LAKSHYA ADVANCED UNIT TEST (LAUT)

00 – 00		Q. Booklet Serial No: 030515	
Test No : 2161	3 Hrs.		

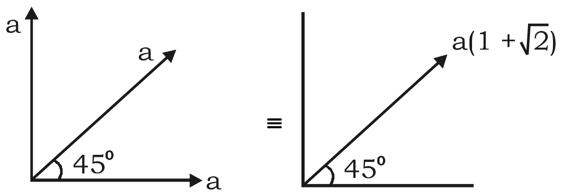
Hints & Solutions

PART A - PHYSICS

SECTION I - MULTIPLE ANSWER CORRECT TYPE

1. **a) the resultant amplitude is $\left(1+\sqrt{2}\right)a$**
   c) the energy associated with the resulting motion is $\left(3+2\sqrt{2}\right)$ times the energy associated with any single motion

   The resultant amplitude can be found by resultant of the amplitudes as shown



$\Rightarrow$ Resultant motion has amplitude $a\left(1+\sqrt{2}\right)$ and at a phase of 45° with first.

Also,

Energy of single motion $\propto a^2$

and

Energy of resultant motion $\propto \left(\sqrt{2}+1\right)^2 a^2$

$$\propto \left(3+2\sqrt{2}\right)a^2$$

2. **c) $V_p < V_{av} < V_{rms}$**
   d) the average kinetic energy of a molecule is $\frac{3}{4}mv_p^2$

   The molecules have speeds ranging from very low values at high values.

$$V_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{\rho}}$$

$$V_{av} = \sqrt{\frac{8P}{\pi\rho}}$$

$$V_P = \sqrt{\frac{2P}{\rho}} \Rightarrow V_P < V_{av} < V_{rms}$$

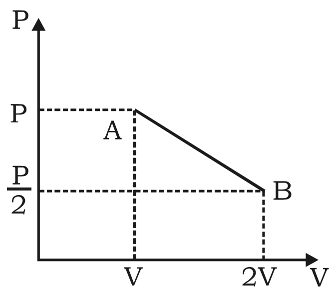
Also,

$$K.E._{av} = \frac{3}{2}PV$$

$$= \frac{3}{2}\left(\frac{1}{2}mV_P^2\right)$$

$$= \frac{3}{4}mV_P^2$$

3. **a) the work done by the gas in the process A to B exceeds the work that would be done by it if the system were taken from A to B along an isotherm.**
   b) In the T - V diagram, the path AB becomes a part of a parabola
   d) In going form A to B the temperature T of the gas first increases to a maximum value and then decreases



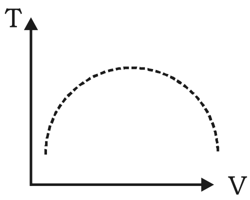
Since the graph is straight line,

$P \propto -V$

$\Rightarrow$ $V^2 \propto -T$ $\Rightarrow$ Parabola

and $P^2 \propto -T$ $\Rightarrow$ Parabola

$\Rightarrow$ V – T curve is a parabola.



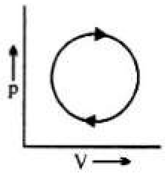
⇒ T first increases and then decreases

→ For isothermal curve, slope $\frac{dP}{dV} = -\frac{P}{V}$

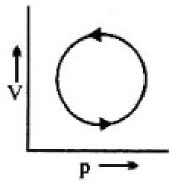
⇒ isothermal curve will lie below AB.

⇒ work done by AB path > work done in isotherm.

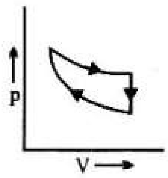
4. a)



b)



c)



For a cyclic process, $\Delta V = 0$

∴ $\Delta Q = \Delta W$

→ For heat to absorbed, the net work done should be by the gas

→ Graph a, b and c have +ve work done by gas.

5. **a) the maximum intensity of radiation will be near the frequency $2v_0$**

c) the total energy emitted will become 16 times

Energy radiated $\propto T^4$

∴ If temperature is doubled.

Energy radiated becomes 16 times.

By Wein's displacement law

$dT = \text{constant}$

or $\frac{T}{V} = \text{constant}$

∴ If temperature is doubled, frequency is also doubled.

6. **a) 80 Hz**

b) 240 Hz

d) 400 Hz

For a closed organ pipe

$l = (2n+1)\frac{d}{4} \Rightarrow d = \frac{4l}{(2n+1)}$

$v = dv$

⇒ $320 = \frac{4(1)}{(2n+1)}v$

⇒ $v = 80(2n + 1)$

⇒ $v = 80, 240, 400, 560$

7. **a) travelling with a velocity of 30 m/s in the negative x-direction**

b) of wavelength πm

c) of frequency $30/\pi$ hertz

d) of amplitude 10^{-4} **m travelling along the negative x-direction**

$y = 10^{-4}\sin(60t + 2x)$

Comparing this with equation of a wave travelling in +ve x direction

$y = A\sin(\omega t - kx)$

⇒ The wave is travelling in –ve X direction with speed

$V = \frac{\omega}{k} = \frac{60}{2} = 30 \text{ m/s}$

⇒ wavelength, $d = \frac{2\pi}{k} = \frac{2\pi}{2} = \pi \text{ m}$

⇒ frequency, $v = \frac{\omega}{2\pi} = \frac{60}{2\pi}$

$= \frac{30}{\pi}$ hertz

→ Amplitude, $A = 10^{-4}$ m

8. **a) the process during the path A → B is isothermal**

b) heat flows out of the gas during the path $B \rightarrow C \rightarrow D$

d) positive work is done by the gas in the cycle ABCDA

For process $A \rightarrow B$, since PV = constant, the excess is isothermal

For path $B \rightarrow C \rightarrow D$ work done by gas is – ve and T decreases i.e. ΔU is also – ve.

$\Delta Q = \Delta U + \Delta W$ is also – ve

heat flows out of the systems

Work done during path $A \rightarrow B \rightarrow C$ is area between the curve ABC $\neq 0$

Work done for cycle ABCDA is area between the ABCDA.

$\Rightarrow \Delta W = +ve$

SECTION II - PARAGRAPH TYPE

9. **d) The force due to gravity and the force due to viscosity of the liquid**

Buoyancy force is the resultant of difference due to pressure.

Therefore, the forces acting other than buoyancy are force due to gravity and due to viscosity of liquid.

10. **b)** $T_0\left(\dfrac{P_0 + \rho_l g(H-y)}{P_0 + \rho_l gH}\right)^{2/5}$

Heat exchange, $\Delta Q = 0$

$\Rightarrow PV^{\upsilon} = k$

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{2/5}$$

$$\Rightarrow T = T_0\left\{\frac{P_0 + \rho g(H-y)}{P_0 + \rho gH}\right\}^{2/5}$$

11. **b)** $\dfrac{\rho_l nR_g T_0}{(P_0 + \rho_l gH)^{2/5}\left[P_0 + \rho_l g(H-y)\right]^{3/5}}$

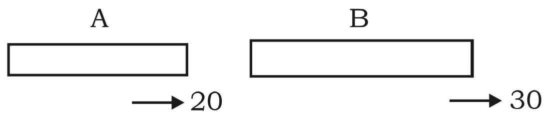
Boyant force = weight of the liquid displaced

$F_B = \rho Vg$

$PV^{\upsilon} = k$

$$\Rightarrow V' = \left\{\frac{P_0 + \rho gH}{P_0 + \rho g(H-y)}\right\}^{3/5} \frac{nRT_0}{(P + \rho gH)}$$

$$F_B = \rho nRT_0\left\{\frac{1}{(P_0 + \rho gH)^{2/3}\left(P_0 + \rho g(H-y)\right)^{3/5}}\right\}$$



12. **b) 360 m/s for passengers in A and 310 m/s for passengers in B**

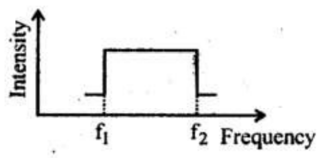
Velocity of sound w.r.t. A,

$V_{S,A} = 340 + 20 = 360$ m/s

Velocity of sound w.r.t. B,

$V_{S,B} = 340 - 30 = 310$ m/s

13. **a)**



$$V = \left(\frac{V + V_0}{V + V_s}\right)^{\upsilon_0}$$

For passenger of train A, $V_0 = V_s$

$\Rightarrow V = V_0$

$\Rightarrow$ Apparent frequency is unchanged

14. **a) 310 Hz**

For passenger of train B

$$V = \upsilon_0\left\{\frac{V - V_0}{V - V_s}\right\} = \upsilon_0\left\{\frac{340-30}{340-20}\right\}$$

$$\Rightarrow V = \upsilon_0\left\{\frac{31}{32}\right\}$$

$$\therefore \Delta V = \Delta\upsilon_0\left\{\frac{31}{32}\right\} = \frac{31}{32}(320)$$

$\Rightarrow$ Speed of apparent frequency = 310 Hz

SECTION III - MATRIX MATCH TYPE

1. **A-P,S, B-Q,R, C-P, D-Q,R**

A) Comparing $V = C_1\sqrt{C_2 - x^2}$ with

$V = \omega\sqrt{A^2 - x^2}$, the particle is executing SHM

B) $V = -kx$

$\Rightarrow$ The object is moving downwards the origin with decreasing speed but cannot cross the origin (stops)

$\Rightarrow$ K.E. of object decreases and it does not change its direction.

C) For this system, the mean position changes (due to psuedo force) but the restoring force is still proportional to '– x'.

$\Rightarrow$ Object executes SHM

D) Escape velocity, $V_e = \sqrt{\frac{2GMe}{Re}}$

$\because$ Velocity given, $v > V_e$, the particle does not come back i.e. doesn't change direction And since it goes against the force of gravitation, its K.E. keeps decreasing.

2. **A-Q, B-P,S, C-S, D-Q,R**

A) For Path Jk, V = constant,

$\Rightarrow \Delta w = 0$

P decreases $\Rightarrow$ T decreases

$\Rightarrow \Delta U < 0$

$\Delta Q = \Delta U + \Delta w$

$\Rightarrow \Delta Q < 0$

B) For path KL, V increases

$\Rightarrow \Delta w = P\Delta V > 0$

V increases, $\Rightarrow$ T increases

$\Rightarrow \Delta U > 0$

$\Delta Q = \Delta U + \Delta w$

$\Rightarrow \Delta Q > 0$

C) For path LM, V = constant $\Rightarrow \Delta w = 0$

P increases $\Rightarrow$ T increases $\Rightarrow \Delta U > 0$

$\Delta Q = \Delta U + \Delta w$

$\Rightarrow \Delta Q > 0$

D) For path MJ, V decreases $\Rightarrow \Delta w < 0$

$\Delta(PV) = (30 - 40) < 0$

$\Rightarrow \Delta T < 0 \Rightarrow \Delta U < 0$

$\Delta Q = \Delta U + \Delta w$

$\Rightarrow \Delta Q < 0$

SECTION IV - INTEGER TYPE

1. **5**

Initially $\frac{1}{2}KA_0^2 = \frac{1}{2}MW^2A^2 = \frac{1}{2}MV_0^2$

$\Rightarrow$ Velocity of block at mean position

$V_0 = \sqrt{\frac{100(10^{-2})}{1}}$

$\Rightarrow V_0 = 1$ m/s

When a block of 3 kg is placed over it, we can conserve the momentum.

$1(V_0) = 4(V) \Rightarrow V = 0.25$ m/s

$\therefore$ For new amplitude, A

$\frac{1}{2}KA^2 = \frac{1}{2}M'V^2$

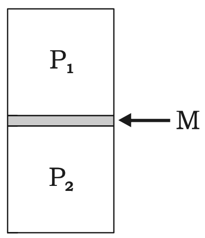
$\Rightarrow 100\,A^2 = 4\left(\frac{1}{16}\right)$

$\Rightarrow A = \sqrt{\frac{1}{400}} = \frac{1}{20}$ m

$\Rightarrow A = 5$ cm

2. **4**

Area = A



For the piston to be in equation

$P_1 + \frac{mg}{A} = P_2$ - (i) at 300 K

and

$P_1' + \frac{mg}{A} + \frac{F}{A} = P_2'$ - (ii) at 500 K

Where F is the extra force required.

Since, n and T are same for both the parts,

$P_1V_1 = P_2V_2$

$\Rightarrow P_2 = 2P_1$ - (iii)

$\because$ equation does not change at 500 K as well

$P_2' = 2P_1'$ - (iv)

$\Rightarrow$ Mg = P1A - (v)

Also, $\because \frac{V}{T}$ for both parts is constant on increasing T.

$\frac{P_1}{T_1} = \frac{P_1'}{T_2}$ and $\frac{P_2}{T_1} = \frac{P_2'}{T_2}$

$\Rightarrow \frac{5}{3}P_1 = P_1'$ and $\frac{5}{3}P_2 = P_2'$ - (vi)

Using (ii), (iii), (iv) and (vi)

$$\frac{5}{3}P_1 + \frac{Mg}{A} + \frac{F}{A} = \frac{5}{3}\left(P_1 + \frac{Mg}{A}\right)$$

$\Rightarrow F = \frac{2}{3}Mg = \frac{2}{3}P_1A$ (using v)

$\Rightarrow F = \frac{2}{3}\left(\frac{nRT}{V}\right)A$

$= \frac{2}{3}\frac{(1)(8.3)(300)(0.1)}{0.1}$

$= 1660$

$\Rightarrow F = 1660 = 415\ n$

$n = 4$

3. **1**

$\Delta Q = \Delta U + \Delta W$

$\Delta Q = (170 + 830)$ J

$= 1000$ J

$= 1$ kJ

4. **9**

$\frac{dT}{dt} = bA(\Delta T) = bA(T - T_0)$

$\Rightarrow 4 = bA(30 - 20)$

$\Rightarrow bA = 0.4$

$\therefore$ at 25^oC

$\frac{dT}{dt} = bA(25 - 20) = -0.7\ (5)$

$= 2^oC/sec$

5. **3**

The amount by heat taken out = heat givenby Al + heat given by Cu

$\Rightarrow Q = Q_1 + Q_2$

$$= \left\{K_1A\frac{(\Delta T)}{l} + K_2A\frac{(\Delta T)}{l}\right\}\Delta t$$

$\left(\because \frac{dQ}{dt} = KA\frac{\Delta T}{l}\right)$

$= (200 + 400)\ \frac{(0.2\times10^{-4})}{20\times10^{-2}}(40) \times 60$

$= 144 = (4k)^2$

$\Rightarrow 9 = K^2$

$\Rightarrow K = 3$

6. **4**

$KE = \Delta Q$

$\Rightarrow \frac{1}{2}m_1v^2 = m_2S(\Delta T)$

$\Rightarrow \frac{1}{2} \times 21 \times 16 = m(4.2)(10)$

$\Rightarrow m = 4$ years

7. **1**

Since the motocyclist doesn't hear any beats Apparent frequency of police horn to motocyclist = Apparent frequency of siren

$\Rightarrow 176\left\{\frac{V - V_m}{V - 20}\right\} = 165\left\{\frac{V + V_m}{V}\right\}$

$\Rightarrow 16\left\{\frac{V - V_m}{V - 20}\right\} = 15\left\{\frac{V + V_m}{V}\right\}$

$\Rightarrow V_m = 20$

$\therefore \frac{V_M}{V_P} = 1$

8. **6**

Let the final temperature is T.

$m_wS_w\Delta T = m_iL + (m_w + m_i)\ S_w\Delta T$

$\Rightarrow 40\ (42)(30) = 10(378) + 50(2.2)(T - 0)$

$5040 = 3780 + 210\ T$

$\Rightarrow T = \frac{1260}{210} = 6^oC$

PART C - MATHS

SECTION I - MULTIPLE ANSWER CORRECT TYPE

1. **b) *a, b, c* are in G.P.**

 d) $(x - a)$ is a factor of $ax^2 + 2bx + c$

 Given that

$$\begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix} = 0$$

 Operating $C_3 \to C_3 - C_1 \alpha - C_2$, we get

$$\begin{vmatrix} a & b & 0 \\ b & c & 0 \\ a\alpha + b & b\alpha + c & -(a\alpha^2 + b\alpha + b\alpha + c) \end{vmatrix} = 0$$

$$\Rightarrow (a\alpha^2 + 2b\alpha + c)\begin{vmatrix} a & b & 0 \\ b & c & 0 \\ a\alpha + b & b\alpha + c & 1 \end{vmatrix} = 0$$

 $\Rightarrow (ac - b^2)(a\alpha^2 + 2b\alpha + c) = 0$

 $\Rightarrow$ either $ac - b^2 = 0$ or $a\alpha^2 + 2b\alpha + c = 0$

 $\Rightarrow$ either a,b,c are in G.P. or $(x - a)$ is a factor of $ax^2 + 2bx + c$

 Hence (b) and (d) are the correct answer.

2. **a) not less than P(A) + P(B) – 1**

 b) not greater than P(A) + P(B)

 c) equal to $P(A) + P(B) - P(A \cup B)$

 We know that

 $P(A \cup B) \le 1$

 $\Rightarrow -P(A \cup B) \ge -1$

 $\therefore P(A \cap B) \ge P(A) + P(B) - 1$ [Using Eqs. (1) and (2)

 Therfore, option (a) is correct. Again,

 $P(A \cup B) \ge 0$

 $\Rightarrow -P(A \cup B) \le 0$

 $\therefore P(A \cap B) \le P(A) + P(B)$ [Using Eqs. (1) and (2)

 Therfore, option (b) is also correct.

 From Eq. (1), option (c) is correct and (d) is not correct.

3. **b) E and F^C (the complement of the even F) are independent**

 c) E^C and F^C are independent

 d) $P(E|F) + P(E^C|F) = 1$

 $P(E \cap F) = (E)\,P(F)$

 Now,

 $P(E \cap F) = P(E) - P(E \cap F) = P(E)\,[1 - P(F)]$

 $= p(E)\,P(F^C)$

 and

 $P(E^C \cap F^C) = 1 - P(E \cup F) = 1 - [P(E) + P(F) - P(E Ç F)$

 $= [1 - P(E)]\,[1 - P(F)] = P(E^C)\,P(F^C)$

 Also,

 $P(E/F) = P(E)$ and $P(E^C/F^C) = P(E^C)$

 $\Rightarrow P(E/F) + P(E^C/F^C) = 1$

4. **a) $P(A/B) \ge \dfrac{P(A) + P(B) - 1}{P(B)}$ $(P(B) \ne 0)$ is always true**

 c) $P(A \cup B) = 1 - P(\bar{A})P(\bar{B})$, if A and B are independent

 a. For any two events A and B,

 $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

 Now we know that

 $P(A \cup B) \le 1$

 $\Rightarrow P(A) + P(B) - P(A \cap B) \le 1$

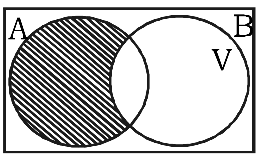
 $\Rightarrow P(A \cap B) \ge P(A) + P(B) - 1$

 $\Rightarrow \dfrac{P(A \cap B)}{P(B)} \ge \dfrac{P(A) + P(B) - 1}{P(B)}$

 $\Rightarrow P(A/B) \ge \dfrac{P(A) + P(B) - 1}{P(B)}$

 Therfore, option (a) is correct.

 b. From Venn's diagram, we can clearly conclude that



 $P(A \cup \bar{B}) = P(A) - P(A \cap B)$

 Therfore, option (b) is incorrect

 c. $P(A \cap B) = P(A) + P(B) - P(A \cap B)$

 $= 1 - P(\bar{A}) + 1 - P(\bar{B}) - P(A)\,P(B)$

 [$\because$ A and B are independent events]

 $= 2 - P(\bar{A}) - P(\bar{B})[1 - P(\bar{A})][1 - P(\bar{B})]$

 $= 2 - P(\bar{A}) - P(\bar{B}) - 1 + P(\bar{A}) + P(\bar{B}) - P(\bar{A})\,P(\bar{B})$

$= 1 - P(\overline{A} \cap \overline{B})$

[Q if A and B are independent, $\overline{A}$ and $\overline{B}$ are also independent]

Therfore, option (c) is the correct statement.

d. For disjoint events,

$P(A \cap B) = P(A) + (P(B)$

Therfore, option (d) is he incorrect.

5. **a) P(E) = 1/3, P(F) = 1/4**

b) P(E) = 1/4, P(F) = 1/3

Let P(E) = x and P(F) = y. According to the equastion,

$P(E \cap F) = \frac{1}{12}$

As E and F are independent events, we have

$P(E \cap F) = P(E)\,P(F)$

$\Rightarrow \frac{1}{12} = xy$

Also, $P(\overline{E} \cap \overline{F}) = P(\overline{E \cup F})$

$= 1 - P(E \cup F)$

$\Rightarrow \frac{1}{2} = 1 - [P(E) + P(F) - P(E)\,P(F)]$

$\Rightarrow x + y = \frac{17}{12}$

Solving Eqs. (1) and (2), we get either $x = 1/3$ and $y = 1/4$ or $x = 1/4$ and $y = 1/3$.

Therfore, option (a) and (b) are correct.

6. **a) $|\omega_1| = 1$**

b) $|\omega_2| = 1$

c) $\text{Re}(\omega_1 \overline{\omega_2}) = 0$

We have,

$|z^1| = |z^2| = 1 \Rightarrow a^2 + b^2 = c^2 + d^2 = 1$

and $\text{Re}(z_1 = \overline{z_{n-2}}) = 0 \Rightarrow \text{Re}\{(a + ib)(c + id)\} = 0$

$ac + bd = 0$

Now from (1) and (2),

$a^2 + b^2 = 1 \Rightarrow a^2 + \frac{a^2c^2}{d^2} = 1 \Rightarrow a^2 = d^2$

Also,

$c^2 + d^2 = 1 \Rightarrow c^2 + \frac{a^2c^2}{d^2} = 1 \Rightarrow b^2 = c^2$

$|\omega_1| = \sqrt{a^2 + c^2} = \sqrt{a^2 + b^2} = 1$ [From (1) and (4)]

and

$|\omega_2| = \sqrt{a^2 + c^2} = \sqrt{a^2 + b^2} = 1$ [From (1) and (4)]

Further,

$\text{Re}(\omega_1 \overline{\omega_2}) = \text{Re}\{(a + ic)(b - id)\} = ab + cd$

$= ab + \left(-\frac{ac}{b}\right)^2$ [From (2)]

$= \frac{ab^2 - ac^2}{b} = 0$ [From (4)]

Also,

$\text{Im}(\omega_1 \overline{\omega_2})\ bc - ad = bc - a$

$\left(-\frac{ac}{b}\right) = \frac{(a^2 + b^2)c}{b} = \frac{c}{b} = \pm 1 \neq 0$

$\therefore\ |\omega_1| = 1,\ |\omega_2| = 1$ and $\text{Re}(\omega_1 \overline{\omega_2}) = 0$

7. **a) zero**

d) purely imaginary

Let $z_1 = a + ib$, $a > 0$ and $b \in R$; $z_2 = c + id$, $d < 0$, $c \in R$.

Given,

$\Rightarrow a^2 + b^2 = c^2 + d^2$

$\Rightarrow a^2 + c^2 = d^2 + b^2$

Now,

$$= \frac{[(a^2 - c^2) + (b^2 - d^2)] + i[(a - c)(b + d) - (a + c)(b - d)]}{(a - c)^2 + (b - d)^2}$$

8. **a) x**

b) x^2

c) x^3

d) x^4

Taking a, b, c commons from C_1, C_2, C_3 respectively and then multiplying from a, b, c in R_1, R_2, R_3 respectively.

$$\therefore\ \Delta = \begin{vmatrix} a^2 + x^2 & a^2 & a^2 \\ b^2 & b^2 + x^2 & b^2 \\ c^2 & c^2 & c^2 + x^2 \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, then

$$\begin{vmatrix} x^2 + a^2 + b^2 + c^2 & x^2 + a^2 + b^2 + c^2 & x^2 + a^2 + b^2 + c^2 \\ b^2 & b^2 + x^2 & b^2 \\ c^2 & c^2 & c^2 + x^2 \end{vmatrix}$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, then

$$\Delta = \begin{vmatrix} x^2+a^2+b^2+c^2 & 0 & 0 \\ b^2 & x^2 & 0 \\ c^2 & 0 & x^2 \end{vmatrix}$$

$$= x^4(x^2 + a^2b^2 + c^2)$$

SECTION II - PARAGRAPH TYPE

9. **a) 12**

Let the matrix be

$$\begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}$$

We have five entries as 1 and remaining four entries as 0. Since Matrix is symmetric, we must have even number of zeros for $i \neq j$. We have two cases.

(i) Two entries in diagonal are zero. We can select two places from three (in diagonal) in $^3C_1$ ways. Hence, the number of matrices is $^3C_2 \times {}^3C_1 = 9$.

(ii) If all the entries in the principal diagonal are 1, we have two '0' and one '1' in upper triangle. Hence, the number of matrices is 3. Therefore, total number of matrices is 12.

10. **b) at least 4 but less than 7**

$$\begin{bmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 1 \end{bmatrix}$$

Either $b = 0$ or $c = 0 \Rightarrow |A| \neq 0$

$\Rightarrow$ two matrices

$$A = \begin{bmatrix} 0 & a & b \\ a & 1 & c \\ b & c & 0 \end{bmatrix}$$

Either $a = 0$ or $c = 0 \Rightarrow |A| \neq 0$

$\Rightarrow$ two matrices

$$A = \begin{bmatrix} 1 & a & b \\ a & 0 & c \\ b & c & 0 \end{bmatrix}$$

Either $a = 0$ or $c = 0 \Rightarrow |A| \neq 0$

$\Rightarrow$ two matrices

$$A = \begin{bmatrix} 1 & a & b \\ a & 1 & c \\ b & c & 1 \end{bmatrix}$$

$a = b = 0 \Rightarrow |A| = 0$

$a = c = 0 \Rightarrow |A| = 0$

$b = c = 0 \Rightarrow |A| = 0$

Therefore, there will be only six matrices.

11. **b) more than 2**

The six matrices A for which $|A| = 0$ are:

$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (inconsistent)

$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ (inconsistent)

$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (infinite solutions)

$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (inconsistent)

$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (inconsistent)

$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ (inconsistent)

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ (inconsistent)

12. **a) 25/216**

$$P(X = 3) = \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\frac{1}{6} = \frac{25}{216}$$

13. **b) 25/36**

Give that

$P(X \le 2) = \frac{1}{6} + \frac{5}{6} \times \frac{1}{6} = \frac{11}{36}$

Hence, the required probability is $1 - (11/36) = 25/36$.

14. **d) 25/216**

For $X \ge 6$, the probabilty is

$$\frac{5^5}{6^6} + \frac{5^5}{6^7} + \ldots\infty = \frac{5^5}{6^6}\left(\frac{1}{1-5/6}\right) = \left(\frac{5}{6}\right)^5$$

For $X > 3$,

$$\frac{5^3}{6^4} + \frac{5^4}{6^5} + \frac{5^4}{6^5} + \ldots\infty = \left(\frac{5}{6}\right)^3$$

Hence, the conditional probability is

$$\frac{(5/6)^5}{(5/6)^3} = \frac{25}{36}$$

SECTION III - MATRIX MATCH TYPE

1. **A-Q, B-P, C-S, D-R**

A) $|A| = 2 \Rightarrow |2A^{-1}| = 2^3/|A| = 4$

B) $|adj(adj(2A))| = |2A|^4 = 2^{12}|A|^4 = 212/2^{12} = 1$

C) $(A + B)^2 = A^2 + B^2$

$\Rightarrow AB + BA = 0$

$\Rightarrow |AB| = |-BA| = -|BA| = -|AB|$

$\Rightarrow |AB| = 0$

$\Rightarrow |B| = 0$

D) Product ABC is not defined.

2. **A-Q, B-S, C-P, D-R**

A) $|z - 1| = |z - i|$

Hence it lies on the perpendicular bisector of the line joining (1, 0) and (0,1) which is a straight line passing through the origin.

B) $|z + \bar{z}| + |z - \bar{z}| = 2$

$\Rightarrow |x| + |y| = 1$

Hence, z lies on a square.

C) Let $xz = x + iy$. Then,

$|z + \bar{z}| + |z - \bar{z}|$

$\Rightarrow |2x| = |2iy|$

$\Rightarrow |x| = |y|$

$\Rightarrow x = \pm y$

Hence, the locus of z is a pair of straight lines.

D) Let $Z = 2/z$. Then,

$$|z| = \left|\frac{2}{z}\right| = \frac{2}{|z|} = \frac{2}{1} = 2$$

This shows that Z lies on a circle with center at the origin and radius 2 units.

SECTION IV - INTEGER TYPE

1. **2**

Here, $\left|\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}\right| = 1$

$\Rightarrow |z_1 - 2z_2| = |2 - z_1\bar{z}_2|$

$\Rightarrow |z_1 - 2z_2|^2 = |2 - z_1\bar{z}_2|$

$\Rightarrow (z_1 - 2z_2)\left(\overline{z_1 - 2z_2}\right) = \left(2 - z_1\bar{z}_2\right)\left(\overline{2 - z_1 z_2}\right)$

$\Rightarrow (z_1 - 2z_2)\left(\bar{z}_1 - 2\bar{z}_2\right) = \left(2 - z_1\bar{z}_2\right)\left(2 - \bar{z}_1 z_2\right)$

$\Rightarrow z_1\bar{z}_1 - 2z_1\bar{z}_2 - 2\bar{z}_1 z_2 + 4z_2\bar{z}_2$

$= 4 - 2\bar{z}_1 z_2 - 3z_1\bar{z}_2 + z_1\bar{z}_1 z_2\bar{z}_2$

$\Rightarrow |z_1|^2 + 4|z_2|^2 = 4 + |z_1|^2 . |z_2|^2$

$\Rightarrow \left(|z_1|^2 - 4\right).\left(1 - |z_2|^2\right) = 0$

But $|z_2| \ne 1$ (given)

$\therefore |z_1|^2 - 4 = 0$

or $|z_1| = 2$

2. **2**

Applying $C_1 \to C_1 + C_2 + C_3$, then

$$f(x) = \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$(\because a^2 + b^2 + c^2 + 2 = 0)$

Now, applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$f(x) = \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 0 & 1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix} = (1-x)^2$$

Hence, degree of $f(x) = 2$

3. **4**

$\because \begin{pmatrix}1 & 2\\2 & 3\end{pmatrix}^2 - \lambda\begin{pmatrix}1 & 2\\2 & 3\end{pmatrix} - \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} = \begin{pmatrix}0 & 0\\0 & 0\end{pmatrix}$

$\Rightarrow \begin{pmatrix}5 & 8\\8 & 13\end{pmatrix} - \begin{pmatrix}\lambda & 2\lambda\\2\lambda & 3\lambda\end{pmatrix} - \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} = \begin{pmatrix}0 & 0\\0 & 0\end{pmatrix}$

$\Rightarrow \begin{pmatrix}4-\lambda & 8-2\lambda\\8-2\lambda & 12-3\lambda\end{pmatrix} = \begin{pmatrix}0 & 0\\0 & 0\end{pmatrix}$

On comparing, we get

$4 - \lambda = 0,\ 8 - 2\lambda = 0,\ 8 - 2\lambda = 0,\ 12 - 2\lambda = 0$

$\therefore \quad \lambda = 4$

4. **6**

$A^3 = A\,.\,A^2 = A(5A - 7I)$ $\quad (\because A^2 = 5A - 7I)$

$= 5A^2 - 7A$

$= 5(5A - 7I) - 7A$

$= 25A - 35I - 7A$

$= 18A - 35I$

Now, $A^4 = A\,.\,A^3 = A(18A - 35I)$

$= 18A2 - 35A$

$= 18(5A - 7I) - 35A$

$= 55A - 126I$

Finally, $A^5 = A\,.\,A^4 = A(55A - 126I)$

$= 55A^2 - 126A$

$= 55(5A - 7I) - 126A$

$= 149\,A - 385I$

$= aA + bI$

$\therefore \quad a = 149$ and $b = -385$

$\therefore \quad \dfrac{3a + b - 2}{10} = 6$

5. **1**

Let E_1, E_2, E_3, E_4, E_5 and E_6 be the events of occurence of 1, 2, 3, 4, 5 anf 6 on the respectively, and let E be the event of getting a sum of numbers equal to 9.

$\therefore\ P(E_1) = \dfrac{1-k}{6};\ P(E_2) = \dfrac{1+2k}{6};\ P(E_3) = \dfrac{1-k}{6};$

$P(E_4) = \dfrac{1-k}{6};\ P(E_2) = \dfrac{1-2k}{6};\ P(E_2) = \dfrac{1+k}{6}$

and $\dfrac{1}{9} \le P(E) \le \dfrac{2}{9}$

Then, $E \equiv \{(3, 6), (6, 3), (4, 5), (5, 4)\}$

Hence,

$P(E) = P(E_3E_6) + P(E_6E_3) + P(E_4E_5) + P(E_5E_4)$

$= P(E_3)P(E_6) + P(E_6)P(E_3) + P(E_4)P(E_5) + P(E_5)P(E_4)$

$= 2P(E_3)P(E_6) + 2P(E_4)P(E_5)$

{Sincce E_1, E_2, E_3, E_4, E_5 and E_6 are independent}

$= 2\left(\dfrac{1-k}{6}\right)\left(\dfrac{1+k}{6}\right) + 2\left(\dfrac{1+k}{6}\right)\left(\dfrac{1-2k}{6}\right)$

$= \dfrac{1}{18}[2 - k - 2k^2]$

Since, $\dfrac{1}{9} \le P(E) \le \dfrac{2}{9}$

$\Rightarrow \dfrac{1}{9} \le \dfrac{1}{18}[2 - k - 2k] \le \dfrac{2}{9}$ ....(i)

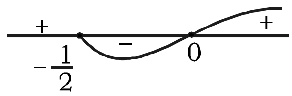
$\Rightarrow 2 \le 2 - k - 2k^2 \le 4$

$\therefore\ 2 - k - 2k^2 \ge 2$

$\Rightarrow -2 + k + 2k^2 \le -2$

$\Rightarrow 2k^2 + k \le 0$

$\Rightarrow k(2k + 1) \le 0$



$\therefore\ -\dfrac{1}{2} \le k \le 0$

Hence, integral value of k is 0 and for k = 0 from Eq. (i),

$$\frac{1}{9} < \frac{2}{9}$$

$\therefore$ Set of intgral values of k = {0}.

$\therefore$ Number of integral solutions of k is 1.

6. **6**

Total numbers of ways in which papers of 4 students, can be cheked by seven teachers $= 7^4$

Now, choosing two teachers out of 7 is $^7C_2 = 21$

The number of ways in which 4 papers can be checked by exactly two teachers = $2^4 - 2 = 14$

$\therefore$ Favourable ways = (21)(14)

$\therefore$ Required probability $= \frac{(21)(14)}{7^4}$

$= \frac{6}{49}$

$\therefore$ $49\text{ A} = 49 \frac{6}{49} = 6$

7. **6**

Let $A = x + iy$. Given,

$|A| = 1 \Rightarrow x^2 + y^2 = 1$

and

$|A + 1| = 1 \Rightarrow (x + 1)^2 + y^2 = 1$

$\Rightarrow x = -\frac{1}{2}$ and $y = \pm \frac{\sqrt{3}}{2}$

$\Rightarrow A = \omega$ or ω^2

$\Rightarrow (\omega)^n = (1 + \omega)n = (-\omega^2)^n$

Therfore, n must be even and divisible by 3.

8. **8**

Let $z = \cos x + i \sin x$, $x \in [0, 2\pi)$. Then,

$1 = \left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right|$

$= \frac{\left|z^2 + \bar{z}^2\right|}{|z|^2}$

$= |\cos 2x + i \sin 2x + \cos 2x - i \sin 2x|$

$= 2\ |\cos 2x|$

$\Rightarrow \cos 2x = \pm 1/2$

Now,

$\cos 2x = 1/2$

$\Rightarrow x_1 = \frac{\pi}{6}, x_2 = \frac{5\pi}{6}, x_3 = \frac{7\pi}{6}, x_4 = \frac{11\pi}{6}$

$\cos 2x = \frac{1}{2}$

$\Rightarrow x_5 = \frac{\pi}{3}, x_6 = \frac{2\pi}{3}, x_7 = \frac{4\pi}{3}, x_8 = \frac{5\pi}{3}$