LAKSHYA ADVANCED UNIT TEST (LAUT)

00 – 00		PHYSICS PAPER II	
Test No : -----	3 Hrs.		

Hints & Solutions

SECTION I - MULTIPLE ANSWER CORRECT

1. **a)** If only switch S_2 is closed then the charge transferred through this switch will be $\frac{Q}{2}$

c) If only switch S_1 is closed then the charge transferred through this switch will be Q

Hint : $\frac{kq}{r} - \frac{k2Q}{2r} + \frac{k(4Q-q)}{3r} = \frac{k2Q}{3r}$

$q - Q + \frac{4Q}{3} - \frac{q}{3} = \frac{2Q}{3} \Rightarrow \frac{2q}{3} = \frac{Q}{3} \Rightarrow q = \frac{Q}{2}$

$\frac{kQ}{2R} + \frac{k3Q}{3R} + \frac{kq'}{2R} = 0$

$q' = -(Q + 2Q) = -3Q$

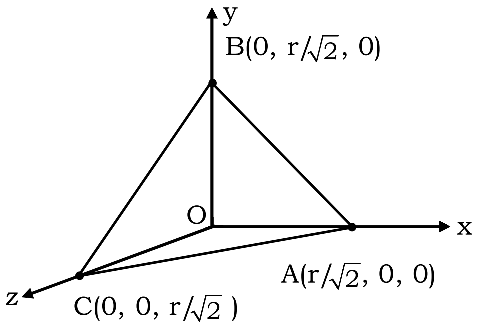
$\therefore \Delta q = Q$

2. **a)** In case I, flux of electric field through ΔABC is equal to $\frac{Q}{8\varepsilon_o}$

c) In **case II**, (-q) charge will cross the centroid of ΔABC

d) In **case II**, The charge (-q) passes through the plane of ΔABC with kinetic energy equals to $\frac{3(\sqrt{3}-\sqrt{2})Qq}{4\pi\varepsilon_o r}$

In case I, ΔABC can be drawn in remaining seven quadreants symmetrically. As total flux passing through complete surface is Q/ε_0 hence the flux through each face will be $Q/8\varepsilon_0$.



In case II, force acting on –q is directed along the line joining origin and centroid of ΔABC initially.

As the charge particle is released from origin it will cross the centroid.

Potential at the origin is equal to $\frac{3KQ}{r/\sqrt{2}} = \frac{3\sqrt{3}KQ}{r}$

and potential at centroid is equal to $\frac{3KQ}{r/\sqrt{3}}$

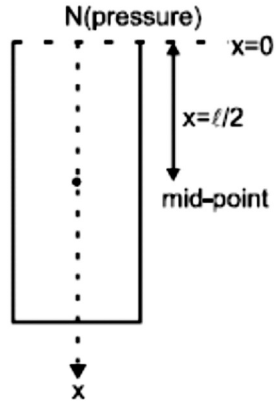
Hence gain in KE of -q charge particle is equal to $\frac{3(\sqrt{3}-\sqrt{2})Qq}{4\pi\varepsilon_o r}$

3. **a)** length of pipe is $\frac{15}{16}$ m

c) pressure amplitude at mid point of pipe is $\frac{P_0}{\sqrt{2}}$

$5\frac{v_0}{4l} = f$

$\frac{5(300)}{4l} = 440$. $\lambda = \frac{15}{16}$ m



$330 = 440 \times \lambda$, $\lambda = 3/4$ m

$P_{(n)} = P_0 \sin Kx$, $K = \frac{2\pi}{\lambda}$, $x = \frac{l}{2}$

$p = P_0 \sin\left(\frac{5\pi}{4}\right) = -\frac{P_0}{\sqrt{2}}$

4. **a)** frequency observed by A is $\frac{3600}{7}$ Hz

c) frequency observed by C is 700 Hz

$C = 325\frac{m}{sec}$

f = 600 Hz

$f_A = \left(\frac{C - V_A}{C + V_S}\right) f = \frac{3600}{7}$ Hz

$f_B = \left(\frac{C + V_B}{C + V_S}\right) f = 600$ Hz

$f_C = \left(\frac{C + V_C}{C - V_S}\right) f = 700$ Hz

5. **a)** transverse displacement of the particle at x = 0.05 m and t = 0.05 s is $-2\sqrt{2}$ cm

c) speed of travelling waves that interfere to produce this standing wave is 2 m/s

d) the transverse velocity of the string particle at $x = \frac{1}{15}$ m and t = 0.1 s is 20 π cm/s

$\frac{\lambda}{4} = 0.1 \Rightarrow \lambda = 0.4$ m

from graph ⇒ T = 0.2 sec. and amplitude of standing wave is 2A = 4 cm.

Equation of the standing wave

$y(x, t) = -2A \cos\left(\frac{2\pi}{0.4}x\right).\sin\left(\frac{2\pi}{0.2}t\right)$cm

$y(x = 0.05, t = 0.05) = -2\sqrt{2}$ cm

$y(x = 0.04, t = 0.025) = -2\sqrt{2} \cos 36^0$

speed $= \frac{\lambda}{T} = 2$m/sec.

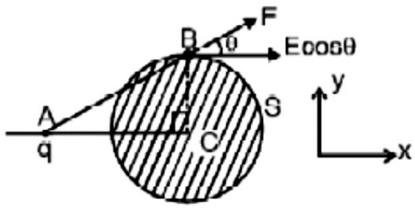
$V_y = \frac{dy}{dt} = -2A \times \frac{2\pi}{0.2}\cos\left(\frac{2\pi x}{0.4}\right)\cdot\cos\left(\frac{2\pi t}{0.2}\right)$

$V_y = (x = \frac{1}{15}$ m, t = 0.1) = 20π cm/sec.

6. **b)** X-component of electric field at the point B only due to conductor is $\frac{-9q}{125\pi\varepsilon_0 R^2}\hat{i}$

c) Electric potential at the point B only due to conductor is $\frac{1}{4\pi\varepsilon_0 R}\left[\frac{3q}{20} + Q\right]$

(i)



At B the tangential component of electric field is zero hence the X component of electric field due to conductor is given

$E_x = -E\cos\theta = -\frac{kq}{\left(\frac{5R}{3}\right)^2} \times \frac{4R/3}{5R/3} = \frac{-9q}{125\pi\varepsilon_0 R^2}$

(ii) The electric potential of whole conductor is same, we assume electric potential due to conductor be $V_{conductor}$

$V_B = V_C = \frac{\frac{kq}{4R}}{3} + \frac{KQ}{R} = \frac{KQ}{5R/3} + V_{conductor}$

or $V_{conductor} = \frac{1}{4\pi\varepsilon_0 R}\left[\frac{3q}{20} + Q\right]$

7. **b)** the magnitude of the acceleration at a distance 3 cm from the point is 27 cm/sec²

c) the motion is simple harmonic about the given fixed point.

$v^2 = 108 - 9x^2$ or $v^2 = 9(12 - x^2)$

We can compare the above expression with $v = \omega = \sqrt{A^2 - x^2}$, which is expression of velocity for S.H.M From this, we will get

$\omega = 3$ & $A = \sqrt{12}$

S.H.M is not a uniformly accelerated motion. Acceleration at a distance 3 cm from the mean position

$a = \omega^2(3cm) = 27cm/s^2$

Maximum displacement from the mean position $= A = \sqrt{12}$

8. **b)** the amplitude of the S.H.M of the particle is 5 units

d) the maximum displacement of the particle from the origin in 9 units

$x = 3 \sin 100 t + 8 \cos^2 50 t$

$= 3 \sin 100 t + \frac{8[1+\cos 100 t]}{2}$

$x = 4 + 3 \sin 100 t + 4 \cos 100 t$

$(x - 4) = 5 \sin (100t + \phi)$ $\{\tan \phi = \frac{4}{3}\}$

Amplitude = 5 units

Maximum displacment = 9 units.

SECTION II - Paragraph Type

9. **c)** $\frac{3}{2}\sqrt{gL}$

At the instant block is in equilibrium position, its speed is maximum and compression in spring is x given by

$kx = mg$ ..... (1)

From conservation of energy

$kx = mg$ ..... (1)

$mg(L + x) = \frac{1}{2}kx^2 + \frac{1}{2}mv^2_{max}$ ..... (2)

from (1) and (2) we get $v_{max} = \frac{3}{2}\sqrt{gL}$

10. **b)** $\frac{\pi}{4}\sqrt{\frac{L}{g}} + \sqrt{\frac{L}{4g}} \sin^{-1}\frac{1}{3}$

$V_{max} = \frac{3}{2}\sqrt{gL}$ and $\omega = \sqrt{\frac{k}{m}} = 2\sqrt{\frac{g}{L}}$

$\therefore A = \frac{V_{max}}{\omega} = \frac{3}{4}L$

Hence time taken t, from start of compression till block reaches mean position is given by

$x = A \sin \omega t_0$ where $x = \frac{L}{4}$

$\therefore t_0 = \sqrt{\frac{L}{4g}} \sin^{-1}\frac{1}{3}$

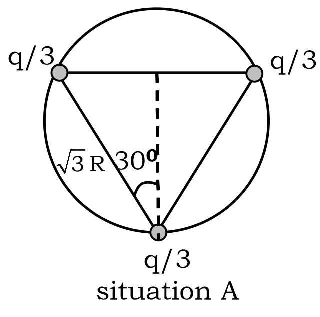
Time taken by block to reach from mean position to bottom most position is $\frac{2\omega}{4\omega} = \frac{\omega}{4}\sqrt{\frac{L}{g}}$

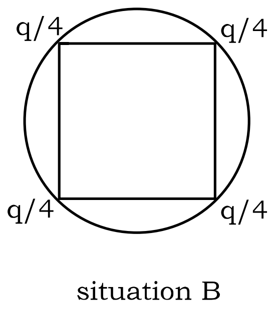
Hence the required time $= \frac{\pi}{4}\sqrt{\frac{L}{g}} + \sqrt{\frac{L}{4g}} \sin^{-1}\frac{1}{3}$

11. **b)** $\frac{1}{1}$

12. **c)** $\frac{1}{12\sqrt{3}\pi\epsilon_0}\frac{q^2}{r}$

(Sol. 11 to 12)





Potentials at the centre

$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$; $V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

Potential energy in situation l is

$U_1 - 3 \times \frac{1}{4\pi\varepsilon_0}\frac{(q/3)^2}{(\sqrt{3}R)} - \frac{1}{12\sqrt{3}\pi\varepsilon_0}\frac{q^2}{R}$

When one charge is removed, the field intencity at the centre is due to the removed charge only.

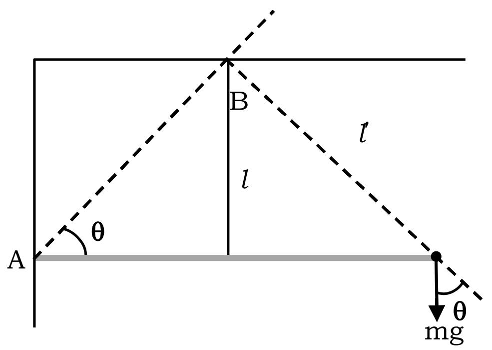
$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q/3}{r^2}$

$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q/4}{r^2}$ $\therefore$ $\frac{E_1}{E_2} = \frac{4}{3}$

SECTION III - Integer Type

1. 4

The bob will execute SHM about a stationary axis possing through AB. If its effective lenght is l' then



$T = 2\pi\sqrt{\frac{l'}{g'}}$

$l' = l/\sin\theta = \sqrt{2}\, l$ (because $\theta = 45$)

$g' = g\cos\theta = g/\sqrt{2}$

$T = 2\pi\sqrt{\frac{2l}{g}} = 2\pi\sqrt{\frac{2\times0.2}{10}} = \sqrt{\frac{2\pi}{5}}$ S.

2. 2

The angular position of pendulum 1 and 2 are (taking angles to the right to reference line xx' to be positive)

$\theta_1 = \theta\cos\left(\frac{4\pi}{T}t\right)$ [where $T = 2\pi\sqrt{\frac{l}{g}}$]

$\theta_2 = -\theta\cos\left(\frac{2\pi}{T}t\right) = \cos\left(\frac{2\pi}{T}t+\pi\right)$

For the string to be parallel for the first time $\theta_1 = \theta_2$

or $\cos\left(\frac{4\pi}{T}t\right) = \cos\left(\frac{2\pi}{T}t+\pi\right)$

$\therefore$ $\frac{4\pi}{T}t = 2n\pi \pm \left(\frac{2\pi}{T}t+\pi\right)$

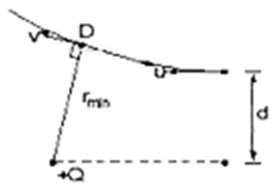
for n = 0, t = $\frac{T}{2}$

for n = 1, t = $\frac{T}{6}, \frac{3T}{2}$

$\therefore$ Both the pendulum are parallel to each other for the first time after t = $\frac{T}{6}, \frac{\pi}{2}\sqrt{\frac{l}{g}}$

3. 1

The path of the particle will be as shown in the figure. At the point of minimum distance (D) the velocity of the particle will be ^ to its position vector w.r.to + Q



$\frac{1}{2}mu^2 + 0 = \frac{1}{2}mv^2 + \frac{kqQ}{r_{min}}$ ....(1)

$\therefore$ Torque on q about Q is zero hence angular momentum about Q will be conserved

$\Rightarrow$ $m\,v\,r_{min} = m\,ud.$ ....(2)

by (2) in (1) $\Rightarrow \frac{1}{2}mu^2 = \frac{1}{2}m\left(\frac{ud}{r_{min}}\right)^2 + \frac{kqQ}{r_{min}}$

$\Rightarrow \frac{1}{2}mu^2\left(1-\frac{d^2}{r_{min^2}}\right) = \frac{mu^2d}{r_{min}}$

{$\therefore$ KQq = mu²d (given)}

$\Rightarrow r^2_{min} - 2r_{min}\,d - d^2 = 0$

$\Rightarrow r_{min} = \frac{2d \pm \sqrt{4d^2+4d^2}}{2} = d(1 \pm \sqrt{2})$

$\therefore$ distance cannot be negative

$\therefore$ $r_{min} = d(1 \pm \sqrt{2})$

4. 5

Flux through ABCD

$f_1 = \overset{1}{E}\,\overset{1}{A}$

$= (x^2\,\hat{i} + y\hat{j}).(-a^2\hat{i})$

= 0 as x = 0

Flux through EFGH

$f_2 = (x^2\,\hat{i} + y\hat{j}).(+a^2\hat{i})$

$= x^2, a^2 = a^4 = 1.0 \times 10^{-4}$ NM²/C

Flux through BCGF

$f_3 = (x^2\,\hat{i} + y\hat{j}).(a^2\,\hat{j})$

$= a^3 = 1.0 \times 10^{-3}$ NM²/C

Flux through EADH

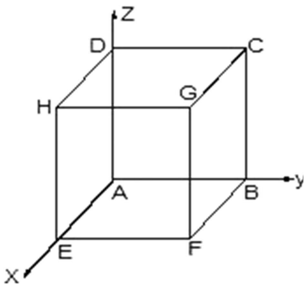
$f_4 = (x^2\,\hat{i} + y\hat{j}).(-a^2\,\hat{j})$ = 0 as y = 0

Flux through ABFE

$f_5 = (x^2\,\hat{i} + y\hat{j}).(-a^2\,\hat{k})$ = 0

Flux through CDHG

$f_6 = 0$



Net flux = $(1.0 \times 10^{-4} + 1.0 \times 10^{-3})$ N–m2/C = 11×10^{-4} N–m²/C

5. 1

For fundamental frequency

$$\mu = \frac{3.2\text{gm}}{40\text{cm}} = \frac{3.2\times10^{-3}}{40\times10^{-2}} = \frac{3.2}{40} = \frac{32}{4000}\ \text{kg/m}$$

$$l = \frac{\lambda}{2}\Big|$$

$\Rightarrow \lambda = 2l$ .... (1)

$$f = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{l}{\mu}}$$

$$\Rightarrow \frac{1000}{64} = \frac{1}{2\times40\times10^{-2}}\sqrt{\frac{T}{32/4000}}$$

$$\Rightarrow \left[\frac{1000}{64}2\times40\times10^{-2}\right]^2\frac{32}{4000} = T$$

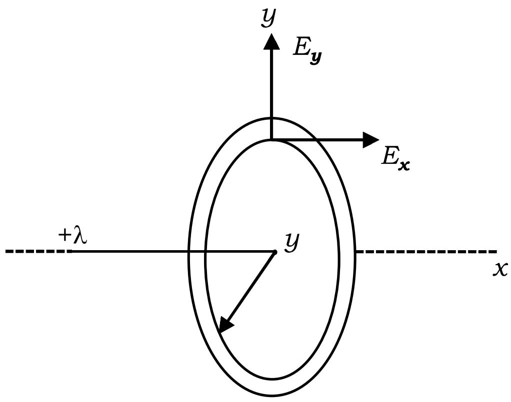
$$\frac{1000}{64}\times\frac{32}{4000} = T \qquad \Rightarrow T = \frac{10}{8}\ \text{N}$$

Now $$y = \frac{\frac{10/8}{10^{-6}}}{\frac{.05\times10^{-2}}{40\times10^{-2}}} = \frac{10^7}{8}\frac{40}{(.05)} = 10^9\text{N/m}^2$$

[{**Ans.** 1×10⁹Nm²]

6. 3

The component of the electrified E_y will noit contribute to flux as angle between and $\vec{A}$ is 90°. Hence, flux due to $\vec{E}$ is



$$d\phi = E_x \times 2\pi y dy$$

$$\phi = \int d\phi = \int_0^R \frac{\lambda}{4\pi\varepsilon.\,y}.2\pi\, y dy$$

$$\phi = \frac{\lambda}{2\varepsilon_0}\int_0^R dy = \frac{\lambda R}{2\varepsilon_0}$$

$$= \frac{6\times1}{2\varepsilon_0} = \frac{3}{\varepsilon_0}$$

7. 4

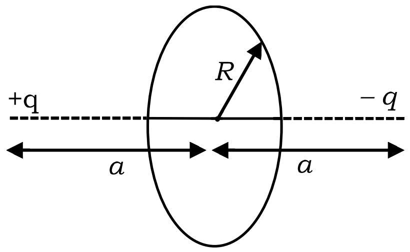
$d\phi = 2E\cos\theta \times 2\pi y dy$

$$= 2 \times \frac{1}{4\pi\varepsilon_0}\frac{q}{(a^2+y^2)}\times\frac{a\times2\pi\, ydy}{(a^2+y^2)^{1/2}}$$

$$\phi = \int d\phi = \frac{qa}{\epsilon_0}\int_0^R \frac{ydy}{(a^2+y^2)^{3/2}}$$

$$\phi = \frac{q}{\varepsilon_0}\left[1 - \frac{1}{\sqrt{1+(R/a)^2}}\right]$$

$$= \frac{8}{\varepsilon_0}\left[1-\frac{1}{2}\right] = \frac{4}{\varepsilon_0}$$



8. 0

$W_{A\to B} = q(V_B - V_A)$

$\int_A^B dV = V_B - V_A = -\int_A^B \vec{E}\cdot\vec{dr}$

$= -\int_{(2,2)}^{(4,1)} (y\hat{i} + x\hat{j}).(dx\hat{i} + dy\hat{j} + dz\hat{k})$

$= -\int_{(2,2)}^{(4,1)} (ydx + xdy)$

$= -\int_{(2,2)}^{(4,1)} d(xy) = [-xy]_{2,2}^{4,1} = 0$

So $W_{AB} = 0$.