PART C - MATHS

61. **d) 149**

$\because \left(x+1+\frac{1}{x}\right)^n = \frac{(1+x+x^2)^n}{x^n}$

$\because$ $(1 + x + x^2)^n$ is of the form $a_0 + a_1x + a_2x^2 + \ldots + a_{2n}x^{2n}$, which contains $(2n + 1)$ terms

Hence, $\left(x+1+\frac{1}{x}\right)^n$ contains $(2n + 1)$ terms

$\therefore$ $2n + 1 = 301$

$\Rightarrow$ $n = 150$

Which is greater than 149.

62. **c) $2i$**

Let $\frac{p}{a}$ = A, $\frac{q}{b}$ = B, $\frac{r}{c}$ = C

$\therefore$ $A + B + C = 1 + i$, $\frac{1}{A}+\frac{1}{B}+\frac{1}{C} = 0$

or $AB + BC + CA = 0$

$\Rightarrow$ $(A + B + C)^2 = (1 + i)^2$

$\Rightarrow$ $A^2 + B^2 + C^2 + 2(AB + BC + CA)$

$\Rightarrow$ $1 + i^2 + 2i = 1 - 1 + 2i$

$\Rightarrow$ $A^2 + B^2 + C^2 + 0 = 2i$

$\Rightarrow$ $\frac{p^2}{a^2}+\frac{q^2}{b^2}+\frac{r^2}{c^2} = 2i$

63. **c) 7**

$\because$ $5^{40} = (5^2)^{20} = (22 + 3)^{20} = 22\lambda + 3^{20}, \lambda \in N$

Also, $3^{20} = (3^2)^{10} = (11 - 2)^{10} = 11\mu + 2^{10}, \mu \in N$

Now, $2^{10} = 1024 = 11 \times 93 + 1$

$\therefore$ Remainder = 1

i.e, $\alpha = 1$

Also, $2^{2003} = 2^3 . 2^{2000} = 8(2^4)^{500} = 8(16)^{500}$
$= 8(17 - 1)^{500} = 8(17\nu + 1), \nu \in N$
$= 8 \times 17\nu + 8$

$\therefore$ Remainder = 8

i.e., $\beta = 8$

$\Rightarrow$ $\beta - \alpha = 8 - 1 = 7$

64. **c) 3**

$S(n) = i^n + i^{-n} = i^n + \frac{1}{i^n} = \frac{i^{2n}+1}{i^n} = \frac{(-1)^n+1}{i^n}$

For (n) = 1, 2, 3, 4, ... values of $S(n)$ are 0, –2, 2, 0, –2, 2, ...

65. **c) 15**

$\because \sum_{i=0}^{m}\binom{10}{1}\binom{20}{m-i} = \sum_{i=0}^{m} {}^{10}C_i . {}^{20}C_{m-i}$

= Coefficient of x^m in the expansion of $(1 + x)^{10} (1 + x)^{20}$

$= {}^{30}C_m$

Which is maximum

$\therefore$ $m = \frac{30}{2} = 15$

66. **a) $\sqrt{3} - 4i$**

Since $\frac{z(\sqrt{3}+4i)}{19} = 1$

$\therefore z = \frac{19}{(\sqrt{3}+4i)} \times \frac{\sqrt{3}-4i}{\sqrt{3}-4i} = \sqrt{3} - 4i$

67. **a) $\binom{30}{10}$**

To find

$({}^{30}C_0)({}^{30}C_{10}) - ({}^{30}C_1)({}^{30}C_{11}) + ({}^{30}C_2)({}^{30}C_{12}) + \ldots + ({}^{30}C_{20})({}^{30}C_{30})$

($\because$ difference of lower suffices = 10)

$\because$ $(1 + x)^{30} = {}^{30}C_0 + {}^{30}C_1x + {}^{30}C_2x^2 + \ldots + {}^{30}C_{20}x^{20} + \ldots + {}^{30}C_{30}x^{30}$ ...(i)

$\Rightarrow$ $(x - 1)^{30} = {}^{30}C_0x^{30} - {}^{30}C_1x^{29} + \ldots + {}^{30}C_{10}x^{20} - {}^{30}C_{11}x^{19} + {}^{30}C_{12}x^{18} + \ldots + {}^{30}C_{30}$ ... (ii)

Multiplying equation (i) and equation (ii) and comparing coefficient of x^{20}, then we get

$({}^{30}C_0)({}^{30}C_{10}) - ({}^{30}C_1)({}^{30}C_{11}) + ({}^{30}C_2)({}^{30}C_{12}) + \ldots + ({}^{30}C_{20})({}^{30}C_{30})$

$= {}^{30}C_{10}$

$[\because (1 + x)^{30}(x - 1)^{30} = (x^2 - 1)^{30} = (1 - x^2)^{30}]$

$= \binom{30}{10}$

68. **a) z**

$$\frac{1+z}{1+\bar{z}} = \frac{z(1+z)}{z+z\bar{z}} = \frac{z(1+z)}{z+|z|^2} = \frac{z(1+z)}{(z+1)} = z$$

($\because |z| = 1$)

69. **d)** $\mathbf{\frac{1}{1-f} - f}$

$\because$ $I + f + f' = \left(5+2\sqrt{6}\right)^n + \left(5-2\sqrt{6}\right)^n$

$= 2k$ (even integer)

$\therefore$ $f + f' = 1$

Now, $(I + f) + f' = \left(5+2\sqrt{6}\right)^n + \left(5-2\sqrt{6}\right)^n$

$= (1)^n = 1$

$\Rightarrow$ $(I + f)(1 - f) = 1$

or $I = \frac{1}{(1-f)} - f$

70. **d)** $\mathbf{|z_1| < |z_3| < |z_2|}$

Since $x_1 < x_2 \Rightarrow \sqrt{x_1^2 + y_1^2} < \sqrt{x_2^2 + y_2^2}$

($\because y_1 = y_2$)

$\therefore$ $|z_1| < |z_2|$ ... (i)

$|z_3| = \frac{1}{2}|z_1 + z_2| < \frac{1}{2}|z_1| + \frac{1}{2}|z_2|$

$< \frac{1}{2}|z_2| + \frac{1}{2}|z_2|$

$\therefore$ $|z_3| < |z_2|$ ... (ii)

From equation (i) and (ii)

$|z_1| < |z_3| < |z_2|$.

71. **d)** **1**

$\because$ a, b, c are in AP

$\Rightarrow$ $2b = a + c$

$\Rightarrow$ $a - 2b + c = 0$

Putting $x = 1$

Required sum $= (1 + a - 2b + c)^{1973}$

$= (1 + 0)^{1973}$

$= 1$

72. **a)** **0**

$\arg(z) + \arg(\bar{z}) = \arg(z) - \arg(z) = 0$.

73. **b)** **1 : 2 : 4**

Coefficient of x^2y^2 in $(x + y + z + t)^4$

$= \frac{4!}{2!2!} = 6$

Coefficient of yzt^2 in $(x + y + z + t)^4$

$= \frac{4!}{1!\,1!\,1!} = 12$

and coefficient of $xyzt$ in

$(x + y + z + t)^4 = \frac{4!}{1!\,1!\,1!\,1!} = 24$

$\therefore$ Required ratio is 6 : 12 : 24

or 1 : 2 : 4

74. **b)** ω

$$\left(\frac{\alpha+\beta\omega+\gamma\omega^2+\delta\omega^2}{\beta+\alpha\omega^2+\gamma\omega+\delta\omega}\right)$$

$$\frac{\omega\left(\alpha+\beta\omega+\gamma\omega^2+\delta\omega^2\right)}{\left(\beta\omega+\alpha\omega^3+\gamma\omega^2+\delta\omega^2\right)}$$

$= \omega$ ($\because \omega^3 = 1$)

75. **b)** **51**

$(x + a)^{100} + (x - a)^{100}$

$= 2\{x^{100} + {}^{100}C_2x^{98}a^2 + {}^{100}C_4x^{96}a^4 + \dots + {}^{100}C_{100}a^{100}\}$

$\therefore$ Number of terms = 51

76. **c)** $\mathbf{3, 1 + 2\omega, 1 + 2\omega^2}$

$(x - 1)^3 - 8 = 0$

$\Rightarrow$ $\left(\frac{x-1}{2}\right)^3 = 1 \Rightarrow \frac{x-1}{2} = (1)^{1/3} = 1, \omega, \omega^2$

$\Rightarrow$ $x - 1 = 2, 2\omega, 2\omega^2$

$\Rightarrow$ $x = 3, 1 + 2\omega, 1 + 2\omega^2$

77. **a)** **2**

${}^4C_3a^3 = 32 \Rightarrow a^3 = 8$

$\therefore$ $a = 2$

78. **c)** $\mathbf{\frac{9^n - 1}{8}}$

Let $x = (1)^{1/n}$

$x^n - 1 = 0$

or $x^n - 1 = (x - 1)(x - \omega)(x - \omega^2) \dots (x - \omega^{n-1})$

$\Rightarrow$ $\frac{x^n - 1}{x-1} = (x - \omega)(x - \omega^2) \dots (x - \omega^{n-1})$

Putting $x = 9$ in both sides, we have

$(9-\omega)(9-\omega^2)(9-\omega^3) \ldots (9-\omega^{n-1}) = \frac{9^n - 1}{8}$

79. **b)** **20**

$\left(1+\frac{1}{1}\right)^n = 64$

$\Rightarrow$ $2^n = 2^6$

$\therefore$ $n = 6$

General term $T_{r+1} = {}^nC_r (x)^{n-r}\left(\frac{1}{x}\right)^r$

$= {}^6C_r x^{6-2r}$

For independent of x,

Let $6 - 2r = 0$

$\therefore$ $r = 3$

$\therefore$ $T_{3+1} = 6C_3 x^0$

$= {}^6C_3$

$= 20$

80. **b)** **2**

$\because$ $|z_1 - z_2| = |z_1 - (z_2 - 3 - 4i) - (3 + 4i)|$

$\geq |z_1| - |z_1 - 3 - 4i| - |3 + 4i|$

$= 12 - 5 - 5 = 2$

$\therefore$ $|z_1 - z_2| \geq 2.$

81. **c)** $\frac{2^{n-1}}{n!}$

$$\frac{1}{n!}\left\{\frac{n!}{1!(n-1)} + \frac{n!}{3!(n-3)!} + \frac{n!}{5!(n-5)!} + \ldots + + \frac{n!}{(n-1)!\,1!}\right\}$$

$= \frac{1}{n!}\{{}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots + {}^nC_{n-1}\}$

$= \frac{2^{n-1}}{n!}$

82. **a)** **3 – (i/2)**

$x = \left(\frac{1+i}{2}\right)$

$\Rightarrow$ $2x - 1 = i \Rightarrow 4x^2 + 1 - 4x = -1$

$\Rightarrow$ $2x^2 - 2x + 1 = 0$

Since

$2x^4 - 2x^2 + x + 3$

$= (2x^2 - 2x + 1)(x^2 + x) + (3 - x^2)$

$= 0 + 3 - \left(\frac{1+i}{2}\right)^2 = 3 - (i/2)$

83. **d)** **32**

$T_{r+1} = {}^{124}C_r (3^{1/2})^{124-r}(5^{1/4})^r$

$= {}^{124}C_r 3^{(124-r)/2} . 5^{r/4}$

For integer terms $r = 0, 4, 8, 12, \ldots, 124$

$\therefore$ Number = 32

84. **a)** **0**

Since, $z = -1 + i\sqrt{3}$

$= 2\left(\frac{-1+i\sqrt{3}}{2}\right)$

$= 2\omega$ (cube root of unity)

$\therefore$ $z^{2n} + 2^n z^n + 2^{2n} = 2^{2n}\omega^{2n} + 2^n . 2^n\omega^n + 2^{2n}$

$= 2^{2n}(\omega^{2n} + \omega^n + 1)$

Let $n = 3m + 1$, $m \in I$

$= 2^{2n}(\omega^2 + \omega + 1) = 0$

85. **c)** $\frac{1}{2} n a_n$

Let $P = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$ … (i)

Replacing r by $n - r$ in equation (i), then

$P = \sum_{r=0}^{n} \frac{(n-r)}{{}^nC_{n-r}}$

$= \sum_{r=0}^{n} \frac{(n-r)}{{}^nC_r}$ … (ii)

$(\because {}^nC_r = {}^nC_{n-r})$

Adding equation (i) and (ii), then

$2P = n\sum_{r=0}^{n}\frac{1}{{}^nC_r} = na_n \qquad \left(\because a_n = \sum_{r=0}^{n}\frac{1}{{}^nC_r}\right)$

$\therefore$ $P = \frac{1}{2} na_n$

86. **d)** i

$\sum_{k=1}^{10}\left(\sin\frac{2\pi k}{11} - i\cos\frac{2\pi k}{11}\right)$

$= -i\sum_{k=1}^{10}\left(\cos\frac{2\pi k}{11} + i\sin\frac{2\pi k}{11}\right)$

$$= -i\sum_{k=1}^{10} e^{i(2\pi k/11)}$$

$$= -i\left\{-1+\sum_{k=0}^{10} e^{i(2\pi k/11)}\right\}$$

= $-i(-1$ + sum of 11, 11th roots of unit)

= $-i(-1 + 0) = i$

87. **a)** $^{n+1}C_{m+1}$

$= {}^{m}C_m + {}^{m+1}C_m + \ldots + {}^{n}C_m$

$= {}^{m+1}C_{m+1} + {}^{m+1}C_m + \ldots + {}^{n}C_m$

$= {}^{m+2}C_{m+1} + \ldots + {}^{n}C_m = {}^{n+1}C_{m+1}$

88. **a)** $\dfrac{n}{3^n - 1}$

Let P = $\displaystyle\sum_{i=0}^{n-1}\frac{\alpha_i}{(3-\alpha_i)} = -\sum_{i=0}^{n-1}\frac{(3-\alpha_i)-3}{(3-\alpha_i)}$

$$= 3\sum_{i=0}^{n-1}\frac{1}{(3-\alpha_i)} - \sum_{i=0}^{n-1} 1$$

$\because \quad z^n - 1 = \displaystyle\sum_{i=0}^{n-1}(z-\alpha_i)$

$\therefore \quad \ln(z^n - 1) = \displaystyle\sum_{i=0}^{n-1}\ln(z-\alpha_i)$

Differentiating both sides w.r.t. z, then

$$\frac{nz^{n-1}}{(z^n-1)} = \sum_{i=0}^{n-1}\frac{1}{(z-\alpha_i)}$$

and put $z = 3$, then

$$\frac{n.3^{n-1}}{(3^n-1)} = \sum_{i=0}^{n-1}\frac{1}{(3-\alpha_i)}$$

From eq. (i) $3\left(\dfrac{n\ .\ 3^{n-1}}{3^{n-1}}\right) - n$

$$= \frac{n\ 3^n}{3^n-1} - n = \frac{n}{3^n-1}$$

89. **c)** $z_1 . z_2$ is unimodular

$$|z_1 + z_2| = \left|\frac{1}{z_1}+\frac{1}{z_2}\right|$$

$$= \left|\frac{z_1+z_2}{z_1z_2}\right|$$

$\Rightarrow \quad |z_1 + z_2|\left[1-\dfrac{1}{|z_1z_2|}\right] = 0$

$\therefore \quad |z_1 z_2| = 1 \quad (\because z_1 \neq -z_2)$

90. **b)** $\left(\dfrac{n(n+1)}{2}\right)^2 + n$

$2(1 + \omega)(1 + \omega^2) + 3(2\omega + 1)(2\omega^2 + 1)$
$+ 4(3\omega + 1)(2\omega^2 + 1) + \ldots + \ldots$
$+ (n + 1)(n\omega + 1)(n\omega^2 + 1)$

nth term $T_n = (n + 1)(n\omega + 1)(n\omega^2 + 1)$

$= (n + 1)(n^2 - n + 1)$

$= n^3 + 1$

$\therefore \quad Sn = \Sigma T_n = \Sigma n^3 + \Sigma 1 = \left\{\dfrac{n(n+1)}{2}\right\}^2 + n$