PART C - MATHS

61. **b)** **N**

$A = \{1, 3, 5, \dots 17\}$

$B = \{2, 4, \dots. 18\}$

N is universal set

$= \{1, 3, 5, \dots.17, 19, 20, \dots.\}$

$A \cup B = \{1, 2, 3, 4 \dots\dots. 18\}$

$(A \cup B) \cap B^c = \{1, 3, 5 \dots\dots 17\}$

$= A$

$\therefore$ $A^c \cup [(A \cup B) \cap B^c] = A^c \cup B$

$= N$ ... Universal set

62. **b)** $\mathbf{A - (B \cap C) = \{1, 2, 4\}}$

$A = \{1, 2, 3, 4\}$

$B = \{2, 3, 5, 6\}$

$C = \{3, 4, 6, 7\}$

$A \cup C = \{2, 3, 4, 5, 6, 7\}$

$B \cap C = \{3, 6\}$

$A - B \cup C = \{1\}$

$A - B \cap C = \{1, 2, 4\}$

63. **b)** **41**

Let n(A) = 794 ... No. of families having

n(B) = 187 ... No. of families having

Out of 1003 families 63 families heither had

$\therefore$ $n(A \cup B) = 1003 - 63$

$= 940$

$\therefore$ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$940 = 794 + 184 - n(A \cap B)$

$n(A \cap B) = 41$

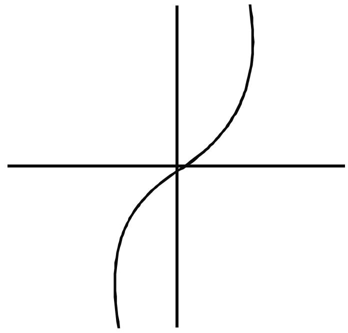
64. **d)** $\left(\mathbf{0}, \frac{\mathbf{3}}{\mathbf{2}}\right]$

$\log\frac{3-x}{x} \geq O$ and $\frac{3-x}{x} > O$

$3 - x > x$ and $3 > x$

$\frac{3}{2} > x$ and $3 > x$

65. **c)** **two points**



$y = x|x|$

$x^2 + y^2 = 5$ - (i)

$2x = 5y$

$x = \frac{5y}{2}$ - (ii)

Solving (i) and (ii) we get two solutions

66. **c)** **there is no *x* satisfying this inequality**

67. **a)** $\mathbf{(-\infty, 0)}$

$|x| - x > 0$

$\therefore$ $x < 0$

68. **b)** $\mathbf{(0, 1) \cup (1, +\infty)}$

69. **d)** **None of these**

$3^{x^2-2} < 3^{\frac{3}{2}|x|-1}$

$x^2 - 2 < \frac{3}{2}|x| - 1$

$2x^2 - 4 < 3\,|x| - 2$

$2x^2 - 2 < 3\,|x|$

70. **c)** $\mathbf{(-1, 0) \cup (1, 2) \cup (2, \infty)}$

$4 - x^2 \neq 0$ & $x^3 - x > 0$

$x \neq 0$ & $x \in (-1, 0) \cup (1, \infty)$

$\therefore$ $x \in (-1, 0), \cup (1,2) \cup (2,\infty)$

71. **b)** $\left[-\frac{\pi}{\mathbf{6}}, \frac{\pi}{\mathbf{6}}\right]$

$A \cap B = \left\{x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]; \frac{1}{2} \leq \sin \leq \frac{1}{2}\right\}$

$= \left[\frac{-\pi}{6}, \frac{\pi}{6}\right]$

72. **d)** $\mathbf{2 - 4a}$

$f(x) = x - x^2$

$f(a+1) = (a+1) - (a+1)^2$

$f(a-1) = (a-1) - (a-1)^2$

$\therefore\ f(a+1) - f(a-1) = 2 - 4a$

73. **c)** $\mathbf{[0, \infty) - \{1\}}$

$y = \sqrt{\dfrac{x}{x+1}}$

$y^2 = \dfrac{x}{x+1}$

$y^2(x+1) = x$

$x = \dfrac{y^2}{1-y^2} \quad \therefore\ y^2 \neq 1 \quad \therefore\ y \neq \pm 1$

Range = $[0, \infty) - [1]$

74. **b)** $\mathbf{\{-1, 0, 1\}}$

75. **a)** $\mathbf{\left\{0, \frac{5}{3}\right\}}$

$4[x - [x]] = x + [x]$

$4x - 4[x] = x + [x]$

$3x = 5[x]$

$[x] = \dfrac{3}{5}x$

76. **a)** $\mathbf{2x - 3}$

$g(x) = x^2 + x - 2$

$\dfrac{1}{2} g[f(x)] = 2x^2 - 5x + 2$

$g[f(x)] = 4x^2 - 10x + 4$

$[f(x)^2] + f(x) - 2 = 4x^2 - 10x + 4 = 0$

$\therefore\ f(x) = 2x - 3, -2x + 2$

77. **c)** $\mathbf{\log\left(\frac{ax^2 - bx + c}{ax^2 + bx + c}\right)}$

$f(x) = \log\left(\dfrac{ax^2 - bx + x}{ax^2 + bx + x}\right)$

$f(-x) = \log\left(\dfrac{ax^2 + bx + x}{ax^2 - bx + x}\right)$

$\therefore\ f(x) = -f(-x)$

78. **b)** $\mathbf{[1, 5]}$

$y = |x-1| + |x-2|$

If $x < 1$, $y = -(x-1) - (x-2)$

$= -2x + 3 \quad \rightarrow$ Decneasing

If $1 < x < 2$, $y = x - 1 - (x-2) = 1$

$\rightarrow$ constant

If $2 \le x \le 3$, $y = x - 1 + x - 2$

$= 2x - 3 \quad \rightarrow$ incneasing

min $f(x) = f(1) = 1$

max $f(x)$ = gneatest among $f(-1)$ f(3)= 5

$\therefore$ Range = [1, 5]

79. **c)** $\mathbf{f[g(x)] = x^2, x \neq 0, h[g(x)] = [g(x)]^2, x \neq 0}$

$f(x) = \dfrac{1}{x}$, $g(x) = \dfrac{1}{x^2}$, $h(x) = x^2$

$f(g(x)) = x^2$

$h(g(x)) = \dfrac{1}{x^4} = [g(x)]^2$

80. **b)** $\mathbf{\left[-1, \frac{-1}{2}\right] \cup \left[0, \frac{1}{2}\right] \cup \{1\}}$

$2\{x\}^2 - 3\{x\} + \geq O$

$\{x\} \geq 1$ or $\{x\} \leq \dfrac{1}{2}$

81. **c)** $\mathbf{\left(1, \frac{7}{3}\right]}$

$f(x) = \dfrac{x^2 + x + 2}{x^2 + x + 1}$

$y = \dfrac{(x^2 + x + 1) + 1}{x^2 + x + 1}$

$y = 1 + \dfrac{1}{\left(x^2 + \frac{1}{2}\right)^2 + \frac{3}{4}}$

$\therefore$ Range $\left(1, \dfrac{7}{3}\right]$

82. **b)** $\mathbf{2f(x)}$

$f\left(\dfrac{2x}{1+x^2}\right) = \log\left(\dfrac{x + \frac{2x}{1+x^2}}{x - \frac{2x}{1+x^2}}\right)$

$= \log\left(\dfrac{1+x}{1-x}\right)^2 = 2f(x)$

83. **d)** **g (f(x))**

(fog) (x) = [x – (x)]

= 0

∴ x – [x] lies in (0,1)

84. **b)** **P ∩ Q = ϕ**

for $x \in R$

$f(x + 1)$ = tan(π (x + 1)) + x + 1 – [x + 1]

= tan (π + πx) + x + 1 – [[x] + 1]

= tan x – [x]

= $f(x)$

∴ period is 1

85. **a)** **one**

86. **a)** **an even function**

$$f(x) = \frac{(1+2^x)^2}{2^x}$$

$$f(x) = \frac{(1+2^{-x})^2}{2^{-x}} = \frac{(2^x+1)^2}{2^x}$$

$f(x)$ = $f(x)$ even

87. **a)** **{(2, 4), (3, 4)}**

A = {x : $x^2 - 5x + 6 = 0$} = {2 : 3}

∴ A × (B ∩ C) = {(2, 4), (3, 4)}

88. **c)** **equivalence relation**

89. **c)** **reflexive and transitive but not symmetric**

i) Let $b = a$, ∴ $a = 2^k a$

∴ k = O ∴ R is retlexing

ii) A = 2^kb ∴ $b = 2^{-k}a$, –k is integer

for a R b, b R a exists

∴ R is symmernc

iii) Let a R b and b R c

∴ a = 2^kb and b R c

a = 2^k, 2^pc

a = 2^{k+p} c

∴ a R c ∴ R is trifficult

90. **c)** $\mathbf{\frac{1}{\sqrt{2}}}$ **or –** $\mathbf{\frac{1}{\sqrt{2}}}$