LAKSHYA ADVANCED UNIT TEST (LAUT)

00 – 00		Q. Booklet Serial No: 280615 PAPER I	
Test No : 2181	3 Hrs.		

Hints & Solutions

PART A - PHYSICS

SECTION I - MULTIPLE ANSWER CORRECT TYPE

1. **a)** $\oint \vec{E}.\vec{ds} = \frac{q_1 + q_2 + q_3}{\varepsilon_0}$

   b) $\oint \left(\vec{E}_1 + \vec{E}_2 + \vec{E}_3\right).\vec{ds} = \frac{q_1 + q_2 + q_3}{\varepsilon_0}$

   c) $\oint \vec{E}_4 . \vec{ds} = 0$

   d) $\oint \vec{E}_2 . \vec{ds} = \frac{q_2}{\varepsilon_0}$

   As from Gauss law, $\oint \vec{E}.\vec{ds} = \frac{q_{enc}}{\varepsilon_0}$

2. **a)** $Q_1 + Q_3 = -Q_2$

   b) $Q_1 = -\frac{Q_2}{4}$

   c) $\frac{Q_3}{Q_1} = 3$

   $V_3 = 0$

   $\Rightarrow \frac{Q_1 + Q_2 + Q_3}{3r} = 0$ - (i)

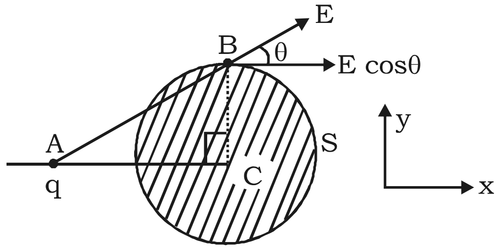
   $V_1 = 0$

   $\Rightarrow \frac{Q_1}{r} + \frac{Q_2}{2r} + \frac{Q_3}{3r} = 0$ - (ii)

   From (i) and (ii) options (a), (b), (c) are correct.

3. **b)** **X-component of electric field at the point B only due to conductor is** $\frac{-9q}{125\pi\varepsilon_0 R^2}\hat{i}$

   c) **Electric potential at the point B only due to conductor is** $\frac{1}{4\pi\varepsilon_0 R}\left[\frac{3q}{20} + Q\right]$



At B the tangential component of electric field is zero hence the X component of electric field due to conductor is given

$$E_x = -E\cos\theta = -\frac{kq}{\left(\frac{5R}{3}\right)^2} \times \frac{4R/3}{5R/3}$$

$$= -\frac{9q}{125\pi\varepsilon_0 R^2}$$

(ii) The electric potential of whole conductor is same, we assume electric potential due to conductor be $V_{conductor}$.

$$V_B = V_C = \frac{kq}{\frac{4R}{3}} + \frac{KQ}{R}$$

$$= \frac{Kq}{5R/3} + V_{conductor}$$

or $V_{conductor} = \frac{1}{4\pi\varepsilon_0 R}\left[\frac{3q}{20} + Q\right]$

4. **a)** **potential at its surface is 150V**

   c) **the electric field on the surface is 1500 V/m**

   d) **the electric potential at its center is 225 V**

   The potential at surface, 5cm from surface and 10cm from surface outwards is

   $V_s = \frac{KQ}{R}$ - (i)

$$\frac{KQ}{R+5} = 100 \quad - \text{(ii)}$$

$$\frac{KQ}{R+10} = 75 \quad - \text{(iii)}$$

From equation (ii) and (iii) ⇒ R = 10 cm

∴ from equation (ii)

$$Q = \frac{100 \times 15 \times 10^{-2}}{9 \times 10^{9}} = \frac{5}{3} \times 10^{-9} C$$

⇒ B is false

$$V_{surface} = \frac{KQ}{R}$$

$$= \frac{100 \times (R+5)}{R} = \frac{100 \times 15}{10}$$

$$= 150\,V \quad \Rightarrow \quad \text{A is true}$$

$$V_{centre} = \frac{3}{2}\frac{KQ}{R} = 225 \text{ volts}$$

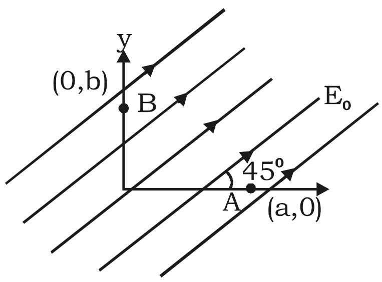
⇒ D is true

$$E_{surface} = \frac{KQ}{R^2} = \frac{150}{10 \times 10^{-2}}$$

$$= 1500\,V/m$$

⇒ C is true

5. **b)** $\mathbf{V_A = V_B}$ **if a = b**
c) $\mathbf{V_A > V_B}$ **if a < b**
d) $\mathbf{V_A < V_B}$ **if a > b**



$$\vec{E} = \frac{E\hat{i}}{\sqrt{2}} + \frac{E\hat{j}}{\sqrt{2}}$$

If a > b then

$$V_A - V_B = -E\,.\,\overrightarrow{BA}$$

$$= -\left(\frac{E}{\sqrt{2}}\hat{i} + \frac{E}{\sqrt{2}}\hat{j}\right) . \left(+\,a\,\hat{i} - \hat{j}\right)$$

$$V_A - V_B = -\frac{Ea}{\sqrt{2}} + \frac{Eb}{\sqrt{2}} = -\text{ve}$$

∴ $V_B > V_A$

if a = b $\quad V_A - V_B = 0$

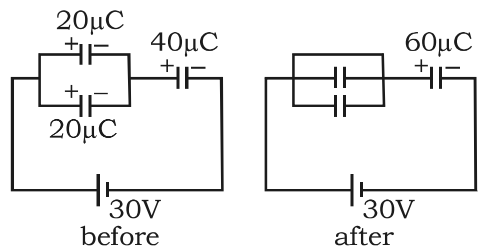
if a < b $\quad V_A - V_B > 0 \quad$ or $V_A > V_B$

6. **a) The charge on the inner surface of the shell will be +3C and it can be distributed uniformly or non uniformly**
d) The net charge on outer surface of the shell will be +7C and its distribution would be uniform

By property of electrostatic shielding in conductors.

7. **a) the amount of charge flown through the battery is 20 μC**
c) the energy supplied by the battery is 0.6 mJ
d) the amount of charge flown through the switch S is 60μC

The charges stored in different capacitors before and after closing the switch S are



The amount of charge flown through the battery is q = 20 mC

∴ Energy supplied by the battery is

$U = qV = (20 \times 10^{-6})(30)J$

$U = 0.6$ mJ

Energy stored in all the capacitors before closing the switch S is

$$U_i = \frac{1}{2}C_{net}V^2 = \frac{1}{2}\left(\frac{4}{3} \times 10^{-6}\right)(30)^2$$

$$= 0.6 \text{ mJ}$$

and after closing the switch

$$U_f = \frac{1}{2}C_{net}V^2 = \frac{1}{2}\left(2 \times 10^{-6}\right)(30)^2$$

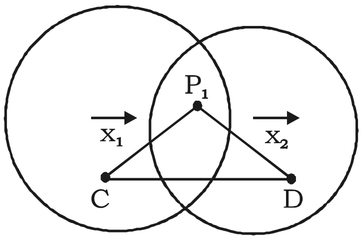
$$= 0.9 \text{ mJ}$$

∴ Heat generated H = $(U_f - U_i)$ = 0.3 mJ and charge flown through the switch is 60μC.

8. **c) The electrostatic field is constant in magnitude.**

d) The electrostatic field has same direction.

At any arbitrary point P in the shaded region.



$\vec{E}_1$ = E.F. due to ρ charge

$= \frac{\rho}{3\epsilon_0}\vec{x}_1$

$= \frac{\rho}{3\epsilon_0}\overrightarrow{C_1P_1}$

$\vec{E}_2$ = E.F. due to $-\rho$ charge

$= \frac{\rho}{3\epsilon_0}\vec{x}_2$

$= \frac{\rho}{3\epsilon_0}\overrightarrow{P_1C_2}$

$\therefore$ net E.F. at P $= \overrightarrow{E_1} + \overrightarrow{E_2}$

$= \frac{\rho}{3E_0}\left(\overrightarrow{C_1P_1} + \overrightarrow{P_1C_2}\right)$

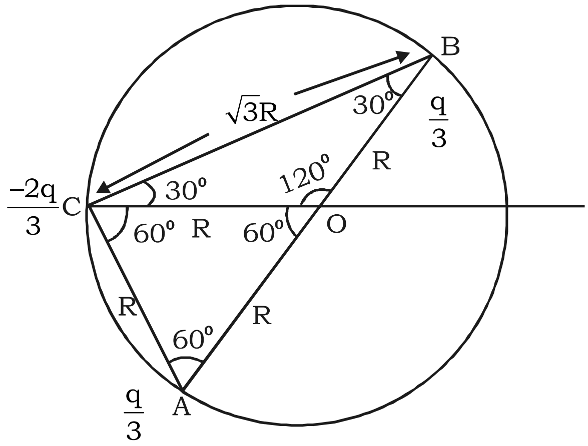
$= \frac{\rho}{3E_0}\overrightarrow{C_1P_2}$

$\because$ $\overrightarrow{C_1C_2}$ = constant

So E.F. in the shaded region is uniform and non zero.

Due to uniform electric field present inside electric potential cannot be constant.

9. **c) The magnitude of the force between the charges at C and B is $\frac{q^2}{54\pi\epsilon_0 R^2}$**



In Δ ACB

$CB^2 = AB^2 - AC^2$

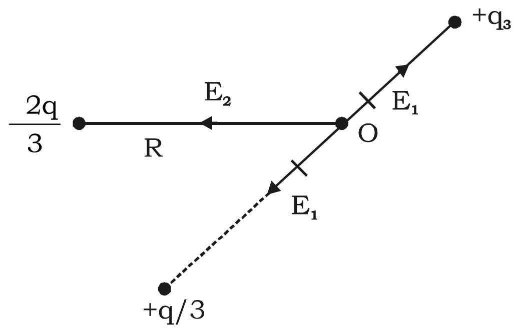
$= qR^2 - R^2$

$CB = \sqrt{3}R$

at point O net Electric field is due to $\frac{-2q}{3}$

only at O $E_{net} = K\frac{2q}{3R^2}$ towards

$= \frac{1}{4\pi\epsilon_0}\frac{2q}{3R^2} = \frac{q}{6\pi\epsilon_0 R^2}$ towards



Potential energy of system $= \frac{Kq}{3(2R)} + \frac{K\frac{q}{3}\left(-\frac{2q}{3}\right)}{R} + \frac{K\frac{q}{3}\left(-\frac{2q}{3}\right)}{\sqrt{3}R} \neq C$

Force between charges at C and B is

$F = \frac{K\frac{q}{3}\left(\frac{2q}{3}\right)}{\left(\sqrt{3}R\right)^2} = \frac{1}{4\pi\epsilon_0}\frac{2q^2}{q}\times\frac{1}{3R^2}$

$$= \frac{q^2}{54\pi \epsilon_0 R^2}$$

Net potential at point O is

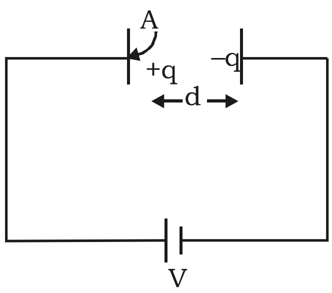
$$V_0 = \frac{Kq}{3R} + \frac{Kq}{3R} - \frac{2q}{3R} = 0$$

10. **a)** $\mathbf{Q = \frac{\epsilon_0 AV}{d}}$

c) $\mathbf{E = \frac{V}{Kd}}$

d) $\mathbf{W = \frac{\epsilon_0 AV^2}{2d}\left[1 - \frac{1}{K}\right]}$

When capacitor is initially charged by a battery then



$$C = \frac{\epsilon_0 A}{d}$$

$$q = CV = \frac{\epsilon_0 A}{d}V \quad \ldots(i)$$

Potential energy stored

$$= U_1 = \frac{1}{2}\frac{q^2}{C}$$

When battery is disconnected and dielectric is inserted then charge on capacitor remains constant but capacitance increases K times due to which potential difference $V' = \frac{q}{KC} = \frac{V}{K}$ decreases K times so electric field

$E = \frac{V'}{d} = \frac{V}{Kd}$ decreases K times.

Work done on the system = loss in potential energy

$$= U_i - U_f$$

$$= \frac{1}{2}\frac{q}{c^2} - \frac{1}{2}\frac{q^2}{KC} = \frac{1}{2}\frac{q^2}{C}\left(1 - \frac{1}{K}\right)$$

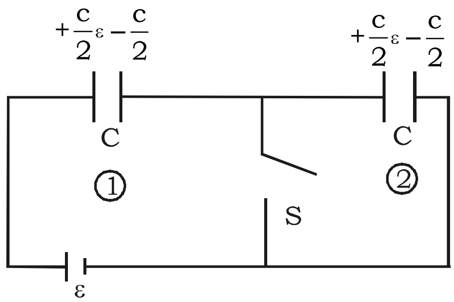
$$= \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)\frac{V^2}{\epsilon_0 A}d\left(1 - \frac{1}{K}\right)$$

$$W = \frac{1}{2}\frac{\epsilon_0 AV^2}{d}\left(1 - \frac{1}{K}\right)$$

SECTION II - MATRIX MATCH TYPE

1. **A-S, B-Q, C-Q, D-P**

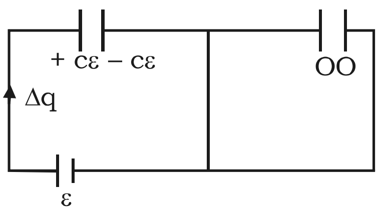
Initially when switch S is open two capacitors are in series.



So charge is distributed as shown in diagram.

$$U_1 = \frac{1}{2}\left(\frac{c}{2}\right)\varepsilon^2 = \frac{1}{4}c\varepsilon^2$$

Finally after switch S is closed capacitor 2 is shorted and gets discharged



So additional charge supplied by battery

$$= \Delta q = c\varepsilon - \frac{c\varepsilon}{2} = \frac{c\varepsilon}{2}$$

$$\text{Final P.E.} = U_2 = \frac{1}{2}c\varepsilon^2$$

$$\Delta U = U_2 - U_1 = \frac{1}{2}c\varepsilon^2 - \frac{1}{4}c\varepsilon^2$$

$= \frac{1}{4}c\varepsilon^2$

Work done by cell = $(\Delta q)\,\varepsilon = \left(\frac{c\varepsilon}{2}\right)\varepsilon$

$= \frac{c\varepsilon^2}{2}$

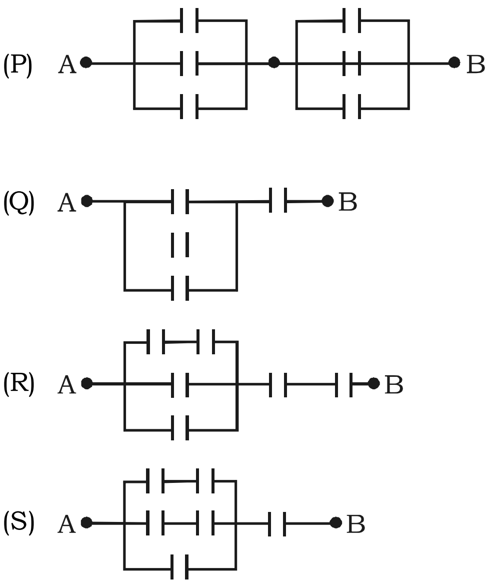
Heat produced = Work done by all – ΔU

$= \frac{c\varepsilon^2}{2} - \frac{1}{4}c\varepsilon^2$

$= \frac{c\varepsilon^2}{4}$

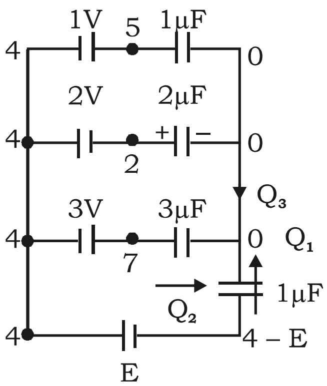
2. **A-S, B-P, C-Q, D-R**

Eq. Circuit of case P



SECTION III - INTEGER TYPE

1. **2**



Potential across 2μF = $\frac{4}{2}$ = 2 volt

$Q_1 + Q_2 + Q_3 = 0$

$(4 - E) \times 1 + 7 \times 3 + [2 \times 2 + 5 \times 1] = 0$

$4 - E + 21 + 9 = 0$

E = 34 volt

2. **9**

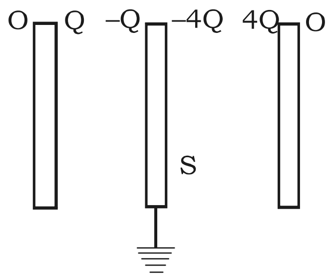
Inseries combination if V kV is applied then 2/3 V and 1/3 V appears across C_1 and C_2 respectively.

So 2/3 V < 6 = > V < 9

Also V/3 < 4 = > V < 12

So common maximum safe value for the combination is 9kV

3. **7**



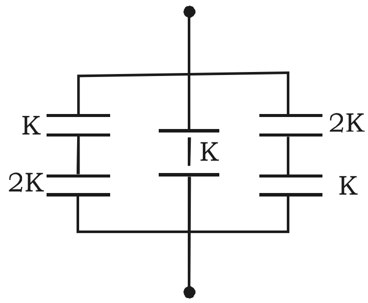
final distribution of charge is as shown

∴ final charge on plate B = – 5Q so charge transfer from earth = – 5Q – 2Q

= – 7Q

∴ charge from to earth = 7Q

4. **6**



Given system can be considered as

$C_{eq} = \frac{11KC_0}{9}$

$$C_0 = \frac{9a^2\varepsilon_0}{d}$$

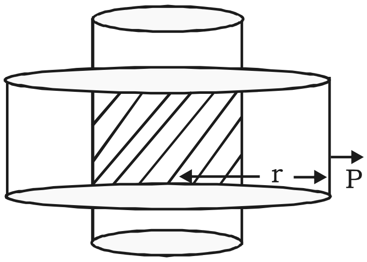
$$K = 2$$

5. **6**

Electric field due to long solid cylinder at point P is $E_1\,(2\pi r l) = \dfrac{\rho \times \pi (R)^2 l}{\epsilon_0}$

$$E_1 (2\pi)(2R) l = \frac{\rho \times \pi R^2 l}{\epsilon_0}$$

$$E_1 = \frac{\rho R}{4E_0} \quad \ldots \text{(i)}$$



Electric field at point P due to solid sphere in like a point charge placed at centre of sphere

$$E_2 = \frac{K\rho \times \frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{(2R)^2} = \frac{1}{4\pi\epsilon_0}$$

$$\rho \times \frac{4}{3}\frac{\pi R^3}{8 \times 4R^2} = \frac{\rho R}{96\epsilon_0} \quad \ldots \text{(ii)}$$

∴ By superposition principle elctric field at point P is

$$E_{net} = E_1 - E_2 = \frac{\rho R}{\epsilon_0}\left(\frac{1}{4} - \frac{1}{96}\right)$$

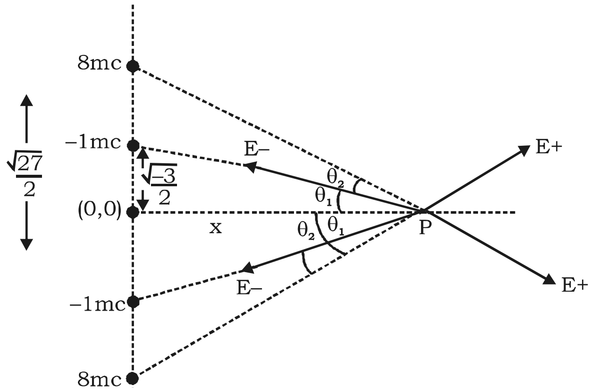
$$= \frac{\rho R}{96\epsilon_0}(23)$$

$$= \frac{\rho R}{96\epsilon_0} 23 = \frac{\rho R 23}{16K\epsilon_0}$$

$$= K = 6$$

6. **3**

Locate point P at which Electric field = 0



$$\Rightarrow 2E_+ \cos\theta_2 = 2E_- \cos\theta_1$$

$$\frac{K8\times10^{-3}}{\left(x^2+\frac{27}{2}\right)}\frac{x}{\sqrt{x^2+\frac{27}{2}}} = \frac{K\times1\times10^{-3}}{\left(x^2+\frac{3}{2}\right)}\frac{x}{\sqrt{x^2+\frac{3}{2}}}$$

$$\Rightarrow \frac{\left(x^2+\frac{27}{2}\right)^{3/2}}{\left(x^2+\frac{3}{2}\right)^{3/2}} = 8$$

$$\Rightarrow \frac{\left(x^2+\frac{27}{2}\right)^{1/2}}{\left(x^2+\frac{3}{2}\right)^{1/2}} = 2$$

$$\Rightarrow \left(x^2+\frac{27}{2}\right) = 4\left(x^2+\frac{3}{2}\right)$$

$$\Rightarrow x = \pm\sqrt{\frac{5}{2}} \quad \ldots \text{(i)}$$

Applying energy conservation between ∞ and point P

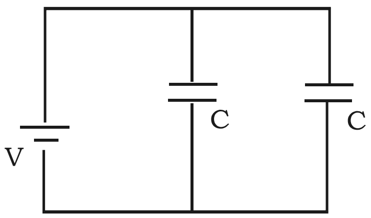
$$\frac{1}{2}mv_0^2 = \frac{K8\times10^{-6}\times0.1\times10^{-6}\times2}{\left(x^2+\frac{27}{2}\right)^{\frac{1}{2}}} - \frac{K\times1\times10^{-6}\times0.1\times10^{-6}\times2}{\left(x^2+\frac{3}{2}\right)^{\frac{1}{2}}}$$

Put $x = \sqrt{\frac{5}{2}}$ and m = 6×10^{-4} kg we get

$V_0 = 3$ m/s

7. **6**

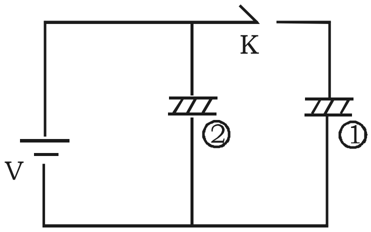
Initially when switch is closed



$U_1 = \frac{1}{2} \times (2c) \times V^2 = CV^2$ ... (i)

q = CV on each capacitor

Finally when switch is open



Charge on capacitor (1) remain unchanged

$\therefore U' = \frac{1}{2}\frac{q^2}{3C} = \frac{1}{6}CV^2$

and on capacitor (2) potential difference = V

$\therefore U'' = \frac{1}{2}(3C)V^2 = \frac{3c}{2}V^2$

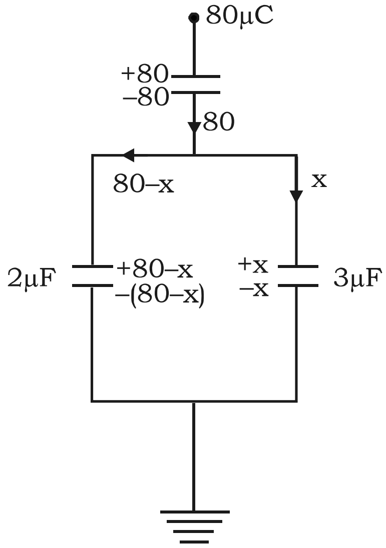
$\therefore U_2 = U' + U'' = \frac{CV^2}{6} + \frac{3CV^2}{2}$

$= \frac{10CV^2}{6}$ ... (ii)

by (i) and (ii)

$\frac{U_1}{U_2} = \frac{CV^2}{10CV^2} \times 6 = \frac{6}{10} \Rightarrow n = 6$

8. **6**



80 μc charge will be distributed on the parallel combination of 2 μf and 3μf.

$\therefore \frac{x}{3} = \frac{80 - x}{2}$

$= 2x = 240 - 3x$

$= 5x = 240$

$= x = 48$

$= x = 6$