LAKSHYA ADVANCED UNIT TEST (LAUT)

00 – 00		Q. Booklet Serial No: 200915	
Test No : 2231	3 Hrs.		

Hints & Solutions

PART A - PHYSICS

SECTION I - MULTIPLE ANSWER CORRECT TYPE

1. **a)** $l^{3/2}$
   d) $(\sin\theta)^{1/2}$

   Potential difference = $\frac{1}{2}$ Bωl^2

   From energy conservation: mg $\frac{l}{2}$ $\sin\theta$

   = $\frac{1}{2}$ Iω^2, where I = $\frac{ml^2}{3}$

   Substituting we get.

   Potential difference $\propto l^{3/2}$

   $\propto (\sin\theta)^{1/2}$

2. **b) zero if both wires slide in opposite directions**
   c) 0.2 mA if both wires slide move toward left

   Each wire can be replaced by a battery whose emf is equal to

   $Blv = 1 \times 4 \times 10^{-2} \times 5 \times 10^{-2}$

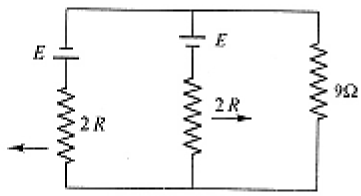
   $= 20 \times 10^{-4}$ V

   The polarity of the battery can be given by Fleming's right hand rule. When both wire move in opposite direction, the circuit diagram looks like as shown in Figure.

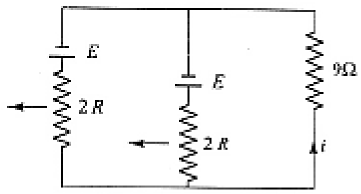
   The effective emf of the two batteries shown in the diagram is zero.

   So, choice (b) is correct and choice(d) is wrong.

   When both wires move towards left, the circuit diagram looks like as shown in the second Figure.



(a)



(b)

   Effective emf of two batteries shown is E (= 20×10^{-4} V) and internal resistance is 1 Ω.

   Hence, current in the circuit is

   $$i = \frac{20\times10^{-4}}{10} = 0.2 \text{ mA}$$

   Hence, choice (c) is correct and choice (a) is wrong.

3. **a)** $I_0 = 40$ **A**
   b) $V_0 = 9.8$ **kV**
   c) $V_1 = 9.8$ **kV**

   The frequency

   $$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\sqrt{\frac{1}{0.6\times10\times10^{-6}}} = 65 \text{ Hz}$$

   The current at resonance is

   $$I_0 = \frac{200\text{ V}}{5\Omega} = 40 \text{ A}$$

   The quantity $V_0 = I_0 \omega L$

   $= 40 \times 0.6 \times 2\pi \times 65$

   $= 9.8$ kV

$$V_1 = \frac{I_0}{\omega C} = \frac{40\times10^5}{2\pi\times65} = 9.8\text{ kV}$$

4. **a) H^+ will be deflected the most**
   c) He^+ and O^{2+} will be deflected equally

$$K = \frac{1}{2}mv^2$$

$$\Rightarrow v = \sqrt{2K/m}, r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$$

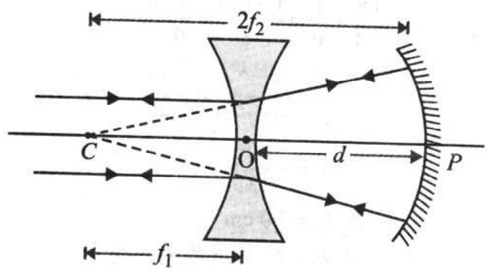
$$r_{H^+} = \frac{\sqrt{2mK}}{eB}, r_{He^+} = \frac{\sqrt{2(4m)K}}{eB} = 2r_{H^+}$$

$$r_{O^{2+}} = \frac{\sqrt{2(16m)K}}{2eB} = 2r_{H^+}$$

Lesser the radius, more will be the deflection.

5. **a) $d = 2|f_2| = |f_1|$**
   c) The beam diameters of the incident and reflected beams must be the same

As is clear from Figure below, the parallel beam will return as a parallel beam, only when beam diverging through concave lens falls normally on concave mirror.



∴ PC = R = 2 f_2 and OC = f_1. As OP = d

∴ $d = 2|f_2| - |f_1|$

Also, the beam diameters of incident and reflected beams must be the same.

6. **a) be farther away than its actual distance**
   d) move faster than its actual speed

Let x be the actual height of the bird above the water surface and y be the apparent height of the bird above the water surface. as seen by the fish.

For light travelling from bird to fish.

$\mu_1 = 1$, $\mu_2 = \mu$ (refractive index of water)

u = – x, v = y

For refraction at plane surface, $\frac{\mu_2}{v} = \frac{\mu_1}{u}$

$$\frac{\mu}{y} = \frac{1}{-x} \quad y = -\mu x$$

or $|y| = \mu x > x$

∴ Bird appears farther away from its actual distance.

Actual speed of bird = $\dot{x}$

Apparent speed of bird = $|\dot{y}| = \mu\dot{x}$

∴ $|\dot{y}| > \dot{x}$

i.e., the bird appears to move faster than its actual speed.

7. **c) 2.8 cm**

$$\frac{\text{Diameter}}{\text{distance}} = \text{constant} = \frac{1.4\times10^9}{10^{11}} = \frac{x}{2}$$

8. **c) 1.5**
   d) 1.6

$$\text{As } \mu \geq \frac{1}{\sin C} > \frac{1}{\sin 45^0} > \frac{1}{1/\sqrt{2}} \geq \sqrt{2} = 1.414$$

∴ possible values of μ are 1.5 and 1.6.

SECTION II - PARAGRAPH TYPE

Paragraph-1

1. **b)** $\mathbf{\frac{\mu_0 iR^2\pi r^2}{2x^3}}$

2. **c)** $\mathbf{\frac{3}{2}\frac{\mu_0\pi iR^2r^2}{x^4}v}$

Magnetic field on the axis of a circular coil is given by

$$B = \frac{\mu_0 iR^2}{2(R^2+x^2)^2}$$

Since x >> R, therefore, magnetic field at the center of the smaller loop is

$$B = \frac{\mu_0 iR^2}{2x^3}$$

Flux linked with coil is

$$\phi = B(\pi r^2) = \frac{\mu_0\pi iR^2r^2}{2x^3}$$

From Faraday's law we have

$$E = \frac{d\phi}{dt} = \frac{3}{2}\frac{\mu_0 \pi i R^2 r^2}{x^4} v$$

Paragraph-2

1. **b) 3.3 cm**

$$\frac{1}{f_v} = (\mu_v - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.6 - 1)\left(\frac{2}{20}\right) = \frac{6}{100}$$

$$f_v = \frac{100}{6} = 16.7 \text{ cm}$$

similarly f_r = 20cm

Chromatic aberration = $f_r - f_v$ = 20 – 16.7 = 3.3cm

2. **b) 80 cm**

$$\frac{1}{f_w} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left(\frac{3/2}{4/3} - 1\right)\left(\frac{1}{20} + \frac{1}{20}\right) = \frac{1}{8} \times \frac{2}{20} = \frac{1}{80}$$

f_w = 80 cm

SECTION III - MATRIX MATCH TYPE

1. **A-C B – C C – AC D - AC**

A) Current in inductor when switch is open:

$$I_0 = \frac{E}{R}$$

Initially induced e.m.f. will be equal to E and finally it is zero. So, energy stored will be zero

B) Same as (i)

C)

D) Here current becomes zero suddenly.

So, $\frac{dI}{dt}$ is large.

2. **A-QR, B-PS, C-PS, D-PS**

Erect and smaller image is formed by a concave lens and a convex mirror. Magnified, erect image is formed in case of a convex lens and a concave mirror. Magnified, inverted image is formed by a convex lens and a concave mirror. Inverted image of same size is formed by a convex lens and a concave mirror.

SECTION IV - INTEGER TYPE

1. **8**

The mutual inductance of solenoid coil system

$$M = \mu_0 n_1 N_2 A_2 = \mu_0 n_1 N_2 \pi r_2^2$$
$$= 4\pi \times 10^{-7} \times 2 \times 10^4 \times 100 \times \pi \times (0.01)^2$$
$$= 8\pi^2 \times 10^{-5} \text{ H}$$

EMF induced in the coil: $e_2 = -M\frac{\Delta i}{\Delta t}$

$$= -8\pi^2 \times 10^{-5} \times \left(\frac{-2-2}{0.05}\right) = 640\,\pi^2 \times 10^{-5} \text{ V}$$

Required change :

$$q = i\,\Delta t = \frac{e}{R}\Delta t = \frac{640 \times \pi^2 \times 10^{-5}}{40\pi^2} \times 0.05 = 8\,\mu C$$

2. 0

$$\langle i \rangle = \frac{\int_{\pi/2\omega}^{3\pi/2\omega} I_m \sin\omega t\, dt}{\frac{3\pi}{2\omega} - \frac{\pi}{2\omega}} = \frac{I_m\left(-\frac{\cos\omega t}{\omega}\right)_{\pi/2\omega}^{3\pi/2\omega}}{\frac{\pi}{\omega}} = 0$$

3. **6**

Case (i), u = – 25 cm, f = 20 cm ; v = ?

$$\frac{1}{f} = -\frac{1}{u} + \frac{1}{v} \quad \therefore \quad \frac{1}{20} = -\frac{1}{-25} + \frac{1}{v}$$

or $\frac{1}{v} = \frac{1}{20} - \frac{1}{25} = \frac{1}{100}$ or v = 100 cm.

$$m_{25} = \frac{v}{u} = \frac{100}{-25} = -4$$

Case (ii), u = –50 cm; f = 20 cm, v = ?

$$\frac{1}{20} = \frac{-1}{-50} + \frac{1}{v} \text{ or } \frac{1}{v} = \frac{1}{20} - \frac{1}{50} = \frac{3}{100}$$

or $v = \frac{100}{3}$ cm

$$m_{50} = \frac{(100/3)}{-50} = -\frac{2}{3}$$

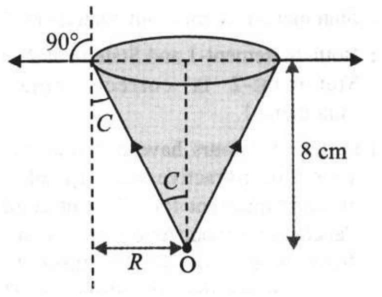
$$\frac{m_{25}}{m_{50}} = \frac{-4}{-2/3} = 6$$

4. **6**

Refer to Figure, $\mu = \frac{1}{\sin C}$

or $\sin C = \frac{1}{\mu} = \frac{1}{5/3} = \frac{3}{5}$

$\therefore \tan C = \frac{3}{4} = \frac{R}{8}$ or $\frac{3 \times 8}{4} = 6$ cm



5. **8**

Here, $A_1 = 15^o$, $\mu_1 = 1.4$; $\mu_2 = 1.75$, $A_2 = ?$

There will be no deviation for the combination of two prisms if net deviation is zero, i.e.,

$\delta_1 + \delta_2 = 0$ or $(\mu_1 - 1)A_1 + (\mu_2 - 1)A_2 = 0$.

or $A_2 = -\frac{(\mu_1 - 1)A_1}{(\mu_2 - 1)} = -\frac{(1.4 - 1)15^o}{(1.75 - 1)} = -8^o$

–ve sign shows that the second prism is placed in opposition to first prism.