## PART C - MATHS

61. **b)** $\boldsymbol{\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=1}$

Let the plane be $\frac{x}{x_1}+\frac{y}{y_1}+\frac{z}{z_1}=1$ which passes through $(a, b, c)$

$\therefore \quad \frac{a}{x_1}+\frac{b}{y_1}+\frac{c}{z_1}=1$

Hence, the locus of $(x_1, y_1, z_1)$ is $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=1$.

62. **a)** **0**

$\vec{a}, \vec{b}, \vec{c}$ are unit coplanar vectors, $2\vec{a}-\vec{b}, \overrightarrow{2b}-\vec{c}$ and $2\vec{c}-\vec{a}$ are also coplanar vectors being the linear combination of $\vec{a}, \vec{b}$ and $\vec{c}$.

Thus, $\left[2\vec{a}-\vec{b}\ 2\vec{b}-\vec{c}\ 2\vec{c}-\vec{a}\right]=0$

63. **d)** $\boldsymbol{xyz = 6k^3}$

Let the variable plane intersect the coordinate axes at A($a$, 0, $b$), B(0, $b$, 0), C(0, 0, $c$).
Then the equation of the plane will be

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \qquad \ldots \text{(i)}$

Let P(α, β, γ) be the centroid of the tetrahedron OABC, then

$\alpha = \frac{a}{4}, \beta = \frac{b}{4}, \gamma = \frac{c}{4}$ or $a = 4\alpha$, $b = 4\beta$, $c = 4\gamma$

$\Rightarrow$ Volume of tetrahedron = $\frac{1}{3}$(Area of ΔAOB) × OC

$\Rightarrow \quad 64k^3 = \frac{1}{3}\left(\frac{1}{2}ab\right)c = \frac{abc}{6}$

$\Rightarrow \quad 64k^3 = \frac{(4\alpha)(4\beta)(4\gamma)}{6}$

$\Rightarrow \quad k^3 = \frac{\alpha\beta\gamma}{6}$

Thus the required locus of P(α,β,γ) is $xyz = 6k^3$.

64. **c)** **A line passing through the points P and Q**

Obviously, the answer is (c); $R(\vec{r})$ moves on PQ

$$\overset{R(\vec{r})}{\overline{P(\vec{p}) \qquad Q(\vec{q})}}$$

65. **a)** **Equally inclined to $\hat{i}$ and $\hat{k}$**

The line of intersection of the two planes will be perpendicular to the normals to the planes. Hence, it is parallel to the vector

$$\left(\hat{i}+2\hat{j}+3\hat{k}\right)\times\left(3\hat{i}+2\hat{j}+\hat{k}\right)=\left(-4\hat{i}+8\hat{j}-4\hat{k}\right)$$

Now, $\left(-4\hat{i}+8\hat{j}-4\hat{k}\right).\hat{i}=-4$

and $\left(-4\hat{i}+8\hat{j}-4\hat{k}\right).\hat{k}=-4$

Hence, the line is equally inclined to $\hat{i}$ and $\hat{k}$.

66. **c)** **–10/3**

Since $\vec{a}_1, \vec{a}_2$ and $\vec{a}_3$ are non-coplanar vectors,

$x + y - 3 = 0 \qquad \ldots \text{(i)}$

$2x - y + 2 = 0 \qquad \ldots \text{(ii)}$

$2x + y + \lambda = 0 \qquad \ldots \text{(iii)}$

From Eqs. (i) and (ii), we get $x = 1/3$, $y = 8/3$.

Therefore, from Eq. (iii), we get $\lambda = -10/3$.

67. **b)** $\boldsymbol{\pm\frac{\hat{i}}{\sqrt{2}}\pm\frac{\hat{j}}{\sqrt{2}}-\frac{\hat{k}}{\sqrt{2}}}$

Let $\vec{r}$ be the new position, then $\vec{r}=\lambda\hat{k}+\mu\left(\hat{i}+\hat{j}\right)$

Also, $\vec{r}.\hat{k}=-\frac{1}{\sqrt{2}} \Rightarrow \lambda=-\frac{1}{\sqrt{2}}$

Also, $\lambda^2 + 2\mu^2 = 1 \Rightarrow 2\mu^2 = \frac{1}{2} \Rightarrow \mu = \pm\frac{1}{2}$

$\therefore \vec{r} = \pm\frac{1}{2}\left(\hat{i}+\hat{j}\right) - \frac{\hat{k}}{\sqrt{2}}$

68. **b)** **$120^0$**

Here, $l + m + n = 0$ and $nm - (m + n) = 0$

Elimination $l$, we get $mn + 2(m + n)^2 = 0$

$\Rightarrow$ $2m^2 + 5mn + 2n^0 = 0$

$\Rightarrow$ $(m + 2n)(2m + n) = 0$

$\therefore$ either $m + 2n = 0$ or $2m + n = 0$ ... (i)

Let the two lines whose DCs satisfy the given relations have their DCs $(l_1, m_1, n_2)$. Then from (i) $m_1 + 2n_1, = 0 \Rightarrow$ $m_1 = -2n_1$ where $l_1 + m_1 + n_1 = 0$

$l_1 = -(m_1 + n_1)$

$l_1 = n_1$

$l_1 = \frac{m_1}{-1} = n_1$ or $\frac{l_1}{1} = \frac{m_1}{-2} = \frac{n_1}{1}$

$= \frac{\sqrt{l_1^2 + m_1^2 + n_1^2}}{\sqrt{1+4+1}}$

$\Rightarrow$ $l_1 = \frac{1}{\sqrt{6}}, m_1 = \frac{-2}{\sqrt{6}}, n_1 = \frac{1}{\sqrt{6}}$ ... (ii)

Again form (i), $2m_2 + n_2$ and $l_2 = -m_2 - n_2$ $= m_2$

$\therefore$ $\frac{l_2}{1} = \frac{m_2}{1} = \frac{-n_2}{2} = \frac{\sqrt{l_2^2 + m_2^2 + n_2^2}}{\sqrt{1+1+4}}$

$= \frac{1}{\sqrt{6}} \Rightarrow l_2 = \frac{1}{\sqrt{6}}, m_2 = \frac{1}{\sqrt{6}}, n_2 = \frac{-2}{\sqrt{6}}$ ... (iii)

From (ii) and (iii)

$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

$= \left(\frac{1}{\sqrt{6}}\right)\left(\frac{1}{\sqrt{6}}\right) + \left(-\frac{2}{\sqrt{6}}\right)\left(\frac{1}{\sqrt{6}}\right) + \left(\frac{1}{\sqrt{6}}\right)\left(-\frac{2}{\sqrt{6}}\right)$

$= \frac{1}{6} - \frac{2}{6} - \frac{2}{6} = \frac{1}{2}$

The acute angle between the two lines

$\Rightarrow$ $\cos\theta = \frac{1}{2} \Rightarrow \theta = 60^0$ and the obtuse angle is $120^0$

69. **c)** **$\sqrt{14}$**

Given $\vec{v}.\hat{u} = \vec{w}.\hat{u}$

and $\vec{v} \perp \vec{w} \Rightarrow \vec{v}.\vec{w} = 0$

Now, $\left|\vec{u} - \vec{v} + \vec{w}\right|^2$

$= \vec{u}^2 + \vec{v}^2 + \vec{w}^2 - 2\vec{u}.\vec{v} - 2\vec{w}.\vec{v} + 2\vec{u}.\vec{w}$

$= 1 + 4 + 9$

So $\left|\vec{u} - \vec{v} + \vec{w}\right| = \sqrt{14}$

70. **c)** **$\sqrt{\frac{2}{3}}$**

Let the edges OA, OB, and OC of the unit cube be along OX, OY and OZ, respectively Since OA = OB = OC = 1 unit $\vec{O}A = i, \vec{O}B = \hat{j}, \vec{O}C = \hat{k}$

Let CM be the perpendicular from the corner C on the diagonal OP. The vector equation of OP is

$\vec{r} = \lambda\left(\hat{i} + \hat{j} + \hat{k}\right)$

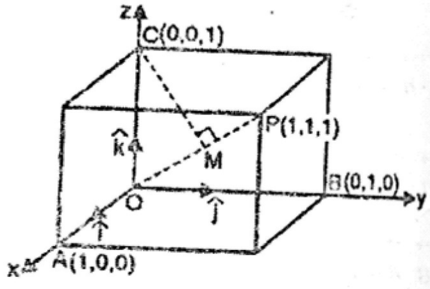
$\therefore$ OM = projection of $\vec{O}C$ on $\vec{O}P$ = $\vec{O}C.\vec{O}P$

$= \hat{k}.\frac{\left(\hat{i}+\hat{j}+\hat{k}\right)}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

Now $OC^2 = OM^2 + CM^2$

$\Rightarrow$ $CM^2 = \left|\vec{O}C\right|^2 - OM^2 = 1 - \frac{1}{3} = \frac{2}{3}$

$\Rightarrow$ $CM = \sqrt{\frac{2}{3}}$



71. **b)** **1**

$$|\vec{a}\times\vec{b}-\vec{a}\times\vec{c}|^2 = |\vec{a}|^2|\vec{b}-\vec{c}|^2-\left(\vec{a}.\left(\vec{b}-\vec{c}\right)\right)^2$$

$$= |\vec{b}-\vec{c}|^2$$

$$= |\vec{b}|^2+|\vec{c}|^2-2|\vec{b}||\vec{c}|\cos\frac{\pi}{3}=1$$

72. **c)** $\boldsymbol{\frac{x-5/2}{9}=\frac{y-1/2}{-1}=\frac{z-2}{-3}}$

Let A′(p, q, r), then

$$\frac{p-1}{3}=\frac{q-2}{-3}=\frac{r+3}{10}$$

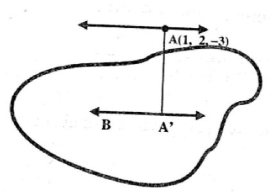
$$= -\frac{\left[3(1)-3(2)+10(-3)-261\right]}{9+9+100}$$

$$=\frac{1}{2}$$

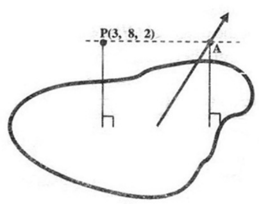
or A′(5/2, 1/2, 2)

Now the image line is parallel to the given line, then its equation is

$$\frac{x-5/2}{9}=\frac{y-1/2}{-1}=\frac{z-2}{-3}$$




73. **d)** **7 units**




Let a general point on the line be A(2λ + 1, 4λ + 3, 3λ + 2).

Let this point lie at the same distance as the point P(3, 8, 2) from the plane $3x + 2y - 2z + 15 = 0$. Therefore,

$$\frac{3.3+2.8-2.2+15}{\sqrt{17}}$$

$$= \frac{3(2\lambda+1)+2(4\lambda+3)-2(3\lambda+2)+15}{\sqrt{17}}$$

$\Rightarrow 36 = 8\lambda + 20 \Rightarrow \lambda = 2$

Therefore, A is (5, 11, 8)

$$PA = \sqrt{(5-3)^2+(11-8)^2+(8-2)^2}$$

$$= \sqrt{4+9+36}=7$$

74. **a)** $\boldsymbol{\vec{a}^2\left(\vec{b}.\,\vec{c}\right)}$

$$\left(\vec{a}\times\vec{b}\right).\left(\vec{a}\times\vec{c}\right)=\left(\vec{a}\times\vec{b}\right).\,\vec{u},$$

where $\vec{u}=\vec{a}\times\vec{c}$

$$\Rightarrow \vec{a}.\left(\vec{b}\times\vec{u}\right)=\vec{a}\left[\vec{b}\times\left(\vec{a}\times\vec{c}\right)\right]$$

$$= \vec{a}.\left[\left(\vec{b}.\vec{c}\right)\vec{a}-\left(\vec{a}.\vec{b}\right)\vec{c}\right]$$

$$= \vec{a}.\left(\vec{b}.\vec{c}\right)\vec{a} \qquad \left[\because \vec{a}.\vec{b}=0\right]$$

$$= \vec{a}^2\left(\vec{b}.\vec{c}\right)$$

75. **a)** **π/6**

$$\frac{1}{4}=\left(\hat{a}\times\hat{b}\right).\left(\hat{a}\times\hat{b}\right)\Rightarrow ab\sin\theta=\frac{1}{2}$$

$\Rightarrow \theta = \pi/6$, as $a = b = 1$.

76. **c)** $\boldsymbol{\vec{a},\vec{b}}$ **and** $\boldsymbol{\vec{c}}$ **cannot be coplanar**

$$\left(\vec{a}\times\vec{b}\right)\times\left(\vec{b}\times\vec{c}\right)=\vec{b}$$

$$\Rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right]\vec{b}=\vec{b}$$

$$\Rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right]=1$$

Therefore, $\vec{a},\vec{b}$ and $\vec{c}$ cannot be coplanar.

77. **b)** $\boldsymbol{\cos^{-1}\frac{1}{3}}$

Take O, one corner of the cube, as origin and OA, OB, OC, the three edges through it, as the axes.

Let OA = OB = OC = $a$.

The coordinates of various corners are shown in the figure.

The four diagonals are AL, BM, CN and OP.

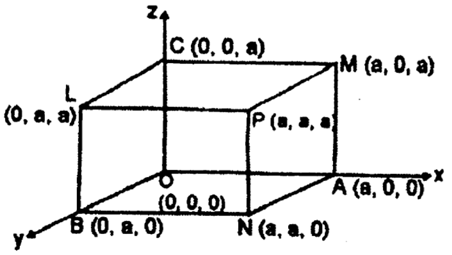
The DRs of the diagonal AL are

$0 - a$, $a - 0$, $a - 0$, i.e. $-a$, $a$, $a$

The DRs of the diagonal OP are
$a-0,\ a-0,\ a-0$, i.e. $a,a,a$
If $\theta$ is the angle between them, then

$$\cos\theta = \frac{|(-a)(a)+(a)(a)+(a)(a)|}{\sqrt{a^2+a^2+a^2}\sqrt{a^2+a^2+a^2}}$$

$$\cos\theta = \frac{a^2}{3a^2} = \frac{1}{3} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{3}\right)$$




78. **d)** $\left[\vec{a}\vec{b}\vec{c}\right] = |\vec{a}|^2$

$\vec{a}\perp\vec{b} \Rightarrow x-y+2=0$

$\vec{a}.\vec{c} = 4 \Rightarrow x+3y=4$

Solving, we get $x=0,\ y=2$

$\Rightarrow \vec{a} = 2\hat{j}+2\hat{k}$

$$\Rightarrow \left[\vec{a}\ \vec{b}\ \vec{c}\right] = \begin{vmatrix} 0 & 2 & 2 \\ 1 & -1 & 1 \\ 1 & 2 & 0 \end{vmatrix} = 8 = |\vec{a}|^2$$

79. **c)** **0**

$\vec{r}.\vec{a}=0, |\vec{r}\times\vec{b}| = |\vec{r}||\vec{b}|$ and $|\vec{r}\times\vec{c}| = |\vec{r}||\vec{c}|$

$\Rightarrow \vec{r}\perp\vec{a},\vec{b},\vec{c}$

$\therefore \left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$

80. **d)** **None of these**

The line of intersection of $\vec{r}.(i+2j+3k)=0$ and $\vec{r}.(3i+3j+k)=0$ **0** will be parallel to $(3i+3j+k)\times(i+2j+3k)$, i.e. $7\hat{i}-8\hat{j}+3\hat{k}$

If the required angle is $\theta$, then

$$\cos\theta = \frac{7}{\sqrt{49+64+9}} = \frac{7}{\sqrt{122}}$$

81. **c)** $\boldsymbol{\vec{b}.\vec{n}=0, \vec{a}.\vec{n}=q}$

We must have $\vec{b}.\vec{n}=0$ and $\vec{a}.\vec{n}=q$

82. **a)** $\boldsymbol{-\frac{2}{3}, 2}$

Here, $2\left|\hat{i}+x\hat{j}+3\hat{k}\right| = \left|4\hat{i}+(4x-2)\hat{j}+2\hat{k}\right|$

$\Rightarrow 2\sqrt{1+x^2+9} = \sqrt{16+(4x-2)^2+4}$

$\Rightarrow 4(x^2+10) = 20+(4x-2)^2$

$\Rightarrow 3x^2-4x-4=0$

$\Rightarrow (x-2)(3x+2)=0$

$\Rightarrow x = 2, \frac{-2}{3}$

83. **c)** $\boldsymbol{\vec{b}.\vec{n}=0, \vec{a}.\vec{n}\neq q}$

We must have $\vec{b}.\vec{n}=0$ and $\vec{a}.\vec{n}\neq q$

84. **d)** $\boldsymbol{\frac{2\sqrt{2}}{3}}$

$\frac{1}{3}|\vec{b}||\vec{c}|\vec{a} = (\vec{a}\times\vec{b})\times\vec{c}$

$= (\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a}$

Comparing, $\frac{1}{3}|\vec{b}||\vec{c}| = -(\vec{b}.\vec{c})$ and $\vec{a}.\vec{c}=0$

$\Rightarrow \frac{1}{3}bc = -bc\cos\theta$

$\Rightarrow \cos\theta = -\frac{1}{3}$

85. **a)** $\left|(\vec{a}-\vec{p}) + \frac{((\vec{p}-\vec{a}).\vec{b})\vec{b}}{|\vec{b}|^2}\right|$

Let Q $(\vec{q})$ be the foot of the altitude drawn from

P$(\vec{p})$ to the line $\vec{r} = \vec{a}+\lambda\vec{b}$.

$(\vec{q}-\vec{p}).\vec{b}=0$ and $\vec{q}=\vec{a}+\lambda\vec{b}$

$\Rightarrow (\vec{a}+\lambda\vec{b}-\vec{p}).\vec{b}=0$

$\Rightarrow (\vec{a}-\vec{p}).\vec{b}+\lambda|\vec{b}|^2=0$

$\lambda = \dfrac{(\vec{p}-\vec{a}).\vec{b}}{|\vec{b}|^2}$

$\Rightarrow \vec{q}-\vec{p}=\vec{a}+\dfrac{((\vec{p}-\vec{a}).\vec{b})\vec{b}}{|\vec{b}|^2}-\vec{p}$

$\Rightarrow |\vec{q}-\vec{p}| = \left|(\vec{a}-\vec{p})+\dfrac{((\vec{p}-\vec{a}).\vec{b})\vec{b}}{|\vec{b}|^2}\right|$

86. **b)** $\vec{s}=\vec{p}+\dfrac{(q-\vec{p}.\vec{n})\vec{n}}{|\vec{n}|^2}$

We have $\vec{s}-\vec{p}=\lambda\vec{n}$ and

$\vec{s}.\vec{n}=q \Rightarrow (\lambda\vec{n}+\vec{p}).\vec{n}=q$

$\Rightarrow \lambda=\dfrac{q-\vec{p}.\vec{n}}{|\vec{n}|^2} \Rightarrow \vec{s}=\vec{p}+\dfrac{(q-\vec{p}.\vec{n})\vec{n}}{|\vec{n}|^2}$

87. **b)** **–37**

Let

$A(20\vec{i}+p\vec{j}), B(5\vec{i}-\vec{j}), C(10\vec{i}-13\vec{j})$

be collinear.

$\vec{AB}=-15\vec{i}-(p+1)\vec{j}$ and

$\vec{AC}=-10\vec{i}-(13+p)\vec{j}$

Since A, B, C are collinear,

$\vec{AB}=x\vec{AC}$ for some scalar $x$

$\Rightarrow -15\vec{i}-(p+1)\vec{j}=x\left[-10\vec{i}-(13+p)\vec{j}\right]$

$\Rightarrow -15=-10x \Rightarrow x=\dfrac{3}{2}$ and

$-(p-1)=-(13+p)\,x=-\dfrac{(13+p)3}{2}$

$\Rightarrow 2p+2=39+3p \Rightarrow p=-37$

88. **c)** $\dfrac{|\vec{p}.\vec{n}|}{|\vec{n}|}$

Let $Q(\vec{q})$ be the foot of the altitude drawn from $p$ to the plane $\vec{r}.\vec{n}=0$

$\Rightarrow \vec{q}-\vec{p}=\lambda\vec{n} \Rightarrow \vec{q}=\vec{p}+\lambda\vec{n}$

Also $\vec{q}.\vec{n}=0 \Rightarrow (\vec{p}+\lambda\vec{n}).\vec{n}=0$

$\Rightarrow \lambda=-\dfrac{\vec{p}.\vec{n}}{|\vec{n}|^2} \Rightarrow \vec{q}-\vec{p}=-\dfrac{(\vec{p}.\vec{n})}{|\vec{n}|^2}\vec{n}$

Thus, the required distance is

$|\vec{q}-\vec{p}|=\dfrac{|\vec{p}.\vec{n}|}{|\vec{n}|}$

89. **c)** **Are collinear**

$\vec{AB}=(3\vec{i}-2\vec{j}+\vec{k})-(2\vec{i}+\vec{j}-\vec{k})$

$=\vec{i}-3\vec{j}+2\vec{k}$

$\vec{BC}=(\vec{i}+4\vec{j}-3\vec{k})-(3\vec{i}-2\vec{j}+\vec{k})$

$=-2\vec{i}+6\vec{j}-4\vec{k}$

$\vec{CA}=(2\vec{i}+\vec{j}-\vec{k})-(\vec{i}+4\vec{j}-3\vec{k})$

$=\vec{i}-3\vec{j}+2\vec{k}$

Since $\vec{AB}=\vec{CA}, |\vec{AB}|=|\vec{CA}|$

90. **d)** $\boldsymbol{x^2+y^2+z^2-ax-by-cz=0}$

Let P(α, β, γ) be the foot of the perpendicular from the origin O to a plane which passes through the point A($a$, $b$, $c$). So OP ⊥ AP

$\Rightarrow \alpha(a-\alpha)+\beta(b-\beta)+\gamma(c-\gamma)=0$

Hence, the locus of (α, β, γ) is $x^2+y^2+z^2-ax-by-cz=0$.