PART C - MATHS

SECTION I - MULTIPLE ANSWER CORRECT TYPE

1. **b)** $\mathbf{C - \cot x + \cot^{-1} x}$

c) $\mathbf{-\tan^{-1} x - \dfrac{\operatorname{cosec} x}{\sec x} + C}$

$$\int \frac{x^2+1-\sin^2 x}{x^2+1}\operatorname{cosec}^2 x\,dx$$

$$= \int\left(1-\frac{\sin^2 x}{x^2+1}\right)\operatorname{cosec}^2 x\,dx$$

$$= \int \operatorname{cosec}^2 x\,dx - \int \frac{1}{x^2+1}dx$$

$$= \int_0^1 \frac{2x^2-3x+3}{(x+1)(x^2+2x+2)}$$

$$= \int_0^1 \frac{2x^2+4x+4-x-1}{(x+1)(x^2+2x+2)}$$

$$= \int_0^1 \frac{2}{x+1} - \frac{1}{(x+1)^2+1}$$

2. **a)** $\mathbf{f(x) = \sin x}$

d) $\mathbf{A = \dfrac{\pi}{2}}$

$$\int \sin^{-1} x \cos^{-1} x\,dx$$

Substituting $t = \sin^{-1} x$

$dx = \cot dt$

$$I = \int t\left(\frac{\pi}{2}-t\right)\cot dt$$

$$= \frac{\pi}{2}\int t\cot dt - \int t^2 \cot dt$$

then use by Parts.

3. **a)** $\mathbf{\dfrac{\pi}{4} + 2\log 2 - \tan^{-1} 2}$

c) $\mathbf{2\log 2 - \cot^{-1} 3}$

4. **a)** $\mathbf{I_5 + I_3 = \dfrac{1}{4}}$

b) $\mathbf{I_8 + I_6 = \dfrac{1}{7}}$

c) $\mathbf{I_6 - I_{10} = \dfrac{2}{63}}$

d) $\mathbf{I_6 + 2I_8 + I_{10} = \dfrac{16}{63}}$

$$I_n = \int_0^{\pi/4} \tan^n x$$

$$= \int_0^{\pi/4} \tan^{n-2} x \sec^2 x - \int_0^{\pi/2} \tan^{n-2} x\,dx$$

$$= \left[\frac{\tan^{n-1} x}{n-1}\right]_0^{\pi/4}$$

$$I_n + I_{n-2} = \frac{1}{n-1}$$

5. **c)** $\mathbf{y = 2x - 4}$

Let $P = \dfrac{dy}{dx}$ so that the given equation can be written as $p^2 - xp + y = 0$. Therefore $y = xp - p^2$. Differentiating with respect to x, we have

$$p = \frac{dy}{dx} = p + x\frac{dp}{dx} - 2p\frac{dp}{dx}$$

$(x - 2p)\dfrac{dp}{dx} = 0 \Rightarrow = \dfrac{dp}{dx} = 0$ so p = const = C(say)

So $y = Cx - C^2$ is a solution, in particular C = 2.

6. **d)** $\mathbf{4e^{3x} + 3e^{-4y} = 7}$

$$\frac{dy}{dx} = e^{3x+4y} = e^{3x}e^{4y} \Rightarrow = e^{-4y}dy = e^{3x}dx$$

Thus $\dfrac{e^{-4y}}{-4} - \dfrac{e^{3x}}{3}$ = const. But $y(0) = 0$,

so, $-\dfrac{1}{4} - \dfrac{1}{3} = C$.

Hence $(e^{-4y}/(-4)) - (e^{3x}/3)$

$= -7/12 \Rightarrow 3e^{-4y} + 4e^{3x} = 7$.

7. **b)** $\mathbf{-3/2}$

Let $f(x) = x^3 + \log_a\left(x + \sqrt{x^2+1}\right)$

$f(-x) = -x^3 + \log_a\left(-x + \sqrt{x^2+1}\right)$

$= -x^3 + \log_a \dfrac{1}{x + \sqrt{x^2+1}}$

$= -x^3 + \log_a\left(x + \sqrt{x^2+1}\right)$

$= -f(x)$

$\therefore \int_{-3/2}^{3/2} f(x)\,dx = 0$

Thus I $= \int_{-3/2}^{3/2} [x]\,dx$

$= \int_{-3/2}^{-1}(-2)\,dx + \int_{-1}^{0} -1dx + \int_{0}^{1} 0\,dx + \int_{-1}^{3/2} 1dx$

$= (-2)[-1 + 3/2] + (-1)[0 + 1] + 0 + 1[3/2-1]$

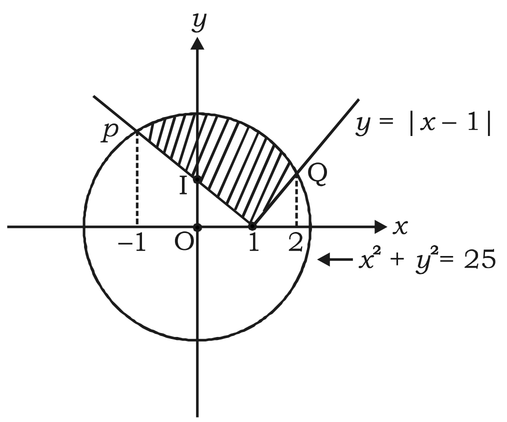
$= -1 - 1 + 1/2$

$= -3/2$

8. **b)** $\left(\dfrac{\mathbf{5\pi - 2}}{\mathbf{4}}\right)$

The curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|$ intersect at (2, 1) and (–1, 2).

Required area



$= \int_{-1}^{2} \sqrt{5 - x^2}\,dx - \int_{-1}^{2} |x - 1|\,dx$

But $\int_{-1}^{2} \sqrt{5 - x^2}\,dx$

$= \left(\dfrac{x}{2}\sqrt{5 - x^2} + \dfrac{5}{2}\sin^{-1}\left(\dfrac{x}{\sqrt{5}}\right)\right)\Bigg]_{-1}^{2}$

$= 1 + \dfrac{5}{2}\sin^{-1}\left(\dfrac{2}{\sqrt{5}}\right) + 1 + \dfrac{5}{2}\sin^{-1}\left(\dfrac{1}{\sqrt{5}}\right)$

$= 2 + \dfrac{5}{2}\sin^{-1}\left(\dfrac{2}{\sqrt{5}}\sqrt{1 - \dfrac{5}{2}} + \dfrac{1}{\sqrt{5}}\sqrt{1 - \dfrac{4}{5}}\right)$

$= 2 + \dfrac{5}{2}\sin^{-1}\left(\dfrac{5}{5}\right)$

$= 2 + \dfrac{5\pi}{4}$

and $\int_{-1}^{2} |x - 1|\,dx = \int_{-2}^{1} |t|\,dt$ [Put $t = x - 1$]

$= \int_{-2}^{0}(-t)\,dt + \int_{0}^{1} t\,dt = \dfrac{t^2}{2}\Bigg]_{2}^{0} + \dfrac{t^2}{2}\Bigg]_{0}^{1}$

$= -0 + 2 + \dfrac{1}{2} = \dfrac{5}{2}$

SECTION II - PARAGRAPH TYPE

Paragraph-1

1. **a)** $\mathbf{\tan^{-1}(2x - 3)^{1/6}}$

$\displaystyle\int \dfrac{(2x-3)^{\frac{1}{2}}}{(2x-3)^{\frac{1}{3}} + 1}\,dx$

$t^6 = (2x - 3)$

$6t^5\,dt = 2dx$

$= 3\displaystyle\int \dfrac{t^8}{t^2+1}\,dt$

$= 3\displaystyle\int t^6 - t^4 - t^2 + 1 + \dfrac{1}{t^2+1}\,dt$

$= 3\left[\dfrac{t^7}{7} - \dfrac{t^5}{5} - \dfrac{t^3}{3} + 1 + \tan^{-1} t\right] + c$

$\therefore$ g(x) $= \tan^{-1}(2x-3)^{\frac{1}{6}}$

2. **d)** **4/3**

I $= \displaystyle\int \dfrac{dx}{(x+2)^2 \sqrt[4]{\left(\dfrac{x-1}{x+2}\right)^3}}$

Let $t^4 = \left(\dfrac{x-1}{x+2}\right)$

$\therefore 4t^3dt = \dfrac{3}{(x+2)^2}\,dx$

I $= \dfrac{4}{3}\displaystyle\int \dfrac{t^3}{t^3}\,dt = \dfrac{4}{3}t + c$

$A = \frac{4}{3}$

Paragraph - 2

1. **b)** $\frac{2n!!}{(2n+1)!!}$

$I_n = \int_0^1 (1-x^2)^n dx$

$= \int_0^1 \left(1 - C_1x^2 + C_2x^4 + \ldots + (-1)^n C_n x^{2n}\right) dx$

$= 1 - \frac{C_1}{3} + \frac{C_2}{5} - \ldots - \frac{(-1)^n}{2n+1}$

But $I_n = \frac{2n!!}{(2n+1)!!}$

2. **a)** $\sum_{k=1}^{n} \frac{1}{k}$

$1 - (1-x)^n = C_1x - C_1x^2 + \ldots + (-1)^{n-1} C_n x^n$

Integrating within the limits 0 to 1, we get

$\int_0^1 \frac{1-(1-x)^n}{x} dx$

$= C_1 - \frac{C_2}{2} + \frac{C_3}{3} + \ldots + \frac{(-1)^{n-1} C_n}{n}$

Thus R.H.S. $= \int_0^1 \frac{1-x^n}{1-x} dx$

$= \int_0^1 \left(1 + x + x^2 + \ldots + x^{n-1}\right) dx$

$= 1 + \frac{1}{2} + \ldots + \frac{1}{n}$

SECTION III - MATRIX MATCH TYPE

1. **A-Q, B-P, C-R, D-S**

$\int \frac{\sec x}{(\sec x + \tan x)^2} dx = \int \frac{\sec x(\sec x + \tan x)}{(\sec x + \tan x)^3} dx$

$= \int \frac{dt}{t^3}$ $(t = \sec x + \tan x)$

$= -\frac{1}{2}(\sec x + \tan x)^{-2} + C$

$\int \frac{\cos x}{(\sin x - 1)(\sin x - 2)} dx$

$= \int \frac{dt}{(t-1)(t-2)} (t = \sin x)$

$= \int \left(\frac{1}{t-2} - \frac{1}{t-1}\right) dt = \log \frac{|\sin x - 2|}{|\sin x - 1|} + C$

$\int \sin^{-1} \frac{2x}{1+x^2} dx = 2\int t \sec^2 t \, dt \, (x = \tan t)$

$= 2\,[t \tan t - \log|\sec t| + C]$

$= 2x \tan^{-1} x - \log(1 + x^2) + C$

Putting $\tan x = y^2$, so that $dx = 2y\, dy/(1 + y^4)$, we have

$\int \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$

$= \int \left(y + \frac{1}{y}\right) \frac{2y}{1+y^4} dy$

$= 2\int \frac{y^2+1}{1+y^4} dy$

$= 2\int \frac{1+1/y^2}{y^2+1/y^2} dy$

$= 2\int \frac{1+1/y^2}{(y-1/y)^2+2} dy$

$= 2\int \frac{du}{u^2+2}$ [where $u = y - 1/y$]

$= 2.\frac{1}{\sqrt{2}} \tan^{-1} \frac{u}{\sqrt{2}} + C$

$= \sqrt{2} \tan^{-1} \left(\frac{\tan x - 1}{\sqrt{2 \tan x}}\right) + C$

2. **A-Q, B-R, C-S, D-P**

Differentiating both sides of (A), we have

$\frac{dy}{dx} = 3Ae^{3x} + 5Be^{5x}$

$= 3y + 2Be^{5x}$

$\Rightarrow \frac{d^2y}{dx^2} = 3\frac{dy}{dx} + 10Be^{5x}$

$= 3\frac{dy}{dx} + 5\left(\frac{dy}{dx} - 3y\right)$

$\Rightarrow \frac{dy^2}{dx^2} - 8\frac{dy}{dx} + 15y = 0$

Differentiating both sides (B) we get

$x\frac{dy}{dx} + y = ae^x - be^{-x} + 2x$

Again differentiating , we have

$$x\frac{d^2y}{dx^2}+2\frac{dy}{dx} = ae^x + be^{-x} + 2 = xy - x^2 + 2$$

Hence a required differential equation is

$$x\frac{d^2y}{dx^2}+2\frac{dy}{dx} - xy + x^2 - 2 = 0$$

Differentiation, (C) we have

$$2Ax + 2By\frac{dy}{dx} = 0 \Rightarrow Ax + By\frac{dy}{dx} = 0 \quad \ldots(i)$$

$$\Rightarrow \quad A + B\left(\frac{dy}{dx}\right)^2 + By\frac{d^2y}{dx^2} = 0$$

(differentiating (i))

$$\Rightarrow \quad A + B\left[\left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2}\right] = 0 \quad \ldots(ii)$$

Multiplying (ii) by x and subtracting from (i) we have

$$Bx\left[y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right] - By\frac{dy}{dx} = 0$$

$$\Rightarrow \quad x\left\{y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right\} = y\frac{dy}{dx}$$

Given $y = a\sin(4x + b)$, we have

$y_1 = 4a\cos(4x + b)$

$\Rightarrow y_2 = -16a\sin(4x + b) = -16y$.

SECTION IV - INTEGER TYPE

1. **2**

An antiderivative of $f(x) = F(x)$

$$= \int\left(\log(\log x) + (\log x)^{-2}\right)dx + C$$

$$= x\log(\log x) - \int\frac{x}{x\log x}dx + \int(\log x)^{-2}dx + C$$

(integrating by parts the first term)

$$= x\log(\log x) - [x(\log x)]^{-1} + \int(\log x)^{-2}dx]$$

$$+ \int(\log x)^{-2}dx + C$$

$= x\log(\log x) - x(\log x)^{-1} + C$

Putting $x = e$, we have $1998 - e = e.0 + e + C$, Thus $C = 1998$

2. **2**

(i) Putting $e^x = t$ in the given integral and taking e^x common in the integrand, we can write the given integral as

$$\int\frac{2t^4+t^3-4t^2+4t+2}{(t^2+4)(t^2-1)^2}dt$$

$$= \int\frac{2t^4-4t^2+2}{(t^2+4)(t^2-1)^2}dt + \int\frac{t^3+4t}{(t^2+4)(t^2-1)^2}dt$$

$$= 2\int\frac{(t^2-1)^2}{(t^2+4)(t^2-1)^2}dt + \int\frac{t(t^2+4)}{(t^2+4)(t^2-1)^2}dt$$

3. **9**

Required area $= \int_0^3\left[(y-2)^2+1-(2y-4)\right]dy = 9$

4. **18**

5. **6**

The equation of the tangent at any point $P(x,y)$

is $Y - y = \frac{dy}{dx}(X - x) \quad \ldots(i)$

Given that

intercept on the x-axis (putting $Y = 0$) = 2 (x – coordinates of P)

$$\Rightarrow -y = \frac{dy}{dx}(2x - x)$$

$$\Rightarrow -\frac{dy}{y} = \frac{dx}{x}$$

Intergrating, we get $xy = c$.

Since the curve passes through (1, 2), $c = 2$.

Hence, the equation of the required curve is $xy = 2$.