PART A - PHYSICS

1. **a)** $\left(\frac{\Delta a + \Delta b}{a - b}\right) \times 100\%$

$x = a - b, \Delta x = \Delta a + \Delta b$

$\therefore \quad \frac{\Delta x}{x} \times 100 = \left(\frac{\Delta a + \Delta b}{a - b}\right) \times 100\%$

2. **b)** **0.18 ± 0.02 cm**

Thickness, $t = \frac{D_0 - D_i}{2} = \frac{4.23 - 3.87}{2}$

$= 0.18$ cm

$\Delta t = \Delta D_0 + \Delta D_i = 0.01 + 0.01 = 0.02$ cm

3. **d)** $\frac{\Delta f}{f} = \frac{\Delta u}{u} + \frac{\Delta v}{v} + \frac{\Delta u}{u + v} + \frac{\Delta v}{u + v}$

$f = \frac{uv}{u + v}$

let, $w = u + v$

$\therefore \quad \Delta w = \Delta u + \Delta v$

so, $\frac{\Delta f}{f} = \frac{\Delta u}{u} + \frac{\Delta v}{v} + \frac{\Delta w}{w}$

$\frac{\Delta f}{f} = \frac{\Delta u}{u} + \frac{\Delta v}{v} + \frac{\Delta u + \Delta v}{u + v}$

$\left(\frac{\Delta f}{f} = \frac{\Delta u}{u} + \frac{\Delta v}{v} + \frac{\Delta u}{u + v} + \frac{\Delta v}{u + v}\right)$

4. **c)** $\mathbf{0.04 \times 10^{-1}}$

$0.82 - 0.816 = 0.004 = 0.04 \times 10^{-1}$

5. **a)** $\mathbf{1.7 \times 10^{-6}\ m^3}$

$v = l^3 =$

$(1.2 \times 10^{-2})^3 = 1.728 \times 10^{-6} \Rightarrow 1.7 \times 10^{-6}\ m^3$

↓ ↓ ↓

2 SF 4 SF 2 SF

6. **a)** **(10 ± 0.01)m**

$l = \sqrt{A} = A^{1/2} = \sqrt{100} = 10$ m

$\frac{\Delta l}{l} = \frac{1}{2}\frac{\Delta A}{A}$ or $\Delta l = \frac{\Delta A}{2A}.l = \frac{0.2}{2 \times 100} \times 10$

$= 0.01$ m

$l = (10 \pm 0.01)$ m

7. **a)** **4 ± 16.25 %**

$R = \frac{V}{I} = \frac{8}{2} = 4\ \Omega$

$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$

$100 \times \frac{\Delta R}{R} = 100 \times \left(\frac{\Delta V}{V} + \frac{\Delta I}{I}\right)$

$= \left(\frac{0.5}{8} + \frac{0.5}{2}\right) \times 100 = 16.25\%$

$R = (4 \pm 16.25)\%$

8. **b)** $\mathbf{4.7 \times 10^{-5}}$

0.38×10^{-5}

4.32×10^{-5}

4.70×10^{-5}

$\therefore \quad 4.7 \times 10^{-5}$

9. **d)** **2 cos θ – sin θ**

$\frac{d}{d\theta}(\cos\theta + 2\sin\theta) = -\sin\theta + 2\cos\theta = 2\cos\theta - \sin\theta$

10. **c)** **2 cos 2θ**

$\frac{d}{d\theta}(\sin 2\theta) = ?$

Let, $t = 2\theta$, $\frac{dt}{d\theta} = 2$

Now, $y = \sin 2\theta = \sin t$

$\frac{dy}{dt} = \cos t$

$\frac{dy}{d\theta} = \frac{dy}{dt} \times \frac{dt}{d\theta} = \cos t.(2) = 2\cos 2\theta$

11. **d)** $\mathbf{e^x (x^5 + 5x^4)}$

$\frac{dy}{dx} = e^x . 5x^4 + x^5 e^x = e^x (x^5 + 5x^4)$

12. **b)** $\mathbf{-4(4x + 5)^{-2}}$

$\frac{dy}{dx} = \frac{(4x+5)(0) - 1(4)}{(4x+5)^2} = \frac{-4}{(4x+5)^2}$

$= -(4x + 5)^{-2}$

13. **a)** $\mathbf{8.63 \times 10^2}$

$15.83 \times 11.06 \times 4.93 = 8.63 \times 10^2$

(4SF) (4SF) (3SF) (3SF)

14. **b)** $\mathbf{1.46 \times 10^{-1}}$

$$\frac{(3SF) \rightarrow 0.363}{(4SF) \rightarrow 2.487} = \underset{(3SF)}{1.46 \times 10^{-1}}$$

15. **d)** $$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l} + \frac{1}{2}\frac{\Delta g}{g}$$

16. **d)** **4%**

$$\rho = \frac{M}{\pi R^2 l}$$

$$\frac{\Delta\rho}{\rho} \times 100$$

$$= \frac{\Delta M}{M} \times 100 + 2\frac{\Delta R}{R} \times 100 + \frac{\Delta l}{l} \times 100$$

$$= \frac{0.003}{0.3} \times 100 + 2 \times \frac{0.005}{0.5} \times 100 + \frac{0.06}{6} \times 100$$

$$\frac{\Delta\rho}{\rho} \times 100 = 4\%$$

17. **a)** **1 + log x**

$$\frac{dt}{dx} = x\left(\frac{1}{x}\right) + \log x\,(1) = 1 + \log x$$

18. **c)** $\mathbf{\cos^2\theta}$

$$y = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\frac{dy}{d\theta} = \sec^2\theta \qquad \text{but } \frac{d\theta}{dy} = \frac{1}{\sec^2\theta} = \cos^2\theta$$

19. **d)** **none of these**

$$\frac{dy}{dx} = x\,(\cos x) + \sin x(1) = x\cos x + \sin x$$

20. **b)** $\mathbf{(\alpha a + \beta b + \gamma c)\%}$

21. **c)** $\mathbf{5x^4}$

$$y = e^{5\ln x} = e^{5\ln x^5} = x^5$$

$$\frac{dy}{dx} = 5x^4$$

22. **c)** $\mathbf{\sec^2\theta}$

$p = \tan\theta + 1$

$$\frac{dp}{d\theta} = \sec^2\theta + 0 = \sec^2\theta$$

23. **c)** $\mathbf{2\pi r}$

$$\frac{dv}{dr} = \pi\,(2r)\,h = 2\pi rh$$

24. **c)** $\mathbf{-x^{-2}}$

$$\frac{d}{dx}(x^{-1}) = -x^{-1-1} = -x^{-2}$$

25. **d)** **– 4 sin x cos x**

$y = \cos 2x - \sin 2x = \cos 2x$

Let $\theta = 2x$ $\quad d\theta/dx = 2$

$y = \cos\theta$

$$\frac{dy}{d\theta} = -\sin\theta \quad \text{or} \quad \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

$$= (-\sin\theta)\,2 = -2\sin 2x$$

$\therefore$ $dy/dx = -2(2\sin x \cos x) = -4\sin x \cos x$

26. **d)** **± 13%**

$$\frac{\Delta x}{x} \times 100 = \left(\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + 3\frac{\Delta c}{c}\right) \times 100$$

$$= \pm 13\%$$

27. **d)** **none of these**

$$\frac{dy}{dx} = 5\left(\frac{1}{x}\right) = \frac{5}{x}$$

28. **a)** **17**

$$v = \frac{ds}{dt} = 3t^2 + 2t + 1$$

$\Rightarrow$ $v = 3(2)^2 + 2(2) + 1 = 17$

29. **b)** **2**

$$\frac{ds}{dt} = 2t + 2$$

$$a = \frac{d^2s}{dt^2} = 2$$

30. **c)** **31**

$$\frac{dz}{dt} = t^2 + t + 1 = 5^2 + 5 + 1 = 31$$

MAHESH TUTORIALS SCIENCE

00 – 00		Q. Booklet Serial No: 070615	
Test No : 1102	3 Hrs.		Q. Booklet Version :

Hints & Solutions

PART C - MATHS

61. **c)** **52**

$$\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$$

$= 14 + 2 = 16$

$\Rightarrow$ $x+\frac{1}{x} = 4$

$$\left(x^3+\frac{1}{x^3}\right) = \left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)$$

$= 4(14 - 1) = 52$

62. **a)** **41 : 17**

$x = 7k$, $y = 5k$

$\Rightarrow$ $3x + 4y : x + 2y = 41:17$

63. **d)** **2**

$(x^2 + y^2) = (x + y)^2 - 2xy = 4 - 2 = 2$

$x^4 + y^4 = (x^2 + y^2)^2 - 2x^2y^2 = 4 - 2 = 2$

64. **b)** **4**

$x^2 + 1$ is always positive and $(x - 2)^2$ is also greater than zero if x not equal to 2 so only $x - 3 > 0 => x > 3$ so $x = 4$

65. **b)** **2**

Let $|x| = t$. Then $t^2 + t - 6 = 0$

$\Leftrightarrow (t + 3)(t - 2) = 0$

$\Rightarrow$ $t = -3, 2$ ($t = -3$ rejected) $t = 2 \Rightarrow x = \pm 2$

66. **a)** $x \in \left(-\frac{3}{2}, \frac{1}{4}\right)$

$9x^2 + 6x + 1 \ge 0 \Leftrightarrow (3x + 1)^2 \ge 0$ (true)

L.H.S. > 0

$\Rightarrow$ R. H.S must be positive $\Leftrightarrow 2 - x > 0$

$\Leftrightarrow x < 2$

$x < 2$ defines feasible region.

Squaring given inequation*,

$9x^2 + 6x + 1 < 4 + x^2 - 4x$

$\Leftrightarrow 8x^2 + 10x - 3 < 0 \Leftrightarrow (4x - 1)(2x + 3) < 0$

$\Leftrightarrow \frac{-3}{2} < x < \frac{1}{4}$

(* the step of taking squaring on both sides is valid only for $x < 2$.)

67. **a)** $\mathbf{4x^2 - 12x - 7 = 0}$

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$\Rightarrow$ $9 = 16 - 4\alpha\beta$

$\Rightarrow$ $\alpha\beta = -7/4$

68. **b)** **0**

Given equation (on expansion) is a cubic equation which has three roots. Out of those, one root is $x = 1$. Other two roots can be obtained from $x^2 + x + 1 = 0$, sum of which is -1,

$\therefore$ Total sum $= 1 - 1 = 0$

69. **a)** **18**

Let α, β, γ be the roots of $x^3 + Px^2 + Qx - 19 = 0$ & α', β', γ' be the roots of $x^3 - Ax^2 + Bx - C = 0$

Given, $\alpha = \alpha' + 1$, $\beta = \beta' + 1$, $\gamma = \gamma' + 1$

Replace x by $x + 1$ in first equation to get

$(x + 1)^3 + P(x + 1)^2 + Q(x + 1) - 19 = 0$

$\Leftrightarrow x^3 + (3 + P)x^2 + (3 + 2P + Q)x + 1 + P + Q - 19 = 0$

Comparing this equation with second (given) equation,

$-A = 3+P$, $B = 3+2P + Q$, $-C = P+Q -19 + 1$

$\Rightarrow$ $-A - B - C = -18$

$\Rightarrow$ $A + B + C = 18$

70. **c)** **5, 4**

$x^2 - 3x + 2 = (x - 1)(x - 2)$.

Let $P(x) = x^4 - px^2 + q$

Using factor theorem, $f(1)=f(2)=0$

$\Leftrightarrow$ $1 - p + q = 0$ & $16 - 4p + q = 0$.

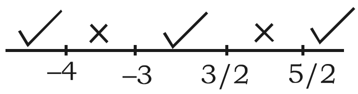
Solving, we get $p = 5$, $q = 4$

71. **d)** $\left(-\infty,-4\right)\cup\left(-3,\frac{3}{2}\right)\cup\left(\frac{5}{2},\infty\right)$

Given inequation $\Leftrightarrow \frac{8x^2+16x-51}{(2x-3)(x+4)} - 3 > 0$

$\Leftrightarrow \frac{8x^2+16x-51-3(2x^2+5x-12)}{(2x-3)(x+4)} > 0$

$\Leftrightarrow \frac{2x^2+x-15}{(2x-3)(x+4)} > 0 \Leftrightarrow \frac{(2x-5)(x+3)}{(2x-3)(x+4)} > 0$



72. **d)** ***p* = 2**

Let α, β be the roots. Given, $\alpha\beta = 1$

$\Leftrightarrow \frac{2}{p} = 1 \Leftrightarrow p = 2$

73. **a)** $-\frac{2}{3}$

Let α, β be the roots. Given, $\alpha + \beta = \alpha\beta$

$\Leftrightarrow \frac{-2}{a} = \frac{3a}{a} \Leftrightarrow a = \frac{-2}{3}$

74. **c)** **1 – *c***

Let $\alpha + 1 = t \Rightarrow \alpha = t - 1$

α satisfies $x^2 - px - (p + c) = 0$

$\Rightarrow \alpha^2 - p\alpha - (p + c) = 0$

$\Rightarrow (t - 1)^2 - p(t - 1) - (p + c) = 0$

$\Leftrightarrow t^2 - (2 + p)\,t + 1 - c = 0.$

(A quadratic equation having roots $\alpha + 1$, $\beta + 1$)

$(\alpha + 1)(\beta + 1) = 1 - c$

75. **b)** $x^2 - x + 3 = 0$

Let $t = \alpha^2 - 4\alpha + 7 = (\alpha^2 - 3\alpha + 5) - \alpha + 2$

∴ α satisfies $x^2 - 3x + 5 = 0$,

∴ $t = -\alpha + 2$

$\Rightarrow \alpha = 2 - t.$

∴ $(2 - t)^2 - 3(2 - t) + 5 = 0$

$\Leftrightarrow t^2 - t + 3 = 0$

(A quadratic equation having roots $-\alpha + 2$, $-\beta + 2$ i.e. $\alpha^2 - 4\alpha + 7$, $\beta^2 - 4\beta + 7$)

76. **a)** $x^2 - 4x + 1 = 0$

If one root of a quadratic equation with rational coefficients is $a + \sqrt{b}$ then other root is $a - \sqrt{b}$. Other root is $2 - \sqrt{3}$.

∴ Product of the roots = 4 – 3 = 1

sum of the roots = $2 + \sqrt{3} + 2 - \sqrt{3} = 4$

Required equation $x^2 - 4x + 1 = 0$

77. **c)** **0**

Subtracting, $2(c - b)\,x + b - c = 0$

$\Rightarrow x = \frac{1}{2}$. Substituting $x = \frac{1}{2}$, we get

$\frac{a}{4} + b + c = 0 \Leftrightarrow a + 4b + 4c = 0$

78. **b)** **zero**

R.H.S must be non-negative. (as L.H.S. is non negative)

$\Leftrightarrow x \ge -\frac{1}{2}$. (feasible region)

squaring given equation,

$x^2 - 6x + 9 = 4x^2 + 4x + 1$

$\Leftrightarrow 3x^2 + 10x - 8 = 0 \Leftrightarrow (3x - 2)\,(x + 4) = 0$

$\Leftrightarrow x = \frac{2}{3}, -4$

($x = -4$ rejected) also $x = \frac{2}{3}$ is not an integer so no integer solution

79. **d)** **distinct and imaginary**

Discriminant $\Delta = 4(a + b)^2 - 8(a^2 + b^2)$

$= -4(a - b)^2 < 0$

80. **d)** **1**

Let α, β be the roots.

$\alpha^2 + \beta^2 = (a - 2)^2 + 2(a + 1) = a^2 - 2a + 6 = (a - 1)^2 + 5 \ge 5$

$\alpha^2 + \beta^2$ takes it's least value (= 5) only when $a = 1$.

81. **c)** **More than two**

By observation $x = a$, $x = b$, $x = c$ satisfy the given equation which is quadratic in nature.

$\Rightarrow$ Given equation is an idenitity as quadratic with more than two roots.

82. **c)** **7**
Discriminant = 25 – 4 k < 0
(for imaginary roots)
$\Leftrightarrow k > \frac{25}{4}$

83. **a)** $\frac{9}{2}$
Let α, β be the roots.
$\alpha - \beta = 3$ (Given)
$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$ 3
$\Leftrightarrow 9 = \left(\frac{k-4}{k-2}\right)^2 + \frac{8}{k-2}$
$\Leftrightarrow (k-4)^2 + 8(k-2) - 9(k-2)^2 = 0$
$\Leftrightarrow 2k^2 - 9k + 9 = 0 \Leftrightarrow (2k-3)(k-3) = 0$
$\Leftrightarrow k = 3, \frac{3}{2}$

84. **b)** **16**
Roots equal so D = 0
$K^2 - 4(4K) = 0$
$\Rightarrow$ $K^2 - 16K = 0$
$\Rightarrow$ K = 0 or 16
But K can't be zero (leading coefficient)
So K = 16

85. **a)** **0, 81**
$x^2 - 2x + 6 = (x-1)^2 + 5 \geq 5$
$\Rightarrow$ Denominator > 0 for all x
$\therefore$ Given inequation
$\Leftrightarrow 6x^2 - 5x - 3 \leq 4(x^2 - 2x + 6)$
$\Leftrightarrow 2x^2 + 3x - 27 \leq 0 \Leftrightarrow (2x+9)(x-3) \leq 0$
$\Leftrightarrow \frac{-9}{2} \leq x \leq 3.$
$\therefore$ $0 \leq 4x^2 < 81$

86. **c)** $2 \pm \sqrt{5}$
Let α, β be the roots then $\alpha = \beta^2$ (given)
$\alpha + \beta = 1 \quad \Leftrightarrow \beta + \beta^2 = 1$ - (i)
$\alpha\beta = -k \quad \Leftrightarrow \quad \beta^3 = -k$ - (ii)
(i) & (ii) $\Rightarrow \beta^3 (1+\beta)^3 = 1$
$\Rightarrow$ $-k(1 - k + 3) = 1$
$\Leftrightarrow$ $k^2 - 4k - 1 = 0 \Rightarrow k = 2 \pm \sqrt{5}$

87. **c)** **no solution**
$x \neq 2$
Given equation $\Leftrightarrow \frac{x^2 - 2x}{x-2} = \frac{2x-4}{x-2}$
(or cancel $\frac{1}{x-2}$ from two sides)
$\Leftrightarrow x^2 - 4x + 4 = 0 \quad \Rightarrow \quad x = 2$ (rejected)

88. **b)** $x^2 + 3x - 4 = 0$
Replace x by $-x$ to get equation with roots of negative sign

89. **b)** $b = 0$
Roots are equal in magnitude and opposite in sign so sum = 0 $\rightarrow b = 0$

90. **a)** **equation**
This is a polynomial equation of degree 10.