1. (A) $\vec{F} \times \vec{a}$ (B) $\vec{v} \times \vec{r}$

(C) $\vec{a} \times \vec{r}$ (D) $\vec{F} \times \vec{r}$

2. (A) The motion of the particle is SHM with time period $T = \frac{2\pi}{3}$ unit

(B) The maximum displacement of the particle from the fixed point is 4 unit

(C) The magnitude of acceleration at a distance 3 units from the fixed point is 27 unit

Sol. $v^2 = 144 - 9x^2$

$2v\frac{dv}{dx} = -18x$

$\therefore v\frac{dv}{dx} = -9x \Rightarrow \omega = 3 \Rightarrow T = \frac{2\pi}{3}$ units

Also, $v^2 = 144 - 9A^2 = 0$

$A = 4$ units

Now, $|a| = \omega^2 x = 9 \times 3 = 27$ units

3. (B) Time period of SHM is independent of a

(C) In mean position elongation in spring is

$\frac{3mg}{2k}$

Sol. Here time period $T = 2\pi\sqrt{\frac{m}{K}}$ independent of g and a and if mean position elongation is x_0 then,

$Kx_0 = m(g + a) = m\left(g + \frac{g}{2}\right)$

$Kx_0 = \frac{3mg}{2}$

$x_0 = \frac{3mg}{2k}$

4. (B) The motion of particle is oscillatory from x = 0.5 m to x = 1.5 m

(C) The motion of particle is simple harmonic

(D) The period of oscillatory motion is $\frac{\pi}{2}$ s

5. (A) Its potential energy varies periodically with frequency 2f

(B) Its kinetic energy varies periodically with frequency 2f

(D) Its total mechanical energy is constant with infinite period

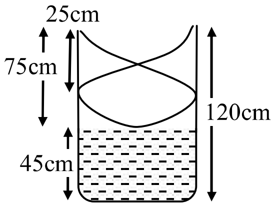
6. (A) minimum length of water column to have the resonance is 45 cm.

(B) the distance between two successive nodes is 50 cm.

(C) the maximum length of water column to create the resonance is 95 cm

Sol. As $V = \nu\lambda$

$\lambda = \frac{V}{\nu} = \frac{340}{340} = 1$ m



first Resonance length

$R_1 = \frac{\lambda}{4} = \frac{1}{4}\text{m} = 25\text{ cm}$

$\therefore R_2 = \frac{3\lambda}{4} = \frac{3}{4}\text{m} = 75\text{ cm}$

$\therefore R_3 = \frac{5\lambda}{4} = \frac{5}{4}\text{m} = 125\text{ cm}$

i.e. third resonance does not establish now H_2O is poured

$\therefore$ minimum length of H_2O column to have the resonance = 45 cm

$\therefore$ Distance between two successive nodes = $\frac{\lambda}{2}$

$= \frac{1}{2}\text{m} = 50\text{ cm}$

and maximum length of H_2O column to create resonance i.e. 120 – 25 = 95 cm.

7. (A) Particle located at E has its velocity in negative x-direction
(B) Particle located at D has zero velocity
(C) Particles located near C are under compression
(D) Change in pressure at D is zero

Sol. Velocity of particle is given by $v_p = -v\left(\frac{dy}{dx}\right)$

Here, v is wave speed and $\frac{dy}{dx}$ the slope.

At point E slope is positive, therefore, v_p will be along negative x-direction. Similarly, slope at D is zero.

Therefore, v_p at D will be zero.

Excess pressure $dP = -B.\frac{dy}{dx}$

At C slope is negative. Therefore, dP is positive i.e., particles located near C are under compression.

At point D, slope is zero i.e., dP = 0

8. **a)** Every particle has a fixed amplitude which is different from the amplitude of its nearest particle.
b) All the particles cross their mean position at the same time.
c) All the particles are oscillating with same amplitude.
d) There is no net transfer of energy across any plane.

Passage No. 1

9. **d)** 2 J

10. (D) $x = 2\cos(2\sqrt{2})t$

$F = -4x \Rightarrow k = 4N/m$

also $a = \frac{F}{m} \Rightarrow a = -8x\ m/s^2$

Now, $\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{4}{0.5}} = 2\sqrt{2}$ rad/s

$\therefore$ P.E. at mean position $= 10 - \frac{1}{2}kA^2$

$= 10 - \frac{1}{2} \times 4 \times 4 = 2$ J

Now at t = 0 $a = -16 m/s^2$ given

$\therefore a = -\omega^2 x$

$16 = \omega^2 x \Rightarrow x = \frac{16}{8} = 2$ m

which is amplitude

$\therefore$equation is $x = 2\cos(2\sqrt{2})t$

Now K.E. at x = 1 = $\frac{1}{2}K(A^2 - x^2)$

$= \frac{1}{2} \times 4 \times 3 = 6$ J

$\therefore$p.E. at x = 1 = 4 J

Passage No. 2

Sol.

11. **a)** $y = 0.1\sin(4\pi(t-1) + 8(x-2))$

The equation of wave moving in negative x-direction, assuming origin of position at x = 2 and origin of time (i.e. intial time) at t = 1 sec.$y = 0.1\sin(4\pi t + 8x)$
Shifting the origin of position to left by 2m, that is, to x = 0. Also Shifting the origin of time backwards
by 1 sec, that is to t = 0 sec. $y = 0.1\sin[(4\pi t + 8(x-2))]$

12. **c)** 0.4π m/s

As given the particle at x = 2 is at mean position at t = 1 sec.
its velocity $v = \omega A = 4\pi \times 0.1 = 0.4\pi$ m/s

INTEGER ANSWER TYPE

1. 2

Sol. Let the speed of the disc be v at any displacement x.
Then, the total mechanical energy of the disc + spring system is

$$E = U + K = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 + \frac{1}{2}I_C\omega^2$$

Where $\omega = \frac{v}{R}$ for rolling and $I_C = \frac{mR^2}{2}$

$$E=\frac{1}{2}kx^2+\frac{3}{4}mv^2$$

Differentiating both sides with x, we have $\frac{dE}{dx}=kx+\frac{3}{2}mv\frac{dv}{dx}$

Where $\frac{vdv}{dx}=a$ and $\frac{dE}{dx}=0$

$$a=\frac{-2k}{3m}x$$

Since, $a=-\omega^2 x,$ we have $\omega=\sqrt{\frac{2k}{3m}}$

2. 2

Sol. $\frac{\delta F}{\delta x}=k_{eff}\left(\text{numerically}\right)$

Let us find δx (static deformation of the system).
Since the particle is in equilibrium,

$$\delta F=2\delta F_s \sin\theta$$

By Pythagoras theorem, $x^2+y^2=l^2$

Taking differentials of both sides, we have $2x\delta x+2y\delta y=2l\delta l$

Since y is constant, $\delta y=0$

$$\delta l=\frac{x}{l}\delta x$$

$$\frac{\delta F}{\delta x}=2k\sin^2\theta$$

$$k_{eff}=2k\sin^2\theta$$

Substituting k_{eff}, we have $T=\frac{\pi}{\sin\theta}\sqrt{\frac{2m}{k}}$

3. 3

Sol. Time period = $\frac{2\pi}{\omega}$

Velocity = $\omega\sqrt{A^2-x^2}=\frac{2\pi}{T}\sqrt{A^2-x^2}$

$$\cos\theta=\frac{A/2}{A}=60^0$$

$$\theta=\frac{\pi}{3}$$

Particle has ternedthrough $\frac{2\pi}{3}$. Thus, time t is

$$\frac{2\pi}{3}=\frac{2\pi}{T}\times t$$

$$t=\frac{T}{3}$$

4. 1

Sol. Velocity = $\omega\sqrt{A^2 - x^2}$

$= \dfrac{2\sqrt{3}\pi A}{T}$ (doubles)

$$\frac{2\sqrt{3}\pi A}{T} = \frac{2\pi}{T}\sqrt{A^2 - x^2}$$

$$A_1 = \frac{\sqrt{13}}{2}A$$

$$\cos\theta = \frac{A/2}{A_1} = \frac{1}{\sqrt{13}} \qquad \left(\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)$$

$$\cos^{-1}\left(\frac{1}{\sqrt{13}}\right) = \omega t$$

$$t = T\cos^{-1}\left(\frac{1}{\sqrt{13}}\right)$$

5. 6

6. 1

7. 8

8. 2

Sol. y = 2a sin kxcosωt

$\therefore$ A = 2a sin kx

as from second harmonic

$\lambda = \ell$

$\therefore$ k = $\dfrac{2\pi}{\lambda} = \dfrac{2\pi}{\ell}$

$\therefore$ A = 2a sin kx = 2sin $\left(\dfrac{2\pi}{\lambda} \times \dfrac{\ell}{8}\right)$

= 2 sin $\dfrac{\pi}{4} = \dfrac{2}{\sqrt{2}}$ mm