LAKSHYA ADVANCED UNIT TEST (LAUT)

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Test No : 2171	3 Hrs.		

Hints & Solutions

PART A - PHYSICS

SECTION I - MULTIPLE ANSWER CORRECT TYPE

1. **a)** $|Q_1| > |Q_2|$
   d) **at a finite distance to the right of Q_2 the electric field is zero**
   Since the number of field lines is directly praportional to charge clearly, $|Q_1| > |Q_2|$
   ∴ there is a location to the right of Q_2 where the electric field is zero (null point)

2. **a)** $E_A^{inside} = 0$
   b) $Q_A > Q_B$
   c) $\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$
   d) $E_A^{on\ surface} < E_B^{on\ surface}$
   The referred generalization is all metals are conductors
   ∴ Their connected they are at the same potential
   ∴ $\frac{1}{4\pi \epsilon_0}\frac{Q_A}{R_A} = \frac{1}{4\pi \epsilon_0}\frac{Q_B}{R_B}$
   Since $R_B < R_A \Rightarrow Q_B < Q_A$
   ∴ $\frac{Q_A}{R_{A^2}} < \frac{Q_B}{R_{B^2}}$
   also $\frac{T_A}{T_B} = \frac{R_B}{R_A}$

3. **a)** **Periodic, for all values of z_0 satisfying $0 < z_0 < \infty$**
   c) **Approximately simple harmonic, provided $z_0 << R$**
   Clearly the motion is periodic and in approximetely simple harmonic if $z_0 << R$.

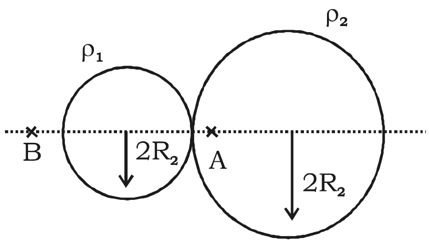
4. **a)** **electric field near A in the cavity = electric field near B in the cavity**
   d) **total electric field flux through the surface of the cavity is q/ε_0**
   The cavity's surface is an equipotential surface.
   ∴ potential at A = potential at B.
   Applying Gauss Law as suggested, the gaussian surface doesn't in clude the induced charge
   ∴ it is q/ε_0

5. **a)** **Total charge within $2R_0$ is q**
   b) **Total electrostatic energy for $r < R_0$ is zero**
   c) **At $r = R_0$ electric field is discontinuous**
   d) **There will be no charge anywhere except at $r = R_0$**
   Since the given potential inside the material is constant all the charge must be at the surface by which there is a spike from zero to $\frac{1}{4\pi \epsilon_0}\frac{Q}{R^2}$ in the electric field at r = R. A gaussian surface with radium 2R will still enclose only q charge.

6. **c)** **If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same**
   d) **The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is equal in value to $(V_B - V_A)$**
   Option (a) is false due to dimensional amalysis option (b) is false due that fact the flux is zero and field non uniform.
   (c) and (d) are true by implications.

7. **b)** $-\frac{32}{25}$

d) 4



If the location is A.

field at A =

$$\left(\frac{1}{4\pi\epsilon_0}\right)\frac{\rho_1\frac{4}{3}\pi R^3}{(2R)^2}-\frac{1}{4\pi\epsilon_0}\frac{\rho_2\left(\frac{4}{3}\pi R^3\right)}{R^2}=0$$

$\therefore \frac{\rho_1}{\rho_2}=\frac{4}{1}$

If the location is B.

Field at B =

$$\frac{1}{4\pi\varepsilon_0}\left(\frac{\rho_1\left(\frac{4}{3}\pi R^3\right)}{(2R)^2}\right)+\frac{1}{4\pi\varepsilon_0}\frac{\rho_2\left(\frac{4}{3}\pi(2R)^3\right)}{(5R)^2}=0$$

$\therefore \frac{\rho_1}{\rho_1}=-\frac{32}{25}$

8. **b)** $\mathbf{F_1 \neq F_2}$

c) $\mathbf{a_1 = a_2}$

As $F = qE$

$\therefore F_1 = 1E$ and $F_2 = 2E$

$\therefore F_1 \neq F_2$

$a_1 = \frac{F_1}{m_1} = \frac{E}{2}$

$a_2 = \frac{F_2}{m_2} = \frac{2E}{4} = \frac{E}{2}$

$\therefore a_1 = a_2$

9. **a)** $\boldsymbol{\sigma_1 = -\sigma_2}$ **in all cases**

c) $\boldsymbol{\sigma_1 = \sigma_2 = 0}$**, if equal charge is given to both the plates**

In all cases, $\sigma_1 = -\sigma_2$

However, when equal charges are given to both the plates, $\sigma_1 = \sigma_2 = 0$; because of induction.

10. **a)** $\mathbf{NC^{-1}\ m^2}$

d) **V–m**

$\phi = E(ds)\cos\theta = (NC^{-1})m^2$

$= N - m\,C^{-1}\,m = JC^{-1}\,m = V\text{-}m$

SECTION II - MATRIX MATCH TYPE

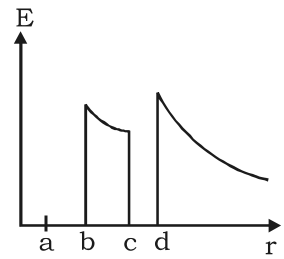
1. **A-S, B-S, C-P, D-S**

(i) $r < a$, $E = 0$, since $Q = 0$ (see figure)

(ii) $a < r < b$, $E = 0$ since $Q = 0$

(iii) $b < r < c$. since $Q = +2q$, $E = \frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$

(iv) $c < r < d$. $E = 0$ since $Q = 0$



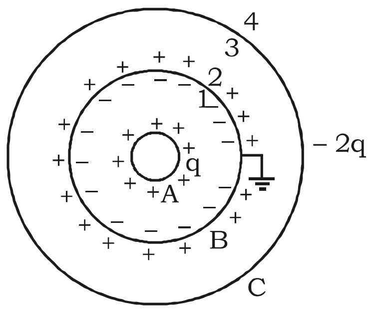
(v) $r > d$. $\frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$, since $Q = +6q$

2. **A-S, B-R,T C-Q, D-P**

Charge induced at the surface of ‘1’

$q_1 = -q$

Any point located on the surface of sphere 2 will have zero potential



$V_P = 0 = V_{A,out} + V_{B,surface} + V_{C,in}$

$$0 = k\frac{(q)}{2R}+k\frac{(-q+x)}{2R}+k\frac{(2q)}{3R}$$

Hence, $x = -\frac{4}{3}q$

Hence, charge appearing on surface 2 is $-\frac{4}{3}q$

and charge appearing on surface 3 = $\frac{4}{3}q$ and charge appearing on surface 4.

$$q_4 = 2q - \frac{4}{3}q = \frac{2q}{3}$$

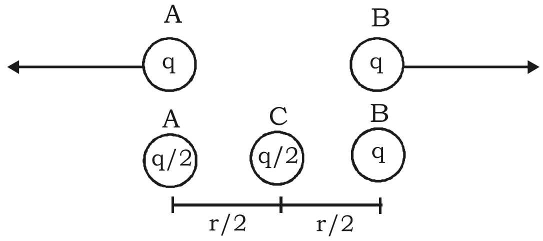
SECTION III - INTEGER TYPE

1. **2**

Let the charge on each sphere be q, then

$$F = k\frac{q^2}{r^2} = 2 \times 10^{-5}$$

$$\Rightarrow \frac{q^2}{r^2} = \frac{2\times10^{-5}}{9\times10^{9}}$$



Where sphere C is touched by sphere A, charge gets equally divided between the two.

$$FCA = k\frac{q^2/4}{r^2/4} = k\frac{q^2}{4}\times\frac{4}{r^2}$$

$$FCB = k\frac{q^2/2}{r^2/4} = k\frac{q^2}{2}\times\frac{4}{r^2} = 2k\frac{q^2}{r^2}$$

$$F_C = F_{CB} - F_{CA} = k\frac{q^2}{r^2} = 2 \times 10^{-5} \text{ N towards A}$$

2. **4**

$$T \sin\theta = F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2} \quad - \quad \text{(i)}$$

$$T \cos\theta = mg \quad - \quad \text{(ii)}$$

From (i) and (ii),

$$\tan\theta = \frac{q^2}{4\pi\varepsilon_0 a^2 mg}$$

$$\frac{a/2}{L} = \frac{q^2}{4\pi\varepsilon_0 a^2 mg}$$

($\because$ for small θ $\tan\theta \approx a/2L$)

$$\Rightarrow \frac{a^3}{L} = \frac{q^2}{2\pi\varepsilon_0 mg}$$

When one of the ball is discharged, the ball come closer and touch each other and again separate due to repulsion. The charge on each ball after touching each other is q/2. Replacing q with q/2 in (iii), we get

$$\frac{b^3}{L} = \frac{(q/2)^2}{2\pi\varepsilon_0 mg} \quad - \quad \text{(iv)}$$

From (iii) and (iv), we get

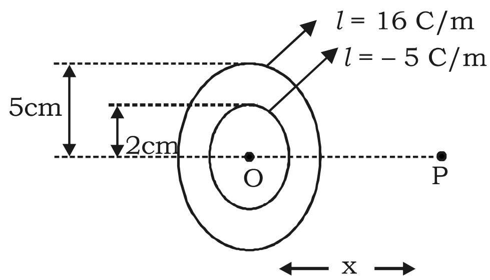
$$\frac{b^3}{a^3} = \frac{1}{4} \Rightarrow \frac{a^3}{b^3} = 4$$

3. **2**

Suppose that the field is zero at the point P, which is at a distance of x centrimetres form the centre O.

Potential at P

$$V_P = \frac{1}{4\pi\varepsilon_0}\left[\frac{16\times2\pi r_1}{\left(r_1^2+x^2\right)^{1/2}} - \frac{5\times2\pi r_2}{\left(r_2^2+x^2\right)^{1/2}}\right]$$



$$V_P = \frac{1}{2\varepsilon_0}\left[\frac{16r_1}{\left(r_1^2+x^2\right)^{1/2}} - \frac{5r_2}{\left(r_2^2+x^2\right)^{1/2}}\right]$$

(where r_1 = 5 cm; r_2 = 2 cm)

Since the electric field at P is zero.

$$E = \frac{dV}{dx} = 0$$

$$\left(\frac{dV}{dx}\right) = 0 = \frac{-x}{2\varepsilon_0}\left[\frac{16r_1}{\left(r_1^2+x^2\right)^{1/2}} - \frac{5r_2}{\left(r_2^2+x^2\right)^{1/2}}\right]$$

which leads to x = 0

$$\text{or} \quad \left(\frac{r_2^2+x^2}{r_1^2+x^2}\right)^{3/2} = \frac{5r_2}{16r_1}$$

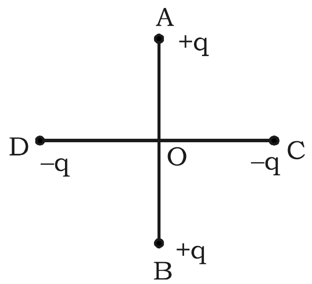
$$= \frac{5}{16}\times\frac{2}{5} = \frac{1}{8}$$

or $\frac{4+x^2}{25+x^2} = \frac{1}{4}$

Solving we get $x = \pm\sqrt{3}$ cm

So we conclude that the potential has a minimum at x = 0 and maximum at $x = \pm\sqrt{3}$ cm or field is zero at these points.

4. **8**



From the figure,

$$AC = \sqrt{(AO)^2+(OC)^2} = \sqrt{3^2+4^2}$$

$= 5m$

similarly BC = 5m

P.E. at C $= 2\times 9\times 10^9 \times \frac{(5\times10^{-5})^2}{5} - 9J$

K.E. at C = 4J

Total energy at C = P.E. + K.E.

$= -9+4$

$= -5J$

P.E. at D $= \frac{-2\times9\times10^9(5\times10^{-5})^2}{r}$

Where AD = BD = r(say) $= \frac{-45}{r}J$

K.E. at D = 0

so the total energy at D is $0 + \frac{-45}{r} = \frac{-45}{r}J$

From the conservation of energy

$\frac{-45}{r} = -5$ or $r = 9$ m

Now, $OD = \sqrt{(AD)^2-(AO)^2}$

$= \sqrt{9^2-3^2} = 6\sqrt{2}\,m$

$OD = 6\sqrt{2}\,m \approx 8$ m

5. **4**

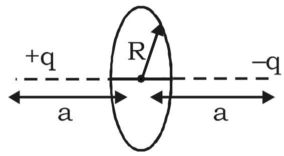
$d\phi = 2E\cos\theta \times 2\pi y\,dy$

$$= 2\times\frac{1}{4\pi\varepsilon_0}\frac{q}{(a^2+y^2)}\times\frac{a\times2\pi y dy}{(a^2+y^2)^{1/2}}$$

$$\phi = \int d\phi = \frac{qa}{\epsilon_0}\int_0^R\frac{ydy}{(a^2+y^2)^{3/2}}$$

$$\phi = \frac{q}{\varepsilon_0}\left[1-\frac{1}{\sqrt{1+(R/a)^2}}\right]$$

$$= \frac{8}{\varepsilon_0}\left[1-\frac{1}{2}\right] = \frac{4}{\varepsilon_0}$$



6. **2**

$$-\int_{l=\infty}^{0}\vec{E}.d\vec{l} = -\int_{l=\infty}^{0}dV$$

$= V_{centre} - V_\infty$

$= V_{centre} - 0 = V_{centre}$

But $V_{centre} = \frac{q}{4\pi\varepsilon_0 R}$

$$= \frac{(9\times10^9)(1.11\times10^{-10})}{0.5} \cong 2V$$

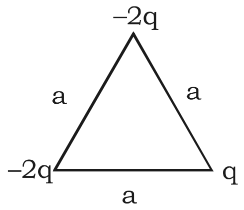
7. **0**

$W = U_f - U_i$

where

$$U_i = \frac{1}{4\pi\varepsilon_0 a}\times[(2q)(-2q)+(-2q)(q)+(q)(-2q)]$$

$= 0$



Also, $U_f = 0$. So, $W = 0$

8. **0**

$W_{A\to B} = q(V_B - V_A)$

$\int_A^B dV = V_B - V_A = \int_A^B \vec{E}.\vec{dr}$

$= -\int_{(2.2)}^{(4.1)} \left(y\hat{i} + x\hat{j}\right).\left(dx\hat{i} + dy\hat{j} + + dz\hat{k}\right)$

$= -\int_{(2.2)}^{(4.1)} (ydx + xdy)$

$= -\int_{(2.2)}^{(4.1)} d(xy) = [-xy]_{2.2}^{4.1} = 0$

so, $W_{AB} = 0$