PART C - MATH

61. **c) 1**

$$\left(\frac{1+i}{1-i}\right)^{200} = \left(\frac{(1+i)^2}{1-i^2}\right)^{200}$$
$$= i^{200}$$
$$= (-1)^{100} = 1$$

62. **b) 0**

$C_1 \to C_1 + C_2$ and use ${}^nC_r = \frac{n}{r} \cdot {}^{n-1}C_{r-1}$ in C_3, we get

$$\Delta = \begin{vmatrix} {}^{n+1}C_r & {}^nC_r & (n+2)\,{}^{n+2}C_r \\ {}^{n+1}C_{r+1} & {}^nC_{r+1} & (n+2)\,{}^{n+1}C_{r+1} \\ {}^{n+1}C_{r+2} & {}^nC_{r+2} & (n+2)\,{}^{n+1}C_{r+2} \end{vmatrix}$$

$$= (n+2)\begin{vmatrix} {}^{n+1}C_r & {}^nC_r & {}^{n+1}C_r \\ {}^{n+1}C_{r+1} & {}^nC_{r+1} & {}^{n+1}C_{r+1} \\ {}^{n+1}C_{r+2} & {}^nC_{r+2} & {}^{n+1}C_{r+2} \end{vmatrix}$$

$= 0$

63. **c) is zero**

$R_1 \to R_1 + R_2 + R_3$ gives $\Delta = 0$

64. **c)** $\sqrt{\dfrac{a^2+b^2}{c^2+d^2}}$

$$(x+iy)(x-iy) = \sqrt{\frac{(a+ib)(a-ib)}{(c+id)(c-id)}}$$
$$= \sqrt{\frac{a^2+b^2}{c^2+d^2}}$$

65. **a)** $\dfrac{\sin 4\theta}{\sin\theta}$

$\Delta = 2\cos\theta\,(4\cos^2\theta - 1) - 2\cos\theta$
$= 4\cos\theta\,(2\cos^2\theta - 1)$
$= 4\cos\theta\cos 2\theta$
$$= \frac{2(2\sin\theta\cos\theta)\cos 2\theta}{\sin 2\theta}$$
$$= \frac{\sin 4\theta}{\sin\theta}$$

66. **a) 9**

Product $= (1-\omega)(1-\omega^2)(1-\omega)(1-\omega^2)$
$= [(1-\omega)(1-\omega^2)]^2$
$= (1-\omega-\omega^2+\omega^3)^2$
$= [3-(1+\omega+\omega^2)]^2$
$= 9$

67. **a) all *m, n, k***

Let $P(x) = x^{3m} + x^{3n+1} + x^{3k+2}$
$P(\omega) = 0$
$\Rightarrow (\omega^3)^m + \omega(\omega^3)^n + \omega^2(\omega^3)^k = 0$
$\Rightarrow 1 + \omega + \omega^2 = 0$ True
$\Rightarrow$ Given expression for all m, n, k is divisible by $x^2 + x + 1$

68. **d) 0**

e = Given determinant at $x = 0$

$$= \begin{vmatrix} 0 & -1 & 3 \\ 1 & 0 & -4 \\ -3 & 4 & 0 \end{vmatrix} = 0$$

69. **b) only the trivial solution**

$$\text{Let } \Delta = \begin{vmatrix} 1 & -2 & 1 \\ k & -1 & 2 \\ 2 & -1 & 1 \end{vmatrix}$$

$= (-1+2) + 2(k-4) + 1(-k+2)$
$= k - 5$
$k \neq 5 \Rightarrow \Delta \neq 0$
$\Rightarrow$ system has only trivial solution.

70. **c) $p = q = 4$**

$x = 1$makes 5 rows proportional
$\Rightarrow (x-1)^4$ is a factor
$C_1 \to C_1 + C_2 + C_3 + C_4 + C_5$
$\Rightarrow x + 4$ is a factor
$p = q = 4$

71. **d) – 4**

Given determinant =

$$e^{i(A+B+C)}\begin{vmatrix} e^{iA} & e^{-i(A+C)} & e^{-i(A+B)} \\ e^{-i(C+B)} & e^{iB} & e^{-i(A+B)} \\ e^{-i(B+C)} & e^{-i(A+C)} & e^{iC} \end{vmatrix}$$

$$= e^{iA}\begin{vmatrix} e^{iA} & -e^{-iB} & -e^{-iC} \\ -e^{-iA} & e^{iB} & -e^{-iC} \\ -e^{-iA} & -e^{-iB} & e^{iC} \end{vmatrix}$$

$$= -e^{i(A+B+C)}\begin{vmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{vmatrix}$$

$= -4$

72. **a)** **0**

$w = \frac{z-1}{z+1} \Rightarrow z = \frac{1+w}{1-w}$

$|z| = 1 \Rightarrow |1 + w| = |1 - w|$

If $w = u + iv$ then

$(1 + u^2) + v^2 = (1 - u)^2 + (-v)^2$

$\Rightarrow u = 0$

73. **b)** **Im(z) = 0**

$\frac{\sqrt{3}}{2} + \frac{i}{2}$ and $\frac{\sqrt{3}}{2} - \frac{i}{2}$ are conjugates of each other .

Let $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 = (u + iv)$ then $z = w + \overline{w}$

$= 2u$

$\Rightarrow$ Im(z) = 0

74. **c)** **a circle**

$\frac{|z_3 - z_1|}{|z_3 - z_2|} = k \Rightarrow z_3$ moves on Appolonius circle.

75. **a)** ***i***

Given expression =

$$-i\underbrace{\sum_{k=1}^{6}\left(\cos\frac{2k\pi}{7} + i\sin\frac{2k\pi}{7}\right)}_{=\text{ sum of complex 7th roots of unity}}$$

$= -i(-1) = i$

76. **a)** **depends on α, β, γ and not on θ**

$\Delta = \begin{vmatrix} \cos\alpha & \sin\alpha & 1 \\ \cos\beta & \sin\beta & 1 \\ \cos\gamma & \sin\gamma & 1 \end{vmatrix}$ represents 2 (area of triangle ABC)

where A(α), B(β), C(γ) lie on unit circle. Rotate ΔABC by angle θ, area remains invariant.

77. **d)** ***abc* = – 1**

$$\begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^3 & c^3+1 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + abc\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$= 0$

$\Rightarrow (abc + 1)(a - b)(b - c)(c - a) = 0$

$abc = -1$

78. **b)** **$(-i - 1)z + (i - 1)\overline{z} = 0$**

Let $z = x + iy$

$\Rightarrow x = \frac{z + \overline{z}}{2}, \quad y = \frac{z - \overline{z}}{2i}$

$x = y \Rightarrow (z + \overline{z})i = (z - \overline{z})$

$\Rightarrow (i - 1)z + (i + 1)\overline{z} = 0$

Multiplying by i,

$(-i - 1)z + (-1 + i)\overline{z} = 0$

79. **c)** **Interior and boundary of an ellipse**

$|z - 1| + |z + 1| = 4$

$\Rightarrow$ z moves on ellipse with foci at 1 and – 1 with major axis = 4

80. **b)** arg (**$z_1 z_2 z_3$**)

$C_1 \to C_1 + C_2 + C_3$ gives

arg z_1 + arg z_2 + arg z_3 is a factor

arg z_1 + arg z_2 + arg z_3 = arg ($z_1 z_2 z_3$)

81. **c)** **three values**

$\begin{vmatrix} a-t & b & y \\ b & c-t & a \\ c & a & b-t \end{vmatrix}$ = determinant of coefficients

For non trivial solution this determinant = 0

$\Rightarrow$ A cubic equation in t.

82. **c)** **the triangle is isosceles**

$$\Delta = \begin{vmatrix} 1 & \sin A & 3\sin A \\ 1 & \sin B & 3\sin B \\ 1 & \sin C & 3\sin C \end{vmatrix} + \begin{vmatrix} 1 & \sin A & -4\sin^3 A \\ 1 & \sin B & -4\sin^3 B \\ 1 & \sin C & -4\sin^3 C \end{vmatrix}$$

$= 0$

$0 = 0 - 64 (\sin A + \sin B + \sin C) (\sin A - \sin B) (\sin B - \sin C) (\sin C - \sin A)$

$\Rightarrow$ $A = B$ or $B = C$ or $C = A$

83. **a)** $\mathbf{\dfrac{3\sqrt{3}}{4}}$

z_1, z_2, z_3 lie on unit circle with centroid at the origin

$\Rightarrow$ z_1, z_2, z_3 are vertices of an equilateral triangle inscribed in unit circle.

$\Rightarrow$ side length of triangle $= \sqrt{3}$

84. **b)** **0**

$x(-3x^2 - 6x - 2x^2 + 6x) + 6(2x + 4 + 3x - 9) - 1(4x - 9x) = 0$

$\Leftrightarrow$ $x(-5x^2) + 6(5x - 5) + 5x = 0$

$5x^3 - 35x + 30 = 0$

$\Leftrightarrow$ $x^3 - 7x + 6 = 0$

$\Leftrightarrow$ $(x-1)(x^2 + x - 6) = 0$

$\Leftrightarrow$ $(x-1)(x+3)(x-2) = 0$

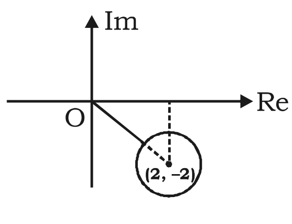
85. **c)** **0**

Given determinant $=$

$$(\alpha\beta + \beta\gamma + \gamma\alpha)\begin{vmatrix} 1 & \beta\gamma & \gamma\alpha \\ 1 & \gamma\alpha & \alpha\beta \\ 1 & \alpha\beta & \beta\gamma \end{vmatrix} = 0$$

(as coef.of $x = \alpha\beta + \beta\gamma + \gamma\alpha = 0$)

86. **a)** $\left(\mathbf{2 - \dfrac{1}{\sqrt{2}}}\right)\mathbf{(1 - \textit{i})}$



Min $|z| = 2\sqrt{2} - 1$

and $\arg z = -\dfrac{\pi}{4}$

87. **d)** **54**

$x^2 + x + 1 = 0 \Rightarrow x = \omega$

$\left(\omega + \dfrac{1}{\omega}\right)^2 = (-1)^2 = 1$

$\left(\omega^2 + \dfrac{1}{\omega^2}\right)^2 = (-1)^2 = 1$ and

$\left(\omega^3 + \dfrac{1}{\omega^3}\right)^2 = 4$ etc. leads to

given expression $= (1 + 1 + 4) + (1 + 1 + 4) + \ldots + (1 + 1 + 4)$ - 9^th^ group

$= 18 + 36$

$= 54$

88. **c)** **a pair of lines**

$\left(\dfrac{z}{|z|}\right)^2 + \dfrac{z}{|z|} + 1 = 0$

$\Rightarrow$ $\dfrac{z}{|z|} = \omega$ or ω^2

89. **d)** $\mathbf{\dfrac{\pi}{4}}$

$|z - 4| = \text{Re } z$ (given)

If $z = x + iy$ then $(x - 4)2 + y^2 = x^2$

$\Rightarrow$ $y^2 = 8(x - 2)$ (parabola)

Perpendicular tangents meet at the directrix

$\Rightarrow$ max. $\arg(z) = \dfrac{\pi}{4}$

90. **a)** $\mathbf{\dfrac{3\pi}{4}}$

$\bar{z} + i\bar{w} = 0$

$\Rightarrow$ $z - iw = 0 \Rightarrow z = iw$

$\arg(zw) = \pi \Rightarrow \arg(iw) + \arg w = \pi$

$\Rightarrow$ $\dfrac{\pi}{2} + 2\arg w = \pi$

$\Rightarrow$ $\arg w = \dfrac{\pi}{4}$