PART C - MATHS

61. **b)** **10**

Equation to the straight line is $y - \sqrt{8} = \tan 135(x + \sqrt{8}) \Rightarrow x + y = 0$

Equation to the circle $x^2 + y^2 = 25$

The line passes through (0, 0) which is the centre of the circle.

$\Rightarrow$ AB is the diameter.

$\therefore$ AB = 2r = 2(5) = 10

62. **d)** $\mathbf{3(x^2 + y^2) = 4a^2}$

Let P (θ) and $Q\left(\theta + \frac{\pi}{3}\right)$ be the points on the circle.

$\therefore$ Angle between OP and OQ = $\frac{\pi}{3}$

$\Rightarrow$ Angle included between the tangents at P and Q = $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Let the tangents at P, Q meet at $R(x_1, y_1)$

$\therefore$ $\tan\frac{\pi}{3} = \frac{OP}{PR} \Rightarrow \sqrt{3} = \frac{a}{\sqrt{S_{11}}} \Rightarrow 3S_{11} = a^2$

$\therefore$ $3\left(x_1^2 + y_1^2 - a^2\right) = a^2$

$\therefore$ Locus is $3(x^2 + y^2) = 4a^2$

63. **b)** **(–1, 1)**

Equation perpendicular to the tangent and through the centre (3, –2) is $3(x - 3) + 4(y + 2) = 0 \Rightarrow 3x + 4y - 1 = 0$. (–1, 1) satisfies the tangent and the normal.

64. **b)** $\mathbf{ly^2 - mxy + nx + a^2l = 0}$

Consider the circle on the line joining $(a, 0)$ and $(-a, 0)$ as diameter. Its equation is $x^2 + y^2 = a^2$

$\therefore$ The pole of $lx + my + n = 0$ w.r. to (1) is

$$P\left(\frac{-a^2l}{n}, \frac{-a^2m}{n}\right)$$

Substituting P satisfies.

65. **a)** **2**

The lines $l_1x + m_1y + n_1 = 0$ and $l_2x + m_2y + n_2 = 0$ are conjugate w.r. to S = 0 if $r^2(l_1l_2 + m_1m_2) = (l_1g + m_1f - n_1)(l_2g + m_2f - n_2)$

$\Rightarrow$ From the circle $r^2 = 1 + 4 + 4 = 9$, $g = -1$, $f = -2$

$\therefore$ $9(2k + 3) = [k(-1) + 3(-2) + 1]\{2(-1) + 1(-2) - 5]$

$\Rightarrow$ $9(2k + 3) = (-k - 5)(-9) \Rightarrow 2k + 3 = k + 5 \Rightarrow k = 2$

66. **c)** **(–35, 15)**

The line touches the circle ⇔ Distance from the centre = radius

$\therefore$ $\frac{3(2) - 4(4) - \lambda}{5} = \sqrt{4 + 16 + 5} \Rightarrow -10 - \lambda = \pm 25 \Rightarrow \lambda = -35, 15$

The line cuts the circle in the distinct points when $-35 < \lambda < 15$

$\therefore$ λ lies in (–35, 15)

67. **d)** $\mathbf{\pm 1}$

Let $P(x_1, y_1)$ be the mid-point of the third side AB.

Then $A = (2x_1, 0)$ and $B = (0, 2y_1)$

$\therefore$ Equation of AB is $\frac{x}{2x_1} + \frac{y}{2y_1} = 1$

$\Rightarrow$ $xy_1 + x_1y - 2x_1y_1 = 0$

The touches the given circle centre (2, 2) and of radius $= \sqrt{4 + 4 - 4} = 2$

⇔ $\left|\frac{2x_1 + 2y_1 - 2x_1y_1}{\sqrt{x_1^2 + y_1^2}}\right| = 2$

Locus of P is $x + y - xy = \pm\sqrt{x^2 + y^2}$

$\therefore$ $\lambda = \pm 1$

68. **d)** $\left(\frac{1}{2}, -\sqrt{2}\right)$

O = (0, 0), A = (1, 0) circle on AB as diameter is

$x(x - 1) + y^2 = 0 \Rightarrow x^2 + y^2 - x = 0$

Equation to the line OA is $y = 0$

$\therefore$ Any circle through O and A is $x^2 + y^2 - x + 2\lambda y = 0$

Centre $P = \left(\frac{1}{2}, -\lambda\right)$, $r_1 = \sqrt{\frac{1}{4} + \lambda^2}$

For circle $x^2 + y^2 = 9$, ⇒ Centre O = (0, 0) and $r_2 = 3$

circles touch internally OP = $r_1 - r_2$

$\Rightarrow$ $\sqrt{\frac{1}{4}+\lambda^2} = 3 - \sqrt{\frac{1}{4}+\lambda^2} \Rightarrow \sqrt{\frac{1}{4}+\lambda^2}$

$= \frac{3}{2} \Rightarrow \lambda^2 = 2 \Rightarrow \lambda = \pm\sqrt{2}$

$\therefore$ Centre P = $\left(\frac{1}{2}, -\sqrt{2}\right)$

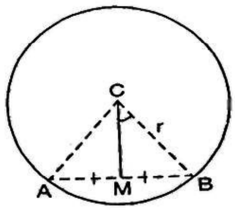
69. **d)** $\boldsymbol{y^2 + 6y - 4x + 9 = 0}$

Shortest distance from the circle is the distance from the centre minus the radius

$\therefore$ $= \left|\sqrt{(x-1)^2+(y+3)^2} - 4\right| = |x-3|$

$y^2 + 6y - 4x + 9 = 0$ is only part of locus

70. **b)** $\boldsymbol{\text{cosec}\frac{\pi}{9}}$



Each side makes an angle of $\frac{2\pi}{9}$ at the centre.

Let the side AB = 2 and CM ⊥ r AB

$\therefore$ $\angle$ BCM = $\frac{1}{2}\left(\frac{2\pi}{9}\right) = \frac{\pi}{9}$

From ΔBCM $\sin\frac{\pi}{9} = \frac{1}{x} \Rightarrow x = \text{cosec}\frac{\pi}{9}$

71. **d)** $\boldsymbol{\left(\frac{5}{2}, \frac{5}{2}\right)}$

Equation to the circle through the points is $(2x + 3y - 6)(9x + 6y - 18) - [2(6) + 3(9)]xy = 0$

$\Rightarrow$ $18x^2 + 18y^2 - 90x - 90y + 108 = 0$

$\Rightarrow$ $2x^2 + 2y^2 - 10x - 10y + 12 = 0$

$\therefore$ centre = $\left(\frac{5}{2}, \frac{5}{2}\right)$

72. **a)** $\boldsymbol{a + b}$

Circle on $\overline{PQ}$ as diameter passes through (0, 0) $\Rightarrow \angle$ POQ = 90^0

Homogenising the curve with the line we have

$ax^2 + 2hxy + by^2 = \left(\frac{lx+my}{n^2}\right)^2$

This pair of lines are ⊥ r $\Leftrightarrow$ Coefficient of x^2 + coefficient of y^2 = 0

$\Rightarrow$ $\left(a - \frac{l^2}{n^2} + b - \frac{m^2}{n^2}\right) = 0 \Rightarrow a + b = \frac{l^2+m^2}{n^2}$

73. **c)** $\boldsymbol{x^2 + y^2 - xx_1 - yy_1 = 0}$

$\angle L = \angle M = 90^0$

$\Rightarrow$ Circle on OP as diameter passes through L and M.

$\therefore$ Equation to the circle on the line joining O(0, 0) and P(x_1, y_1) as diameter is $(x - 0)(x - x_1) + (y - 0)(y - y_1) = 0 \Rightarrow x^2 + y^2 - xx_1 - yy_1 = 0$

74. **c)** $\boldsymbol{|a| < 4}$

Equation of chord can be written as $x \pm y = \pm a$

So, length of perpendicular from the centre (0, 0) of the circle to the chord < radius

i.e, $\left|\frac{\pm a}{\sqrt{2}}\right| < \sqrt{8} \Rightarrow |a| < 4$

75. **a)** $\boldsymbol{\left(a^2 - x_1\right)^2 x^2 + \left(a^2 - y_1^2\right)y^2 - 2xyx_1y_1 = 0}$

Equation to the chord of contact of P is $xx_1 + yy_1 - a^2 = 0$ Homogenising the circle with this equation $x^2 + y^2 - a^2 \left(\frac{xx_1+yy_1}{a^2}\right) = 0$

$a^2(x^2 + y^2) - (xx_1 + yy_1)^2 = 0$

$\Rightarrow \left(a^2 - x_1^2\right)x^2 - 2xyx_1y_1 + \left(a^2 - y_1^2\right)y^2 = 0$

76. **c)** $\boldsymbol{\frac{35\sqrt{7}}{32}}$

PA = $\sqrt{S_{11}} = \sqrt{16+16-25} = \sqrt{7}$

$r = 5 = $ OA

PO = $\sqrt{16+16} = \sqrt{32}$. Let $\angle$ APO = θ

From Δ AOP : $\sin\theta = \frac{5}{\sqrt{32}}$, $\cos\theta = \frac{\sqrt{7}}{\sqrt{32}}$

Δ APB = $\frac{1}{2}$ AP.PB sin 2θ

$= \frac{1}{2}\left(\sqrt{7}\right)\left(\sqrt{7}\right)2\sin\theta\cos\theta$

$= 7\left(\frac{5}{\sqrt{32}}\right)\left(\frac{\sqrt{7}}{\sqrt{32}}\right) = \frac{35\sqrt{7}}{32}$

77. **d)** **both a and c**

Let equation of the required circle be $(x-4)^2 + (y-3)^2 = r^2$ ... (1)

If the circle (1) touches the circle $x^2 + y^2 = 1$, the distance between the centres (4, 3) and (0, 0) of these circles is equal to the sum or difference of their radii, r and 1.

$\Rightarrow$ $\sqrt{4^2+3^2} = 1 \pm r \Rightarrow r \pm 1 = 5$

$\Rightarrow$ r = 4 or 6 so that the equations of the required circles from (1) are $x^2 + y^2$. $8x - 6y + 9 = 0$ and $x^2 + y^2 - 8x - 6y - 11 = 0$

78. **b)** $\boldsymbol{p^2 = q^2}$

Equation of the given circle can be written as $(x-p)^2 + (y-q)^2 = p^2$, so that the centre of the circle is (p, q) and its radius is p. This shows that $x = 0$ is a tangent to the circle from the origin. Since tangents from the origin are perpendicular, equation of the other tangent must be y = 0, which is possible if $q = \pm p$ or $p^2 = q^2$.

79. **d)** **(–6, –9)**

The centre of the given circle is (1, –2) and its radius is $\sqrt{1+4+93} = \sqrt{98}$. Since the sides of the square inscribed in the circle are parallel to the coordinate axes, let the equation of the sides be $x = a$, $y = c$, and $y = d$. Now the length of each diagonal of the square is equal to the diameter of the circle, i.e., $2\sqrt{98}$. Hence the length of each side of the square is 14, and the perpendicular distance of each side from the centre (1, –2) of teh circle is 7. This gives $a = -6$, $b = 8$, $c = -9$ and $d = 5$, so the vertices of the square are (8, 5), (8, –9), (–6, 5) and (–6, –9).

80. **c)** $\boldsymbol{x^2 + y^2 - ax = 0}$

Let (h, k) be the centre of the required circle. Then (h, k) being the mid point of the chord of the given circle, its equation is

$hx + ky - a(x+h) = h^2 + k^2 - 2ah$

Since it passes through the origin, we have

$-ah = h^2 + k^2 - 2ah \Rightarrow h^2 + k^2 - ah = 0$

Hence locus of (h, k) is $x^2 + y^2 - ax = 0$.

81. **a)** $\boldsymbol{2ax + 2by = a^2 + b^2 + k^2}$

Let equation of the circle through (a, b) be $x^2 + y^2 + 2gx + 2fy + c = 0$

Then $a^2 + b^2 + 2ga + 2fb + c = 0$ ... (1)

Since (1) cuts the circle $x^2 + y^2 = k^2$ orthogonally, we have ... (2)

$2g \times 0 + 2f \times 0 = c - k^2 \Rightarrow c = k^2$

So that from (2), we get $a^2 + b^2 + 2ga + 2fb + k^2 = 0$ and the locus of the centre $(-g, -f)$ of (1) is $2ax + 2by - (a^2 + b^2 + k^2) = 0$.

82. **d)** $\boldsymbol{x^2 + y^2 - 2gx + 2gy + g^2 = 0}$ **where** $\boldsymbol{g = \frac{2}{(\cos\alpha - \sin\alpha - 1)}}$

Equation of a circle which touches the coordinate axes can be written as $(x-g)^2 + (y+g)^2 = g^2$, g being a variable. If this circle touches the line $x\cos a + y\sin a = 2$, we get

$g\cos a \pm g\sin a - 2 = \pm g$

$\Rightarrow$ $g = \frac{2}{\cos\alpha \pm \sin\alpha \pm 1}$

83. **c)** $\boldsymbol{x^2 + y^2 - 4x - 4y = 0}$

Since the centre of the required circle lies on $x + y = 4$, let (g, 4 – g) be this centre. Since the circle passes through the origin, let its equation be

$x^2 + y^2 - 2gx - 2(4 - g)y = 0$

As this circle cuts the given circle orthogonally, we have

$2 \times g \times 2 - 2(4 - g) = 4$

$\Rightarrow 6g = 12 \;\Rightarrow g = 2$

So that equation of the required circle is $x^2 + y^2 - 4x - 4y = 0$.

84. **b)** $\boldsymbol{\cot\frac{\theta}{2} = \frac{\sqrt{S_1}}{\sqrt{g^2 + f^2 - c}}}$

Let PA and PB be the tangents from P (x_1, y_1) to the given circle with centre C $(-g - f)$, such that $\angle APB = \theta$. Then $\angle APC$

$= \frac{\theta}{2}$ figure. Therefore, from ΔPAC, we get

$$\cot\frac{\theta}{2} = \frac{PA}{AC} = \frac{\sqrt{S_1}}{\sqrt{g^2 + f^2 - c}}$$

$$\Rightarrow \tan\frac{\theta}{2} = \frac{\sqrt{g^2 + f^2 - c}}{\sqrt{S_1}}$$

$$\Rightarrow \quad \theta = 2\tan^{-1}\left(\frac{\sqrt{g^2 + f^2 - c}}{\sqrt{S_1}}\right)$$

85. **d)** $\boldsymbol{(x - 3)^2 + y^2 = 9}$

The point $\left(3 + \frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$ does not satisfy the given circles is (a) and (c).

$\therefore$ (a) and (c) can not be the correct choices,

The centre of the circle given in (b) is $\left(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$, which does not lie on $x - y = 3$.

Hence (b) cannot be correct choice.

The centre of the circle given in (d) is (3, 0), which lie on $x - y = 3$,

then radius

$$= \sqrt{\left(3 + \frac{3}{\sqrt{2}} - 3\right)^2 + \left(\frac{3}{\sqrt{2}} - 0\right)^2}$$

$$= \sqrt{\frac{9}{2} + \frac{9}{2}} = \sqrt{9} = 3$$

Hence, the correct answer is (d)

86. **d)** **none of these**

Since PQ is the chord of contact of the tangents from the origin 0 to the circle

$x^2 + y^2 + 2gx + 2fy + c = 0$

equation of PQ is ... (1)

$gx + fy + c = 0$ ... (2)

An equation of a circle through the intersection of (1) and (2) is given by

$x^2 + y^2 + 2gx + 2fy + c \pm \lambda(gx + fy + c) = 0$ ... (3)

If the circle (3) passes through O, the origin, then $c + \lambda c = 0$, i.e., $\lambda = -1$; and the equation of the cirle (3) becomes

$x^2 + y^2 + gx + fy = 0$

Centre of this circle is $\left(\frac{-g}{2}, \frac{-f}{2}\right)$, and hence it is the circumcentre of the triangle OPQ.

87. **a)** **(1, 1)**

Let equation of the circle passing through the given points be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ... (1)

Then $1 + t^2 + 2g + 2ft + c = 0$ ... (2)

$t^2 + 1 + 2gt + 2f + c = 0$ ... (3)

and $t^2 + t^2 + 2gt + 2ft + c = 0$ ... (4)

From (2) and (3), we have

$2g(1 - t) + 2f(t - 1) = 0$

$\Rightarrow 2(t - 1)(f - g) = 0$

That is, $f = g$, because $t \neq 1$, otherwise the three given points would be the same. Again, from (3) and (4), we get

$t^2 + 1 + 2gt + 2g + c = 0$ ... (5)

and $2t^2 + 2gt + 2gt + c = 0$ ... (6)

Subtracting these, we get

$t^2 - 1 + 2g(t - 1) = 0 \Rightarrow 2g = -(t + 1)$

Finally, from (6), we get $c = -2t^2 + 2t(t +$

1) = $2t$, so that the equation of the circle (1) can be written as

$x^2 + y^2 - (t+1)x - (t+1)y + 2t = 0$

which passes through (1, 1) for all values of t, while the points given by (b), (c) and (d) do not satisfy this equation.

88. **b)** $\sqrt{c\left(g^2 + f^2 - c\right)}$

Since OA = OB and CA = CB, the diagonal OC divides the quadrilateral OACB in two equal right-angled triangles, OAC and OBC figure. Therefore, the area of the quadrilaterlal OACB is

2(area of triangle OAC) = $2 \times \frac{1}{2}$ OA × AC

$= \sqrt{0 + 0 + 2g \times 0 + 2f \times 0 + c}$

$\sqrt{g^2 + f^2 - c} = \sqrt{c\left(g^2 + f^2 - c\right)}$

89. **b)** $x^2 + y^2 + 5x - 4y + 4 = 0$

As the required circle touches the y-axis at (0, 2), let its equation be

$(x - a)^2 + (y - 2)^2 = a^2$ or

$x^2 + y^2 - 2ax - 4y + 4 = 0$

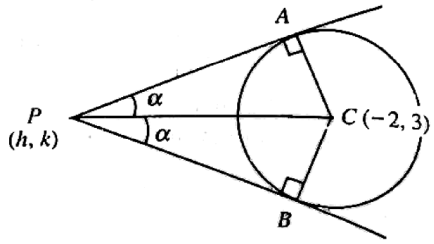
This circle meets the x-axis at the points where $x^2 - 2ax + 4 = 0$, which gives two values of x, say x_1, x_2 with $x_1 < x_2$, such that $x_1 + x_2 = 2\alpha$ and $x_1x_2 = 4$. Now, we are given that $|x_1 - x_2| = 3$.

∴ $(x_1 + x_2)^2 - 4x_1x_2 = (x_1 + x_2)^2 = 9$

⇒ $4\alpha^2 - 16 = 9 \Rightarrow \alpha^2 = \frac{25}{4} \Rightarrow \alpha = + \frac{5}{2}$

Hence the equation of the required circle is $x^2 + y^2 \pm 5x - 4y + 4 = 0$

90. **d)** $x^2 + y^2 + 4x - 6y + 9 = 0$



Let PA and PB be the tangents drawn from the point P(h, k) to the given circle with centre C(–2, 3). So that

∠ APB = 2α and ∠ APC = ∠ CPB = α

∠ PAC = ∠ PBC = 90^0

From triangle PCA,

⇒ $\sin\alpha = \frac{CA}{CP}$ and

CA = $\sqrt{4 + 9 - \left(9\sin^2\alpha + 13\cos^2\alpha\right)}$

= 2 sin α

⇒ CP = 2

⇒ $4 = h^2 + k^2 + 4h - 6k + 13$

The locus of P(h, k) is therefore

$x^2 + y^2 + 4x - 6y + 9 = 0$