XII - MATHS - SOLUTIONS

SECTION I (Multiple Answer Correct)

1. **a)** $a = 0, b \neq 0$
   c) $a \neq 0, b = 0$

$\because \quad \dfrac{dy}{dx} = \dfrac{ax+h}{by+k}$

$\Rightarrow (by + k)dy = (ax + h)dx$

On integrating, we get

$\int(by+k)dy = \int(ax+h)$

$\Rightarrow \dfrac{by^2}{2} + ky\dfrac{ax^2}{2} + hx + c$,

where c is an arbitrary constant.

For parabola, one of the two terms x^2 or y^2 is zero.

Therefore, either $\dfrac{b}{2} = 0$, $a \neq 0$ or $\dfrac{a}{2} = 0$, $b \neq 0$

$\Rightarrow b = 0, a \neq 0$ or $a = 0, b \neq 0$

2. **a)** Circle
   b) Straight line

$\because \quad y\left(\dfrac{dy}{dx}\right)^2 + (x-y)\dfrac{dy}{dx} - x = 0$

$\Rightarrow \quad \left(y\dfrac{dy}{dx} + x\right)\left(\dfrac{dy}{dx} - 1\right) = 0$

$\Rightarrow y\dfrac{dy}{dx} + x = 0$ or $\dfrac{dy}{dx} - 1 = 0$

$\Rightarrow y\,dy + x\,dx = 0$ or $dy - dx = 0$

$\Rightarrow 2x\,dx + 2y\,dy = 0$ or $dx - dy = 0$

On integrating, we get

$x^2 + y^2 = c_1$ or $x - y = c_2$,

where c_1, c_2 are arbitrary constants.

3. **b)** Domain of $y(x)$ is $(-\infty, -2) \cup (3, \infty)$
   d) $a = -5\ln\left(\dfrac{12}{7}\right)$

$\because \dfrac{dy}{dx} - \dfrac{y}{x} + \dfrac{5x}{(x+2)(x-3)} = 0$

$\therefore$ I.F = $e^{\int -\frac{1}{x}dx} e^{-\ln x =} e^{\ln\left(\frac{1}{x}\right)} = \dfrac{1}{x}$

Solution is $y \cdot \left(\dfrac{1}{x}\right) = \displaystyle\int \dfrac{1}{x} \cdot \dfrac{5x}{(x-2)(x-3)}dx + c$

$\Rightarrow \quad \dfrac{y}{x} = \displaystyle\int \dfrac{(x+2)-(x-3)}{(x+2)(x-3)}dx + c$

$\Rightarrow \quad \dfrac{y}{x} = \displaystyle\int\left(\dfrac{1}{(x-3)} - \dfrac{1}{(x+2)}\right)dx + c$

$\Rightarrow \quad \dfrac{y}{x} = \ln(x-3) - \ln(x+2) + c$

$\Rightarrow \quad \dfrac{y}{x} = \ln\left(\dfrac{x-3}{x+2}\right) + c$

$\therefore \quad y = x\left[\ln\left(\dfrac{x-3}{x+2}\right) + c\right]$ ...(i)

It passes through (4, 0) then

$0 = 4\left[\ln\left(\dfrac{1}{6}\right) + c\right]$

$0 = -\ln 6 + c$

$\therefore \quad c = \ln 6$

From (i), we get

$y = x\ln\left(6\left(\dfrac{x-3}{x+2}\right)\right)$

The point (5, a) lies on it, then

a = 5 $\ln\left(\dfrac{12}{7}\right)$ and domain of $y(x)$ is defined when, $\dfrac{x-3}{x+2} > 0$

$\therefore\ x \in (-\infty, -2) \cup (3, \infty)$

4. **b)** Least value of $f(x)$ in $\left[\dfrac{\pi}{4}, \dfrac{\pi}{3}\right]$ is $\dfrac{3\sqrt{3}}{2\pi}$
   c) $\int_0^{\pi/2} f(x)dx > 1$
   d) $f(x)$ is an even function

The given differential equation can be written as

$\dfrac{dy}{dx} - y\cot x = -\dfrac{\sin^2 x}{x^2}$

$\because \quad$ I.F = $e^{\int -\cot dx} e^{-\ln\sin x =} e^{\ln\left(\frac{1}{\sin x}\right)} = \dfrac{1}{\sin x}$

$\because$ Solution is

$y \cdot \left(\dfrac{1}{\sin x}\right) = \displaystyle\int -\dfrac{1}{x^2}dx + c$

$\Rightarrow \quad \dfrac{y}{\sin x} = \dfrac{1}{x} + c$ ...(i)

Given, $x \to \infty$, $y \to 0$, then

$0 = 0 + c \Rightarrow c = 0$ ($\because$ $-1 < \sin\infty < 1$)

From (i),

$y = \dfrac{\sin x}{x}$

$\Rightarrow f(x) = \dfrac{\sin x}{x}$

$\because\ f(-x)=\dfrac{-\sin x}{-x}=f(x)$

$\Rightarrow$ f(x) is an even function.

$\lim\limits_{x\to\infty} f(x)=\lim\limits_{x\to\infty}\dfrac{\sin x}{x}=0$

$\therefore\quad f(x)=\dfrac{\sin x}{x}$ is decreasing

$\therefore$ Least value $= f\left(\dfrac{\pi}{3}\right)=\dfrac{\sin(\pi/3)}{\pi/3}=\dfrac{3}{\pi}\cdot\dfrac{\sqrt{3}}{2}=\dfrac{3\sqrt{3}}{2\pi}$

also $\quad f(x)>f\left(\dfrac{\pi}{2}\right)$

$\Rightarrow\quad f(x)>\dfrac{1}{(\pi/2)}$

$\therefore\ \int_0^{\pi/2} f(x)dx>\dfrac{2}{\pi}\mathbf{0}\int_0^{\pi/2} dx$

$\Rightarrow \int_0^{\pi/2} f(x)dx>1$

5. **a)** If $y(a) = -\dfrac{\pi}{2}$, then a = 0, 2

d) $\sin y \in \left[-1,-\sqrt{\dfrac{2}{3}}\right]\cup\left[\sqrt{\dfrac{2}{3}},1\right]$

We have $\quad \dfrac{dy}{dx}=\dfrac{\sin y+x}{\sin 2y-x\cos y}$

$\Rightarrow\quad \cos y\dfrac{dy}{dx}=\dfrac{\sin y+x}{2\sin y-x}\quad$ ...(i)

Put sin y = $v \Rightarrow$ cos y $\dfrac{dy}{dx}=\dfrac{dv}{dx}$,

Then equation (i) reduced to

$\dfrac{dv}{dx}=\dfrac{v+x}{2v-x}$

$\Rightarrow\quad 2v\,dv - x\,dv = v\,dx + dx$

$\Rightarrow\quad 2v\,dv = (v\,dx + x\,dx) + x\,dx$

$\Rightarrow\quad 4v\,dv = 2d\,(vx) + 2x\,dx$

On integrating, we get

$2v^2 = 2v\,x + x^2 + c$

or $\quad 2\sin^2 y = 2x\sin y + x^2 + c$

For x = 0, $y=\dfrac{\pi}{2}$, then

$2 = 0 + 0 + c$

$\therefore\quad c = 2$, then

$2\sin^2 y = 2x\sin y + x^2 + 2$

$\Rightarrow\quad (x + \sin y) = 3\sin y - 2 \geq 0$

$\therefore\quad \sin^2 y \geq \dfrac{2}{3}$

or $\quad \sin y \in \left[-1,-\sqrt{\dfrac{2}{3}}\right]\cup\left[\sqrt{\dfrac{2}{3}},1\right]$

6. **b)** $\pi/4$

d) $3\pi/4$

7. **a)** $\hat{j}-\hat{k}$

d) $-\hat{j}+\hat{k}$

8. **a)** $\left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right)\vec{a}-\vec{b}$

b) $\dfrac{1}{|\vec{a}|^2}\left\{|\vec{a}|^2\,\vec{b}-\left(\vec{a}\cdot\vec{b}\right)\vec{a}\right\}$

c) $\dfrac{\vec{a}\times\left(\vec{a}\times\vec{b}\right)}{|\vec{a}|^2}$

9. **a)** Parallel to $(y-z)\hat{i}+(z-x)\hat{j}+(x-y)\hat{k}$

b) Orthogonal to $\hat{i}+\hat{j}+\hat{k}$

c) Orthogonal to $(y+z)\hat{i}+(z+x)\hat{j}+(x+y)\hat{k}$

d) Orthogonal to $x\hat{i}+y\hat{j}+z\hat{k}$

10. **a)** $\left(\vec{c}\times\vec{a}\right)\times\vec{b}=\vec{0}$

c) $\vec{b}\times\left(\vec{c}\times\vec{a}\right)=\vec{0}$

d) $\left(\vec{c}\times\vec{a}\right)\times\vec{b}=\vec{b}\times\left(\vec{c}\times\vec{a}\right)=\vec{0}$

SECTION - II (Matrix Match Type)

1. **Ans : A-P,R, ; B-P,R ; C-P,S ; D-P,Q,S**

(A) :

$\because\ y = z^m \Rightarrow \dfrac{dy}{dx} = m\,z^{m-1}\dfrac{dz}{dx}$, then

differential equation

$2x^4 y\dfrac{dy}{dx} + y^4 = 4x^6$ reduces to

$2x^4 \cdot z^m \cdot m\,z^{m-1}\dfrac{dz}{dx} + z^{4m} = 4x^6$

$\Rightarrow\quad \dfrac{dz}{dx}=\dfrac{4x^6 z^{4m}}{2mx^4 z^{2m-1}}$

For homogeneous differential equation

$$4m = 6$$

$$m = \frac{3}{2}; \text{ then}$$

$4(m^2 + m) = 4\left(\frac{9}{4}+\frac{3}{2}\right) = 15 = 3 \times 5$

(B) :

$\because \quad y = (A + Bx)e^{3x}$

or $e^{-3x} . y = A + bx$

Differentiating both sides w.r.t. x, then

$$e^{-3x}\frac{dy}{dx} + y.(-3)e^{-3x} = B$$

After differentiating both side w.r.t. x, then

$$e^{-3x}\left(\frac{d^2y}{dx^2} - 3\frac{dy}{dx}\right) + \left(\frac{dy}{dx} - 3y\right)\left(-3e^{-3x}\right) = 0$$

Dividing by e^{-3x}, we get

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$ but given $\frac{d^2y}{dx^2} + m\frac{dy}{dx} + ny = 0$

On comparing, we get

$m = -6$, $n = 9$, then $n - m = 15 = 3 \times 5$

(C) :

$\because \quad y = Ax^m + Bx^{-n}$

$\Rightarrow \frac{dy}{dx} = Amx^{m+1} - Bnx^{-n-1}$

$\Rightarrow \frac{d^2y}{dx^2} = Am(3-1)x^{m-1} + Bn(n+1)x^{-n-2}$

Putting these values in

$x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 12y = 0$

$\Rightarrow Am(m - 1)x^{m-2} + Bn(n + 1)x^{-n-2} + 2Amx^m$ $-2Bnx^{-n} - 12(Ax^m + B^{-n}) = 0$

$\Rightarrow [m(m = 1) - 12]\, Ax^m + [n(n - 1) - 12]\, Bx^{-n} = 0$

We have $m(m + 1) = 12$

$\Rightarrow {}^{m+1}C_2 = 6, \quad {}^nP_2 = 12$

$\therefore \quad {}^{m+1}C_2 + {}^nP_2 = 18 = 3 \times 6$

(D) :

$\because \quad y = A + B e^{mx} + C e^{nx}$

$\Rightarrow A + B e^{mx} + C e^{nx} - y = 0$

Differentiating both sides w.r.t. x, then

$$Bme^{mx} + Cne^{nx} - \frac{dy}{dx} = 0 \quad ...(i)$$

Differentiating (i) w.r.t. x then

$$Bm^2e^{mx} + Cn^2e^{nx} - \frac{d^2y}{dx^2} = 0 \quad ...(ii)$$

Differentiating (i) w.r.t. x then we get

$$Bm^3e^{mx} + Cn^3e^{nx} - \frac{d^3y}{dx^3} = 0 \quad ...(iii)$$

Eliminating B and C from (i), (ii) and (iii), we have

$$\begin{vmatrix} 1 & 1 & \frac{dy}{dx} \\ m & n & \frac{d^2y}{dx^2} \\ m^2 & n^2 & \frac{d^3y}{dx^3} \end{vmatrix} = 0$$

$$\Rightarrow (n-m)\frac{d^3y}{dx^3} - \left(n^2 - m^2\right)\frac{d^2y}{dx^2} + mn(n-m)\frac{dy}{dx} = 0$$

or $\frac{d^3y}{dx^3} - (n-m)\frac{d^2y}{dx^2} + nm\frac{dy}{dx} = 0 \quad (\because n - m \neq 0)$

But given $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} - 35\frac{dy}{dx} = 0$

On comparing, we get

$m + n = -2, \qquad mn = -35$

$m = -7, n = 5$ or $\quad m = 5, n = -7$

$(\because m, n \in 1)$

$\Rightarrow |m - n| = 12 = 3 \times 4 = 2 \times 6$

2. **Ans : A-R ; B-S ; C-Q ; D-P**

SECTION III (Integer Type)

1. **2**

The given equation can be written as

$$\frac{dy}{dt} + 2ty = t^2 \quad ...(i)$$

Here I.F. $= e^{\int 2t\,dt} = e^{t^2}$

$\therefore$ Solution of equation (i) is

$$y \cdot \left(e^{t^2}\right) = \int t^2 \cdot e^{t^2} \cdot dt + c$$

$$= \int \underset{\text{I}}{\frac{t}{2}} \cdot \underset{\text{II}}{\left(e^{t^2} \cdot 2t\right)} dt + c$$

$$= \frac{t}{2} \cdot e^{t^2} - \frac{1}{2}\int e^{t^2} dt + c$$

$$\frac{y}{t} = \frac{1}{2} - \frac{1}{2} \cdot \lim_{t\to\infty} \frac{\int e^{t^2} dt}{t \cdot e^{t^2}} + \frac{c}{t \cdot e^{t^2}}$$

$$\lim_{t\to\infty} \frac{y}{t} = \frac{1}{2} - \frac{1}{2}\lim_{t\to\infty} \frac{\int e^{t^2} dt}{t \cdot e^{t^2}} + 0$$

$$= \frac{1}{2} - \frac{1}{2}\lim_{t\to\infty} \frac{e^{t^2}}{t \cdot 2te^{t^2} + e^{t^2} \cdot 1}$$

$$= \frac{1}{2} - \frac{1}{2}\lim_{t\to\infty} \frac{1}{\left(2t^2 + 1\right)}$$

$=\frac{1}{2}-0$

$=\frac{1}{2}$

$\lim_{t\to\infty}\frac{t}{y}=\frac{1}{\lim_{t\to\infty}\frac{y}{t}}=\frac{1}{\frac{1}{2}}=2$

2. **6**

The given differential equation can be written can be written as

$$y'(x)=y(x)+\int_0^1 y(x)dx \qquad ...(i)$$

Differentiating both sides w.r.t x, then

or $y''(x) = y'(x)$

or $\frac{y''(x)}{y'(x)}=1$

or $\int\frac{y''(x)}{y'(x)}dx=\int dx$

or $\ln y'(x) = x + \ln c$

or $\ln\left(\frac{y'(x)}{c}\right)=x$

or $y'(x) = c\, e^{c}$ ...(ii)

and $y(x) = c\, e^{x} + d$

$\Rightarrow$ $y(0) = c + d = 1$ $(\because y(0) = 1)$

$\therefore$ $y(x) = c\, e^{x} + 1 - c$ ...(iii)

From (i), (ii) and (iii), we get

$$c e^{x}=c e^{x}+1-c+\int_0^1\left(c e^{x}+1-c\right)dx$$

$\Rightarrow$ $c-1=\left[c e^{x}+(1-c)x\right]_0^1$

$\Rightarrow$ $c - 1 = ce + 1 - c - c$

$\therefore$ $c=\frac{2}{3-e}$ ...(iv)

From (iii),

$$y(1)=ce+1-c=\frac{e+1}{3-e} \qquad \text{[From (iv)]}$$

$$=\frac{2\cdot718+1}{3-2\cdot718}=\frac{3\cdot718}{0\cdot282}=13\cdot18$$

$[y(1) - 7] = [6.18] = 6$

3. **4**

4. **9**

5. **5**

6. **9**

7. **6**

8. **1**