MAHESH TUTORIALS SCIENCE

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Hints & Solutions

PART A - PHYSICS

1. **d)** $\mathbf{6x^2 (4x^3 - 5)^{-1/2}}$

$\frac{d}{dx}(4x^3 - 5)^{1/2}$

apply chain rule

$= \frac{1}{2}(4x^3 - 5)^{-1/2} \cdot \frac{d}{dx}(4x^3 - 5)$

$= 6x^2 (4x^3 - 5)^{-1/2}$

2. **c)** $\mathbf{3x^2 \cos x^3}$

$\frac{d}{dx}(\sin x^3) = \cos x^3 \cdot \frac{d}{dx} x^3$

$= 3x^2 \cos x^3$

3. **d)** $\mathbf{\frac{-y}{x}}$

$xy = c^2$

apply uv rule

$x\frac{dy}{dx} + y\frac{dx}{dx} = 0$

$\frac{dy}{dx} = -\frac{y}{x}$

4. **a)** **2**

$\int_0^{\pi/2} (\sin x + \cos x) dx$

$= \int \sin x \, dx + \int \cos x \, dx$

$= -\cos x\Big|_0^{\pi/2} + \sin x\Big|_0^{\pi/2}$

5. **b)** $\mathbf{\frac{(\ln x)^2}{2} + c}$

$I = \int \frac{\ln x}{x} dx$

Let $\ln x = t$

$\frac{1}{x} dx = dt$

$I = \int t \, dt$

$= \frac{t^2}{2} + c$

$= \frac{(\ln x)^2}{2} + c$

6. **b)** $\mathbf{\tan x - \cot x + c}$

$I = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$

$= \int \sec^2 x \, dx + \int \operatorname{cosec}^2 x \, dx$

$= \tan x - \cot x + c$

7. **a)** **900 $\mathbf{cm^3/sec}$**

$v = x^3$

$\frac{dv}{dt} = 3x^2 \frac{dx}{dt}$

$= 3 \times (10)^2 \times 3$

$= 900 \text{ cm}^3/\text{sec}$

8. **c)** $\mathbf{\frac{1}{31}\left(5\hat{i} + 6\hat{j} - 30\hat{k}\right)}$

for perpendicular $\vec{A} \cdot \vec{B} = 0$

and $|\vec{B}| = 1$

So, $\frac{1}{31}\left(5\hat{i} + 6\hat{j} - 30\hat{k}\right)$

9. **b)** $\mathbf{\vec{A}}$ **and** $\mathbf{-\vec{A}}$ **are collinear**

Collinear vector have same line of vector

So, $\theta = 0$ or $\theta = 180^0$

So, $\vec{A}$ and $-\vec{A}$ are collinear

10. **c)** $\frac{\mathbf{10}}{\mathbf{7}}\hat{\mathbf{i}}-\frac{\mathbf{15}}{\mathbf{7}}\hat{\mathbf{j}}+\frac{\mathbf{30}}{\mathbf{7}}\hat{\mathbf{k}}$

$\vec{A}=3\hat{i}-2\hat{j}-2\hat{k}$

$\vec{B}=3\hat{i}+2\hat{j}$

So $\vec{C}=\vec{A}+\vec{B}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & -2 \\ 3 & 2 & 0 \end{vmatrix}$$

$= 4\hat{i}-6\hat{j}+12\hat{k}$

this is perpendicular to both $\vec{A}$ and $\vec{B}$
So required vector

$$= 5\left(\frac{4\hat{i}-6\hat{j}+12\hat{k}}{\sqrt{16+36+144}}\right)$$

$$= \frac{10}{7}\hat{i}-\frac{15}{7}\hat{j}+\frac{30}{7}\hat{k}$$

11. **a)** $\frac{\mathbf{9}}{\mathbf{29}}\left(\mathbf{2\hat{i}-3\hat{j}-4\hat{k}}\right)$

$\vec{A}=4\hat{i}-3\hat{j}+2\hat{k}$

$\vec{B}=-2\hat{i}+3\hat{j}+4\hat{k}$

Projection of $\vec{A}$ on $\vec{B}$

$$= \frac{\vec{A}.\vec{B}}{(B)^2}\vec{B}$$

$$= \frac{-8-9+8}{(4+9+16)}\left(-2\hat{i}+3\hat{j}+4\hat{k}\right)$$

$$= \frac{9}{29}\left(2\hat{i}-3\hat{j}-4\hat{k}\right)$$

12. **a)** **m = 4, n = –4**

$\vec{A}=4\hat{i}-3\hat{j}+4\hat{k}$

$\vec{B}=n\hat{i}+3\hat{j}-m\hat{k}$

For $\vec{A}\,||\,\vec{B}$

$$\frac{4}{n}=-\frac{3}{3}=-\frac{4}{m}$$

So $n=-4$

$m=4$

13. **a)** $\vec{\mathbf{A}}.\left(\vec{\mathbf{B}}+\vec{\mathbf{A}}\right)$

For Dot product $\vec{A}.\vec{B}$

$\vec{A}+\vec{B}$ is vector and $\vec{A}$ is vector so $\vec{A}.\left(\vec{A}+\vec{B}\right)$ is meaningful

14. **c)** $\frac{\mathbf{3\pi}}{\mathbf{3}}$

$\vec{A}=-12\hat{i}+9\hat{j}-6\hat{k}$

$\vec{B}=4\hat{i}-3\hat{j}+2\hat{k}$

$$\cos\theta = \frac{\vec{A}.\vec{B}}{\left(\vec{A}\right)\left|\vec{B}\right|}$$

$$= \frac{-48-27-12}{\sqrt{144+81+36}\sqrt{16+9+4}}$$

$\cos\theta = -1$

$\theta = \pi$

15. **d)** **9**

$\vec{A}=p\hat{i}+9\hat{j}-p\hat{k}$

$\vec{B}=-18\hat{i}+9\hat{j}-p\hat{k}$

for $\vec{A}\perp\vec{B}\Rightarrow\vec{A}.\vec{B}=0$

$-18p+81+p^2=0$

$p=\pm 9$

16. **c)** **Scalar product is anti-commutative**

Scalar product is commutative

$\vec{A}.\vec{B}=\vec{B}.\vec{A}$

17. **d)** **20**

$\left|\vec{A}\right|=3$

$\left|\vec{B}\right|=5$

So $\vec{A}.\vec{B}\in[-15, 15]$

18. **d)** $\frac{\mathbf{2\pi}}{\mathbf{3}}$

$\left|\vec{A}\right|=\left|\vec{B}\right|=a$

$$\left|\vec{A}+\vec{B}\right|=\sqrt{a^2+a^2+2a^2\cos\theta}=1 \quad \dots \text{(i)}$$

$$\left|\vec{A}-\vec{B}\right|=\sqrt{a^2+a^2-2a^2(\cos\theta)}=\sqrt{3} \quad \dots \text{(ii)}$$

Divide (i) ÷ (ii)

$$\frac{1}{3}=\frac{1+\cos\theta}{1-\cos\theta}$$

$3(1 + \cos\theta) = 1 - \cos\theta$

$\cos\theta = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}$

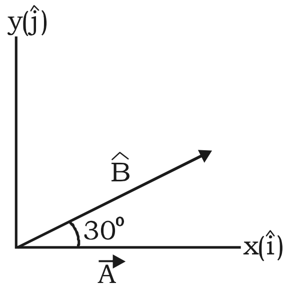
19. **b)** $\mathbf{6\hat{k}}$

$\vec{A} = 4\hat{i}$

$\vec{B} = \frac{3\sqrt{3}}{2}\hat{i} + \frac{3}{2}\hat{j}$

$\vec{A} \times \vec{B} = 6\hat{k}$

$\therefore$ $\vec{A} = 4\hat{i}$



20. **a)** $|\vec{\mathbf{A}}| = |\vec{\mathbf{B}}|$

$\vec{C}$ and $\vec{D}$ are perpendicular to each other

Then $\vec{C}.\vec{D} = 0$

$(\vec{A} + \vec{B}).(\vec{A} - \vec{B}) = 0$

$A^2 - \vec{A}.\vec{B} + \vec{B}.\vec{A} - B^2 = 0$

$A^2 = B^2$

$(\because \vec{A}.\vec{B} = \vec{B}.\vec{A})$

$\therefore$ $|\vec{A}| = |\vec{B}|$

21. **a)** $\mathbf{\frac{1}{2}\cot\frac{x}{2}}$

$y = \log\left(\sin\frac{x}{2}\right),$

$\frac{dy}{dx} = \left(\frac{1}{\sin\frac{x}{2}}\right)\left(\cos\frac{x}{2}\right)\frac{1}{2} = \frac{1}{2}\cot\frac{x}{2}$

22. **b)** $\mathbf{\frac{\pi}{4}}$

$\int_0^{\pi/2} \sin^2 x\,dx = \frac{1}{2}\int_0^{\pi/2}(1 - \cos 2x)\,dx$

$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi/2}$

$= \frac{\pi}{4}$

23. **c)** $\mathbf{\frac{1}{2}}$

$|\vec{a_1} + \vec{a_2}|^2 = 3,$

$a_1^2 + a_2^2 + 2\vec{a_1} \bullet \vec{a_2} = 3$

$1 + 1 + 2\,\vec{a_1} \bullet \vec{a_2} = 3,$

$\vec{a_1} \bullet \vec{a_2} = 1/2$

$(\vec{a_1} - \vec{a_2}) \bullet (2\vec{a_1} + \vec{a_2}) = 2a_1^2 - a_2^2 - \vec{a_1} \bullet \vec{a_2}$

$= 2 - 1 - \frac{1}{2} = \frac{1}{2}$

24. **b)** **5, 13**

Let $\vec{A}$ and $\vec{B}$ be the two forces with $|\vec{A}| < |\vec{B}|$. Given $|\vec{A}| + |\vec{B}| = 18$. The resultant force $\vec{R}$ has a magnitude R given by (here **θ** is the angle between vectors A and B)

$R = (A^2 + B^2 + 2AB\cos\theta)^{\frac{1}{2}} = 12$(given)

or

$A^2 + B^2 + 2AB\cos\theta = 144$ ...(i)

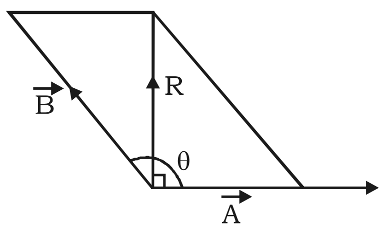
The angle **α** subtended by the resultant vector R with force A is given by

$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$

Given $\alpha = 90^\circ$. Therefore,

$\tan 90^\circ = \frac{B\sin\theta}{A + B\cos\theta}$

Since $\tan 90^\circ = \infty$, we have $A + B\cos\theta = 0 \Rightarrow \cos\theta = -\frac{A}{B}$



Now $A + B = 18$ .... (ii)

For equation (i),

$$A^2 + B^2 - 2AB\left(\frac{A}{B}\right) = 144$$

$A^2 - B^2 = (A + B)(A - B) = 144$

$A - B = 8$ .... (iii)

From (ii) and (iii), A = 13.

Therefore B = 18 – A = 18 – 13 = 5

Thus the magnitudes of the two forces are 13 units and 5 units.

25. **c)** **3**

$\vec{F_1} + \vec{F_2} + \vec{F_3}$

$= \left(p\hat{i} + 3\hat{j} - \hat{k}\right) + \left(-5\hat{i} + 2\hat{j}\right) + \left(6\hat{i} - \hat{k}\right)$

$= (p+1)\hat{i} + 3\hat{j}$,

$\left|\vec{F_1} + \vec{F_2} + \vec{F_3}\right| = \sqrt{(p+1)^2 + 9}$,

$(p + 1)^2 + 9 = 25$

$(p + 1)^2 = 16$, $p + 1 = \pm 4$,

p = 3 or –5

26. **a)** $\mathbf{10\left(\hat{i} + \hat{j} + \hat{k}\right)}$

$\vec{A} = \vec{A_x}\hat{i} + \vec{A_y}\hat{j} + \vec{A_z}\hat{k} = p\left(\hat{i} + \hat{j} + \hat{k}\right)$

$A_x = A_y = A_z = p$,

$3p^2 = \left(10\sqrt{3}\right)^2 \Rightarrow p = 10$

$\Rightarrow A = 10\left(\hat{i} + \hat{j} + \hat{k}\right)$

27. **c)** π

$\vec{P} \times \vec{Q} = -\vec{Q} \times \vec{P}$. Hence the angle between $\vec{P} \times \vec{Q}$ and $\vec{Q} \times \vec{P}$ is π.

28. **b)** $\vec{Q}$ **is a null vector**

Given $\vec{P} + \vec{Q} = \vec{P} - \vec{Q}$

$\therefore \vec{Q} = -\vec{Q}$

$\Rightarrow 2\vec{Q} = 0 \text{ or } \vec{Q} = 0$

That is, $\vec{Q}$ is the negative of itself.

This is possible only when $\vec{Q} = 0$

29. **d)** **none of these**

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

Hence the value of α does not turn out to be $\frac{\theta}{2}$.

30. **c)** **The magnitude of the vector is doubled and its direction is reversed**

PART B - CHEMISTRY

31. **c)** **kinetic energy of photo-electrons**

$KE_{max} = h\upsilon - h\upsilon_0$

$\upsilon\uparrow \quad KE_{max}\uparrow$

32. **c)** $\mathbf{\dfrac{27}{5}}$

$$E_{n_1 z} = -13.6 \times \frac{z^2}{n^2}$$

$$\underset{\substack{\downarrow \\ \text{Higher} \\ \text{orbit}}}{E_{n_2}} - \underset{\substack{\downarrow \\ \text{lower} \\ \text{orbit}}}{E_{n_1}} = 13.6 \times z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{E_2 - E_1}{E_3 - E_2} = \frac{13.6 \times z^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)}{13.6 \times z^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right)}$$

$$= \frac{\frac{3}{4}}{\frac{5}{36}} = \frac{3}{4} \times \frac{36}{5}$$

$$= \frac{27}{5}$$

33. **d)** **Zero**

Visible region means any higher orbit to 2nd orbit.

34. **a)** **0**

no. of angular modes = l

for 35, $l = 0$

35. **d)** **s**

spin quantum no. is derived from spectral evidence

36. **c)** $\mathbf{3 \to 1}$

$$\begin{array}{ll} 5 \; \underline{\qquad\quad} & \\ & \big\} \; \Delta E = 0.306 \text{ eV} \\ 4 \; \underline{\qquad\quad} & \\ & \big\} \; \Delta E = 0.65 \text{ eV} \\ 3 \; \underline{\qquad\quad} & \\ & \big\} \; \Delta E = 1.5 \text{ eV} \\ 2 \; \underline{\qquad\quad} & \\ & \big\} \; \Delta E = 10.2 \text{ eV} \\ 1 \; \underline{\qquad\quad} & \end{array}$$

or

$$\begin{array}{ll} \underline{\qquad\qquad} & E_5 = -0.544 \\ \underline{\qquad\qquad} & E_4 = -0.85 \\ \underline{\qquad\qquad} & E_3 = -1.5 \text{ ev} \\ \underline{\qquad\qquad} & E_2 = -3.4 \text{ ev} \\ \underline{\qquad\qquad} & E_1 = -13.6 \text{ ev} \end{array}$$

Energy different b/w 2 consecutive orbits decreases with increase in orbit no.

$\Delta E_{5\to3} = (0.306 + 0.65)$ ev.

$\Delta E_{4\to1} = (0.65 + 1.9 + 10.2)$ ev

$\Delta E_{3\to1} = (1.9 + 10.2)$ev

$\Delta E_{5\to4} = 0.306$ ev

37. **a)** $\mathbf{E_1 > E_3 > E_2}$

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mkE}}$$

$$\Rightarrow KE = \frac{h^2}{\lambda^2 \times 2m}$$

$$\Rightarrow KE \propto \frac{1}{m}$$

as h is constant and λ is same

38. **b)** $\boxed{\uparrow} \quad \boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}$

Fact

after complete filling of s–subshell, then only p–subshell can be filled in same shell.

39. **a)** **[Ar] $4s^0 3d^1$**

$\mu_m = \sqrt{n(n+2)} \Rightarrow 1.73 = \sqrt{n(n+2)}$

$\Rightarrow n = 1$ one unpaired electron.

40. **a)** **s > p > d > f**

fact

41. **b)** **1.54 Å**

$$\text{covalent radius} = \frac{\text{Bond length}}{2}$$

42. **a) acidic, basic**

$$\mathrm{M_1 - O - H}$$
$$\underbrace{3.4 \quad 3.5}_{\Delta EN\,=\,0.1} \quad \underbrace{3.5 \quad 2.1}_{1.4}$$

ΔEN → difference in electronegativity

O – H is more polar as difference in EN is more
Hence acidic

$$\mathrm{M_1 - O - H}$$
$$\underbrace{1.2 \quad 3.5}_{2.3} \quad \underbrace{3.5 \quad 2.1}_{1.4}$$

M_2 – O is more polar, as difference in EN is more
Hence Basic

43. **c) zero**
fact

44. **b) Be^-**
EC of Be → $1s^2 2s^2$
Be^- → unstable because Be is stable due to full filled s– subshell
Li^+ → Stable due to duplet
C^- → stable due to formation of half filled p–subshell.

45. **d) 10^{11} kg/cc**
radius of a nucleus of mass no A
$= 1.3 \times 10^{-15} \times A^{1/3}$ m
$= 1.3 \times 10^{-13} \times A^{1/3}$ cm
mass of a nucleus = A u
$= A \times 1.66 \times 10^{-24}$ gm
$= A \times 1.66 \times 10^{-27}$ kg
density of nucleus

$$= \frac{A \times 1.66 \times 10^{-27}\,\text{kg}}{\left(1.3 \times 10^{-13} \times A^{\frac{1}{3}}\right)^3 \text{cm}^3} \approx 10^{11}\ \text{kg/cm}^3$$

46. **a) 2.0×10^{-20} J**

$$\text{KE per atom} = \frac{4.4 \times 10^{-19} - 4 \times 10^{-19}}{2}$$
$= 0.2 \times 10^{-19}$ J
$= 2 \times 10^{-20}$ J

47. **c) $E_n = -\dfrac{1313.3}{n^2}\,\text{kJ mol}^{-1}$**

$$E_{n,H} = -13.6 \times \frac{1}{n^2}\ \text{eV}$$

Now, 1 ev/ particle ≈ 96.4 KJ/mol

$$\therefore \quad E_{n,H} = -13.6 \times \frac{1}{n^2} \times 96.4\ \text{KJ/mol}$$
$$\approx -\frac{1311}{n^2}\,\text{kJ mol}^{-1}$$

48. **a) $n_4 \rightarrow n_1$**
min wavelength means max energy
$$E = \frac{hc}{\lambda}$$
hence, $n_4 \rightarrow n_1$

49. **b) r/3**

$$r_{n_1 z} = \frac{n^2}{z} \times r_{H,1st\,orbit}$$
$$= \frac{1}{3} \times r$$
$$= \frac{r}{3}$$

50. **c) $\dfrac{1}{2m}\sqrt{\dfrac{h}{\pi}}$**

$$\Delta x \,.\, \Delta p = \frac{h}{4\pi} \Rightarrow (\Delta x)^2$$
$$= (\Delta p)^2 = \frac{h}{4\pi}$$
$$\Delta p = \Delta x = \sqrt{\frac{h}{4\pi}}$$
$$\Delta p = \Delta(m\,v) = m\,\Delta v = \sqrt{\frac{h}{4\pi}}$$
$$\Rightarrow \Delta v = \frac{1}{2m}\sqrt{\frac{h}{\pi}}$$
(uncertainty in velocity)

51. **d) n^2**
no of orbitals in energy level (orbit no. n) = n^2

52. **d) s-orbital**
for s–orbital, $l = 0$
$m = 0$

53. **b)** **Energy released when an electron is added to an isolated atom in the gaseous state**
Fact/Definition

54. **c)** **$S^{2-} > Cl^{-} > K^{+} > Ca^{2+}$**
For an isoeletronic species, with $\uparrow$ in atomic no (z), atomic radius $\downarrow$

55. **c)** **C < N > O**
N has half filled electronic configuration. Hence stable
$\therefore$ difficult to remove the electron

56. **c)** **Zr and Hf have about the same radius**
Fact

57. **a)** **–5.1 eV**
$Na \rightarrow Na^{+} + e$ 5.1 eV $\rightarrow$ IE
$Na^{+} + e \rightarrow Na$ –5.1 eV
(Electron gain enthalpy)

58. **c)** **HF > HCl > HBr > HI**
thermal stability $\propto \dfrac{1}{\text{size of atom}}$
(down the group)

59. **c)** **$NH_3 < PH_3 < AsH_3 < SbH_3$: Increasing acid strength**
acidic strength $\propto$ size of atom attached to H-atom. (down the group)

60. **a)** **s-and p-block**
Fact

PART C - MATHS

61. **b)** $-\dfrac{4}{5}$

$\theta \in$ IV Q

$$\frac{5\tan(\pi+\theta)+4\cos(\pi+\theta)}{5\sec(2\pi-\theta)-4\cot(2\pi+\theta)}$$

$$\Rightarrow \frac{5(\tan\theta)-4\cos\theta}{5\sec\theta-4\cot\theta}$$

$$= \frac{5\left(-\frac{4}{3}\right)-4\left(\frac{3}{5}\right)}{5\left(\frac{5}{3}\right)-4\left(\frac{-3}{5}\right)} = \frac{-4}{5}$$

62. **c)** **0**

$$\frac{\left(\sqrt{3}+2\sin A\right)^3}{(1-2\cos A)^3}+\frac{(1+2\cos A)^3}{\left(\sqrt{3}-2\sin A\right)^3}$$

$$\Rightarrow \frac{\left(3-4\sin^2 A\right)^3+\left(1-4\cos^2 A\right)^3}{(1-2\cos A)^3\left(\sqrt{3}-2\sin A\right)^3}$$

$$\Rightarrow \frac{\left(3-4\sin^2 A\right)^3-\left(3-4\sin^2 A\right)^3}{(1-2\cos A)^3\left(\sqrt{3}-2\sin A\right)^3}$$

$\Rightarrow 0$

63. **b)** **II**

$\operatorname{cosec}\theta - \cot\theta = 5$

$\operatorname{cosec}\theta + \cot\theta = \dfrac{1}{5}$

$\Rightarrow \operatorname{cosec}\theta = \dfrac{13}{5}$ $\cot\theta = \dfrac{-12}{5}$

$\theta =$ II Q

64. **b)** $-a^2 + b^2 + c^2$

$(a\tan\theta + b\sec\theta)^2$

$= a^2\tan^2\theta + b^2\sec^2\theta + 2ab\sec\theta\tan\theta$

$\Rightarrow a^2\sec^2\theta - a^2 + b^2 + b^2\tan^2\theta$
$+ 2ab\sec\theta + \tan\theta$

$\Rightarrow (a\sec\theta + \text{b}\tan\theta)^2 - a^2 + b^2$

$\Rightarrow c^2 - a^2 + b^2$

65. **a)** $-2\operatorname{cosec}\theta$

$$\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}+\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$$

$$\Rightarrow \left|\frac{1-\cos\theta}{\sin\theta}\right|+\left|\frac{1+\cos\theta}{\sin\theta}\right|$$

$\Rightarrow \cot\theta - \operatorname{cosec}\theta - \cot\theta - \operatorname{cosec}\theta$

$\theta \in$ III Q

$\Rightarrow -2\operatorname{cosec}\theta$

66. **c)** **16mn**

$(m^2 - n^2)^2 = (4\sin\theta + \tan\theta)^2$

$= 16\sin^2\tan^2\theta$

$mn = \tan^2\theta - \sin^2\theta$

$mn = \tan^2\theta\sin^2\theta$

$(m^2 - n^2)^2 = 16mn$

67. **b)** **– 1**

$21\theta = \pi$

$$\frac{\sin 23\theta - \sin 7\theta}{\sin 2\theta + \sin 14\theta} = \frac{-\sin 2\theta - \sin 14\theta}{\sin 2\theta + \sin 14\theta}$$

-1

68. **a)** $x > 0$

$\cos 10^0 > \sin 10^0$

$\Rightarrow x > 0$

69. **a)** **1**

$\alpha + \beta = 90^0$, $\alpha - \beta = 30^0$

$\Rightarrow \alpha = 60^0$ $B = 30^0$

$\tan(\alpha + 2\beta)\tan(2\alpha + \beta)$

$\Rightarrow \tan(120^0)\tan(150^0)$

$\Rightarrow 1$

70. **b)** $1-\dfrac{1}{2}\left(a^2-1\right)^2$

$\sin\theta + \cos\theta = a$

$1 + 2\sin\theta\cos\theta = a^2$

$\sin\theta\cos\theta = \dfrac{a^2-1}{2}$

$\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$

$$= 1-\frac{\left(a^2-1\right)^2}{2}$$

71. **c)** **4**

$$\frac{\sin x+\cos x}{\cos^3 x}$$

$= a\tan^3 x + b\tan^2 x + c\tan x + d$

take $x = 45^0$

$a + b + c + d = 4$

72. **a)** **acute angled**

cot A cot B cot C > 0

$\Rightarrow$ cot A > 0, cot B > 0, cot C > 0

As two obtuse angles cant exist in triangle Δ is acute.

73. **b)** **–1**

sin 120 cos 150 + cos 120 sin 150

$\Rightarrow \sin(120^0 + 150^0)$

$\Rightarrow \sin 270^0 = -1$

74. **d)** **7**

$$\tan\left(\frac{\pi}{4}+\theta\right)+\tan\left(\frac{\pi}{4}-\theta\right)=3$$

$$\tan^2\left(\frac{\pi}{4}+\theta\right)+\tan^2\left(\frac{\pi}{4}-\theta\right)+2=9$$

$$\tan^2\left(\frac{\pi}{4}+\theta\right)+\tan^2\left(\frac{\pi}{4}-\theta\right)=7$$

75. **a)** **4**

$$\frac{\cos 10^0-\sqrt{3}\sin 10^0}{\sin 10^0\cos 10^0}$$

$$\Rightarrow \frac{2\left(\cos 60\cos 10-\sin 60\sin 10^0\right)}{\sin 10\ \cos 10}$$

$$\Rightarrow \frac{4\cos 70}{\sin 20}$$

$\Rightarrow 4$

76. **d)** $\mathbf{2\cos\frac{\pi}{32}}$

$$\sqrt{2+\sqrt{2+\sqrt{2+2\cos\frac{\pi}{4}}}}$$

$$\Rightarrow 2\cos\frac{\pi}{32}$$

77. **d)** **4**

$$\frac{\sqrt{3}\cos 20-\sin 20}{\cos 20\ \sin 20}$$

$$\Rightarrow \frac{2\left(\sin 60\cos 20-\cos 60\sin 20\right)}{\cos 20\ \sin 20}$$

$$\Rightarrow \frac{4\sin 40}{\sin 40}$$

$= 4$

78. **a)** $\mathbf{\frac{-627}{725}}$

$$\tan\left(\frac{\alpha}{2}\right)+\tan\left(\frac{\beta}{2}\right)=\frac{26}{8}$$

$$\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{15}{8}$$

$$\tan\left(\frac{\alpha+\beta}{2}\right)=\frac{-26}{7}$$

$$\cos\left(\frac{\alpha+\beta}{2}\right)=\frac{1-\left(\frac{26}{7}\right)^2}{1+\left(\frac{26}{7}\right)^2}=\frac{-627}{-725}$$

79. **b)** **tan6θ**

$$\frac{\sin 3\theta+\sin 9\theta+\sin 5\theta+\sin 7\theta}{\cos 3\theta+\cos 9\theta+\cos 7\theta+\cos 5\theta}$$

$$\Rightarrow \frac{2\sin 6\theta\left(\cos 3\theta+\cos\theta\right)}{2\sin 6\theta\left(\cos 3\theta+\cos\theta\right)}$$

$\Rightarrow \tan 6\theta$

80. **c)** **11**

$$32\sin\frac{A}{2}\sin\frac{5A}{2}$$

$\Rightarrow 16(\cos 2A - \cos 3A)$

$\Rightarrow 16(2\cos^2 A - 1 - 4\cos^3 A + 3\cos A)$

81. **a)** **0**

$$2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$=\sqrt{3}(2)\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$$

$\Rightarrow \cot\left(\frac{A-B}{2}\right) = \sqrt{3}$

$\Rightarrow A - B = 60^0$

$\Rightarrow A = B + 60^0$

$3A = 3B + 180^0$

$\sin 3A + \sin 3B = \sin(180 + 3B) + \sin 3B = 0$

82. **a)** $\boldsymbol{\frac{x}{y}}$

$\frac{\sin A}{\cos A} = \frac{x \sin B}{1 - x\cos B}$

$\sin A - x \sin A \cos B = x \sin B \cos A$

$\sin A = x \sin(A + B)$

$\frac{\sin A}{\sin B} = \frac{x}{y}$

83. **c)** **2**

$80 - 10 = 70$

$\frac{\tan 80 - \tan 10}{1 + \tan 80 \tan 10} = \tan 70$

$\frac{\tan 80 - \tan 10}{\tan 70} = 2$

84. **a)** $\boldsymbol{\frac{56}{33}}$

$\cos(\alpha + \beta) = \frac{4}{5} \Rightarrow \tan(\alpha + \beta) = \frac{3}{4}$

$\sin(\alpha + \beta) = \frac{5}{13} \Rightarrow \tan(\alpha + \beta) = \frac{5}{12}$

$\tan(2\alpha) = \frac{\tan(\alpha+\beta) + \tan(\alpha+\beta)}{1 - \tan(\alpha+\beta)\tan(\alpha+\beta)}$

$\Rightarrow \frac{56}{33}$

85. **c)** **–1**

$\cos \alpha + \cos \beta = 0$

$\sin \alpha + \sin \beta = 0$

squaring and adding

$2 + 2\cos(\alpha - \beta) = 0$

$\cos(\alpha - \beta) = -1$

86. **a)** **1**

$A + B + C = \frac{\pi}{2}$

$\Rightarrow \tan A \tan B + \tan B \tan C + \tan C \tan A = 1$

87. **a)** $\boldsymbol{a/b}$

$\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{a+b}{a-b}$

componendo and dividendo

$\frac{\sin(\alpha+\beta) + \sin(\alpha+\beta)}{\sin(\alpha-\beta) - \sin(\alpha+\beta)} = \frac{2a}{2b}$

$\frac{\tan\alpha}{\tan\beta} = \frac{a}{b}$

88. **b)** **1**

$57 - 12 = 45$

$\frac{\tan 57 - \tan 12}{1 + \tan 57 \tan 12} = 1$

$\tan 57 - \tan 12 - \tan 57 \tan 12 = 1$

89. **b)** $\boldsymbol{\frac{2a}{a-1}}$

$\frac{2a}{a-1} = \frac{2\frac{\tan 3A}{\tan A}}{\frac{\tan 3A}{\tan A} - 1}$

$= \frac{2\tan 3A}{\tan 3A - \tan A}$

$\Rightarrow \frac{2\sin 3A \cos A}{\sin 2A}$

$\Rightarrow \frac{\sin 3A}{\sin A}$

90. **a)** $\boldsymbol{\frac{-3}{\sqrt{130}}}$

$\frac{\pi}{2} < \frac{\alpha-\beta}{2} < \frac{3\pi}{2}$

squaring and adding

$2\cos(\alpha - \beta) = \frac{(21)^2 + (27)^2}{65^2} - 2$

$\cos(\alpha - \beta) = -\frac{56}{65}$

$\Rightarrow \cos\left(\frac{\alpha-\beta}{2}\right) = \frac{-3}{\sqrt{130}}$