PHYSICS SOLUTIONS

1. (D)
Net gravitational field inside a shell is zero. Hence, net force on the particle will be zero i.e., the particle stays at rest in its original position.

2. (B)
The gravitational potential at the centre of the ring is

$$V = -\frac{GM}{R}$$

irrespective of the distribution of mass

Since, $W_{c\to\infty} = m\left(V_\infty - V_C\right)$

$\Rightarrow W_{c\to\infty} = m\left[0 - \left(-\frac{GM}{R}\right)\right]$

$\Rightarrow W_{c\to\infty} = \frac{GMm}{R}$

3. (B)
angular momentum conservation :- mvr= constant

4. (D)
Since, $T = \frac{2\pi r}{v}$

$\Rightarrow \frac{2\pi R}{v_0} = 2$ and $\frac{2\pi R'}{v_0'} = 16$

This is possible when $R' = 4R$ and $v_0' = \frac{v_0}{2}$

5. (D)
Distance of second satellite from the centre of earth becomes four times. Since , according to Kepler's Laws we have

$T^2 \propto r^3$

$\Rightarrow \quad T \propto r^{3/2}$

So, time period of second satellite must become $\left(4\right)^{3/2} = 8$ times.

Hence, $T' = 8T = \left(8\right)\left(90\right) = 720\,\text{min}$

6. (C)
$\Delta E = -\frac{GMm}{2\left(3R\right)} - \left(-\frac{GMm}{R}\right) = \frac{5GMm}{6R}$

7. (C)
$V = -\int_0^R \frac{G\left(2\pi x\right)dx\,\sigma}{\sqrt{x^2+r^2}} = -2\pi\sigma G\left[\sqrt{R^2+r^2} - r\right]$

8. (D)
$\frac{mv^2}{r} = \frac{GMm}{r^2} \Rightarrow V \propto \frac{1}{\sqrt{r}}$

9. (B)

$$P.E = -\frac{GMm}{(R+h)}$$

$$= -\frac{GMm}{R}\left(1+\frac{h}{R}\right)^{-1}$$

$$= -\frac{GMm}{R}+\frac{GMm}{R^2}h$$

$$= -\frac{GMm}{R}+mgh$$

10. (B)

$$T^2 \propto R^3 \quad \Rightarrow \quad T_2^2 = T_1^2\left(\frac{R_2}{R_1}\right)^3$$

$$T_2^2 = (365)^2\left[\frac{1}{2}\right]^3$$

$$T_2 = 365\frac{1}{2\sqrt{2}}$$

= 129 days

11.

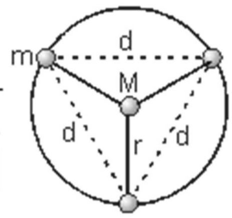
Solution:-

The distance between the orbiting stars is $d = 2r\cos 30^0 = \sqrt{3}r$.

The net inward force on orbiting stars is

$$\frac{Gm^2}{d^2}\cos 30° + \frac{GMm}{r^2} + \frac{Gm^2}{d^2}\cos 30° = \frac{mv^2}{r}$$

$$\therefore \quad G\left[\frac{m}{\sqrt{3}}+M\right] = \frac{4\pi^2 r^3}{T^2} \quad or \quad T = 2p\sqrt{\frac{r^3}{G\left(M+\frac{m}{\sqrt{3}}\right)}}$$



12.

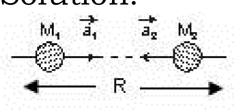
Solution:-

In the circular motion around the Earth, the centripetal force on the satellite is a gravitational force.

Therefore, $V^2 = GM/R$, where M is the mass of the Earth, R is the radius of the orbit of satellite and G is the universal gravitational constant. Therefore, the kinetic energy increases with the decrease in the radius of the orbit. The gravitational potential energy is negative and decreases with the decrease in radius.

13.

Solution:-



$a_1 = \frac{GM_1M_2}{R^2}/M_1,\ a_2 = \frac{GM_1M_2}{R^2}/M_2$

acceleration of M1 w.r.t. M2

$a_{rel} = a_1 + a_2$

$= \frac{G(M_1+M_2)}{R^2} = \frac{GM}{R^2}$

14.

Solution:-

The maximum possible velocity for satellite to be in elliptic orbit should be such that satellite should not escape.

$\therefore\ V_{escape} = V_{orbital}$

$\therefore$ required value of $\eta = \sqrt{2}$

15.

Solution:

$E = \frac{-dv}{dx}$ here $dx = dr\sin\theta$

Solution:

$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R_1}$

$-\frac{GM}{R} + \frac{v^2}{2} = -\frac{GM}{R_1}$

$\frac{v_e^2}{2} + \frac{v^2}{2} = -v_0^2$

$\frac{v^2}{2} = \frac{v_e^2}{2} - v_0^2 \Rightarrow V = \sqrt{V_e^2 - 2V_0^2}$

16.

Solution:

$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R_1}$

$-\frac{GM}{R} + \frac{v^2}{2} = -\frac{GM}{R_1}$

$\frac{v_e^2}{2} + \frac{v^2}{2} = -v_0^2$

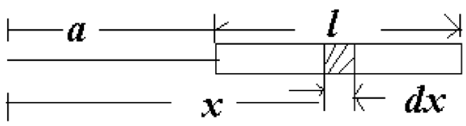
$\frac{v^2}{2} = \frac{v_e^2}{2} - v_0^2 \Rightarrow V = \sqrt{V_e^2 - 2V_0^2}$

17.

Sol:-

$dm = \frac{m}{l}dx$

$U = -GMm\int_a^{a+l}\frac{dx}{x} = -Gm\frac{M}{l}lh(1+\frac{l}{a})$



18.

Sol:- Conceptual

19.

Solution:-

In given stars system

$$\frac{Mv^2}{R} = \frac{GMM}{(2R)^2} \Rightarrow v = \sqrt{\frac{GM}{4R}}$$

To escape a particle of mass m from the centre of mass of the system,

$$\frac{1}{2}mv_c^2 \geq \frac{2GMm}{R} \quad \Rightarrow \quad v_c \geq 2\sqrt{\frac{GM}{R}} \quad \Rightarrow \quad v_c \geq 4V$$

20.

Solution:-

$$-\frac{GMm}{2(R+h)} = -\frac{GMm}{(R+2R)}$$

So h = R/2

21.

Solution:- (d)

Field vector and displacement vectors are perpendicular to work done zero.

22. (B)

$$g^1 = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

$$0.01\ g = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

$$1+\frac{h}{R} = 10$$

H = 9R

23. (A)

$$\frac{3}{5}g = g - R\omega^2$$

$$R\omega^2 = \frac{2}{5}g$$

$$\omega = \sqrt{\frac{2}{5}\frac{g}{R}}$$

24. (B)

$$\Delta U = -\frac{GMm}{2R} - \left(-\frac{GMm}{R}\right)$$

$$= \frac{GMm}{2R} = \frac{1}{2}mgR$$

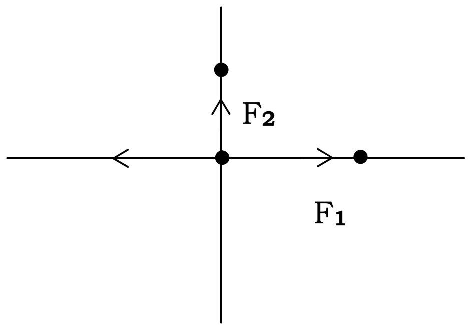
25. (A)

$$mR\omega^2 = \frac{GMm}{R^n}$$

$$\omega^2 \propto \frac{1}{R^{n+1}}$$

$$T \propto R^{\left(\frac{n+1}{2}\right)}$$

26. (A)



$$f_{net} = \sqrt{f_1^2 + f_2^2}$$

$$= \sqrt{2}\frac{GMm}{(0.2)^2}$$

$$= \frac{6.67\times10^{-11}}{4\times10^{-2}}\left(\hat{i}+\hat{j}\right)$$

$$= 1.67\times10^{-9}\left(\hat{i}+\hat{j}\right)$$

27. (A)

Force inside

$F \propto R$

So $\frac{f_1}{f_2} = \frac{r_1}{r_2}$

Outside $f \propto \frac{1}{r_2}$

28. (D)

E_{net} = 0 in side the smaller shell

So point D $\Rightarrow$ F = 0

29. (D)

m $\leftarrow x \rightarrow$ $\leftarrow (d - x) \rightarrow$ M

•   •   •

p

$\leftarrow$ d $\rightarrow$

For $\vec{E} = 0$

$$\frac{Gm}{x^2} = \frac{GM}{(d-x)^2}$$

$$\frac{m}{M} = \left(\frac{x}{d-x}\right)^2$$

$$\sqrt{\frac{m}{M}} = \frac{x}{d-x} \quad \Rightarrow x = \frac{\sqrt{m}d}{\sqrt{M}+\sqrt{m}}$$

Potential = $-\frac{GM}{x} - \frac{GM}{(d-x)}$

30. (B)

For satellite

$$V \propto \frac{1}{\sqrt{r}}$$