqrt{5}}$

i.e. θ lies in first quadrant

$\therefore x^{1/3}\left(\frac{3}{\sqrt{5}}\right) = 3 \Rightarrow x = 5\sqrt{5}$

Case II :

$\sin\theta = -\frac{1}{\sqrt{5}}$, $\cos\theta = -\frac{2}{\sqrt{5}}$

$\theta = 2n\pi + (\pi + \alpha)$, where $0 < \alpha < \frac{\pi}{2}$ and

$\sin\alpha = -\frac{1}{\sqrt{5}}$

i.e. θ lies in 3rd quadrant

$\therefore x^{1/3}\left(-\frac{3}{\sqrt{5}}\right) = 3 \Rightarrow x = -5\sqrt{5}$

11. **c)** 0

12. **d)** 2

Let the four integers be $a - d$, a, $a + d$, $a + 2d$ where a and d are integers and $d > 0$, Now

$a + 2d = (a-d)^2 + a^2 + (a+d)^2$

$\Rightarrow 2d^2 - 2d + 3a^2 - a = 0$

$\therefore\ d = \frac{1}{2}\left[1 \pm \sqrt{1+2a-6a^2}\right]$

Since d is a positive integer, so

$1 + 2a - 6a^2 > 0$

$\Rightarrow \frac{1-\sqrt{7}}{6} < a < \frac{1+\sqrt{7}}{6} \Rightarrow a = 0$

$\therefore\ d = 1$ or 0

But since $d > 0 \Rightarrow d = 1$

$\therefore$ Numbers are $-1, 0, 1, 2$.

SECTION III - Integer Type

1. **1** $(\sin x)^3 + p^3 + 1 = 3p \sin x$

$\Rightarrow \sin x + p + 1 = 0$ or $\sin x = p = 1$

$p = -(1 + \sin x)$

Hence, only one value of p ($b > 0$) is possible which is given by $p = 1$

2. **7** $2 < \frac{4x - 66 + 64 - x}{10}$

$\therefore\ 2 < \frac{3x}{10}$

$\therefore\ x > \frac{20}{3}$

$\therefore$ Least integer = 7

3. **1** $x = 1 \times 10^0 + 1 \times 10^1 + 1 \times 10^2 + 1 \times 10^3 + \ldots + 1 \times 10^{19}$

$= 1 + 10 + 10^2 + \ldots + 10^{19}$

$= \frac{10^{20} - 1}{9}$

$z = 2(10^0 + 10^1 + 10^2 + \ldots + 10^{19})$

$= \frac{2\left(10^{10} - 1\right)}{9}$

$y = \frac{3\left(10^{10} - 1\right)}{9} = \frac{10^{10} - 1}{3}$

$\therefore\ \frac{x - y^2}{z} = \frac{\frac{10^{20} - 1}{9} - \frac{1020 - 2.10^{10} + 1}{9}}{\frac{2}{9}\left(10^{10} - 1\right)}$

$= \frac{2 \times 10^{10} - 2}{2\left(10^{10} - 1\right)} = 1$

4. **3** $(0.625)^{4-3x} \geq (1.6)^{x(x+8)}$

$\Rightarrow \left(\frac{5}{8}\right)^{4-3x} \geq \left(\frac{5}{8}\right)^{x(x+8)}$

$\Rightarrow 3x - 4 \geq x^2 + 8x$

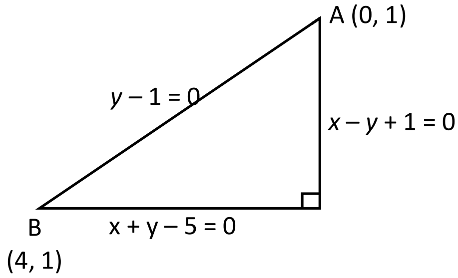
$\Rightarrow x^2 + 5x + 4 \leq 0$

$\Rightarrow -4 \leq x \leq -1$

$\therefore$ domain is $[-4, -1]$

$\therefore\ b - a = -1 - (-4) = 3$

5. **4**



Since the triangle is right angled, so the circumcentre will be the mid point of hypotenuse (2, 1)

$\therefore\ a = 2, b = 1 \Rightarrow a + 2b = 4$

6. **2** Let $t = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots$ and

$S = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots$

$S - t = \left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\right) - \left(1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\right)$

$= \frac{2}{2^2} + \frac{2}{4^2} + \frac{2}{6^2} + \ldots$

$= \frac{2}{2^2}\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\right)$

$S - t = \frac{S}{2}$

$\therefore\ \frac{S}{2} = t \Rightarrow \frac{S}{t} = 2$

7. **2** $\text{Log}_5[\log_3(18x - x^2 - 77)] > 0$ and

$18x - x^2 - 77 > 0$

$\Rightarrow \log_3(18x - x^2 - 77) > 1$ and

$x^2 - 18x + 77 < 0$

$\Rightarrow 18x - x^2 - 77 > 3$ and $(x - 11)(x - 7) < 0$

$\Rightarrow x^2 - 18x + 80 < 0$ and $7 < x < 11$

$\Rightarrow (x - 8)(x - 10) < 0$ and $7 < x < 11$

$\Rightarrow 8 < x < 10$ and $7 < x < 11$

$\Rightarrow x \in (8, 10)$

$\therefore\ a = 8$ and $b = 10 \Rightarrow b - a = 2$

8. **4** Since, $-2 \leq \sin x - \sqrt{3} \cos x \leq 2$

$\Rightarrow -2 \leq \frac{4m - 6}{4 - m} \leq 2$

Or $-1 \leq \frac{2m - 3}{4 - m} \leq 1$

If $\frac{2m - 3}{4 - m} \leq 1 \Rightarrow \frac{2m - 3 - (4 - m)}{4 - m} \leq 0$

$\Rightarrow \frac{3m - 7}{m - 4} \geq 0$

Also $-1 \le \frac{2m-3}{4-m} \Rightarrow \frac{m+1}{m-4} \le 0$

$\therefore \; m \in \left[-1, \frac{7}{3}\right]$

$\therefore$ Possible integers are –1, 0, 1, 2