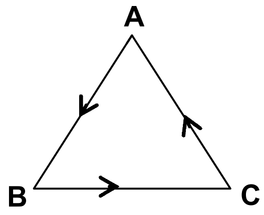
PART C - MATHS

61. **c)** $-\frac{3l^2}{2}$



$\overrightarrow{AB} + \overrightarrow{BC} + .\ \overrightarrow{CA} = 0$

Squaring we get

$AB^2 + BC^2 + CA^2 + 2$

$\left(\overrightarrow{AB}.\overrightarrow{BC} + .\ \overrightarrow{AB}.\overrightarrow{CA} + \overrightarrow{BC}.\overrightarrow{CA}\right) = 0$

$\therefore\quad \overrightarrow{AB}.\overrightarrow{BC} + .\ \overrightarrow{AB}.\overrightarrow{CA} + \overrightarrow{BC}.\overrightarrow{CA} = -\frac{3l^2}{2}$

62. **a)** $\bar{a}\times\bar{b}$

$\left|\vec{a}\ \vec{b}\ \vec{i}\right|\vec{i} + \left|\vec{a}\ \vec{b}\ \vec{j}\right|\vec{j} + \left|\vec{a}\ \vec{b}\ \vec{k}\right|\vec{k}$

$= \left(\vec{a}\times\vec{b}.\vec{i}\right)\vec{i} + \left(\vec{a}\times\vec{b}.\vec{j}\right)\vec{j} + \left(\vec{a}\times\vec{b}.\vec{k}\right)\vec{k}$

$= \bar{a}\times\bar{b}$

63. **d)** $\bar{a}.\left(\bar{a}+\bar{b}+\bar{c}\right)$

$\vec{a} = x_1\left(\vec{b}\times\vec{c}\right) + x_2\left(\vec{c}\times\vec{a}\right) + x_3\left(\vec{a}\times\vec{b}\right)$

$\vec{a}\ .\ \vec{a} = x_1,\ \vec{a}\ .\ \vec{b} = x_2, \vec{a}\ .\ \vec{c} = x_3$

$\Rightarrow\quad x_1 + x_2 + x_3 = \bar{a}.\left(\bar{a}+\bar{b}+\bar{c}\right)$

64. **c)** $\left|\sin\frac{\phi}{2}\right|$

$\left(\frac{1}{2}\left|\vec{x}-\vec{y}\right|\right)^2 = \frac{1}{4}\left(\vec{x}-\vec{y}\right).\left(\vec{x}-\vec{y}\right)$

$= \frac{1}{4}\left(x^2 + y^2 - 2\vec{x}.\ \vec{y}\right)$

$= \frac{1}{4}\left[2 - 2\cos\phi\right] = \sin^2\frac{\phi}{2}$

$\Rightarrow\quad \frac{1}{2}\left|\vec{x}-\vec{y}\right| = \left|\sin\frac{\phi}{2}\right|$

65. **a)** $2\left[\vec{a}\,\vec{b}\,\vec{c}\right]\vec{r}$

Given $= \left|\vec{a}\ \vec{b}\ \vec{c}\right|\vec{r} - \left|\vec{a}\ \vec{b}\ \vec{r}\right|\vec{c} + \left|\vec{b}\ \vec{c}\ \vec{a}\right|\vec{r}$

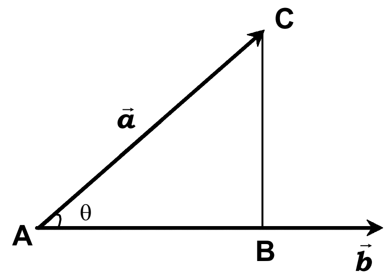
$- \left|\vec{b}\ \vec{c}\ \vec{r}\right|\vec{a} + \left|\vec{c}\ \vec{a}\ \vec{b}\right|\vec{r} - \left|\vec{c}\ \vec{a}\ \vec{r}\right|\vec{b}$

$= 3\left[\vec{a}\ \vec{b}\,\vec{c}\right]\vec{r} - \left[\vec{a}\ \vec{b}\ \vec{c}\right]\vec{r}$

$$\left[\text{since } \vec{r} = \frac{\left(\vec{r}\cdot\vec{b}\times\vec{c}\right)\vec{a} + \left(\vec{r}\cdot\vec{c}\times\vec{a}\right)\vec{b} + \left(\vec{r}\cdot\vec{a}\times\vec{b}\right)\vec{c}}{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}\right]$$

$= 2\left[\vec{a}\ \vec{b}\ \vec{c}\right]\vec{r}$

66. **c)** $|\vec{a}_1| + |\vec{a}_1\times\vec{b}|$



$\vec{a}_1 = \vec{a} - \left(\vec{a}\cdot\vec{b}\right)\vec{b}$

$\Rightarrow\quad \overrightarrow{BC} = \vec{a}_1$

$\vec{b}_1 = \vec{a}\times\vec{b}$

$\Rightarrow\quad \left|\vec{b}_1\right| = \left|\vec{a}\times\vec{b}\right| = \left|\vec{a}\right|\left|\vec{b}\right|\left|\sin\theta\right| = \left|\sin\theta\right|$

$\left|\vec{b}_1\right| = \left|\vec{a}_1\right|$ also $\vec{a}_1\cdot\vec{b} = 0$

$\Rightarrow\quad \left|\vec{b}_1\right| = \left|\vec{a}_1\right| + \left|\vec{a}_1\cdot\vec{b}\right|$

67. **c)** $\frac{6}{\sqrt{14}}$

$\vec{r} = \left(3+\lambda\right)\vec{i} + \left(2\lambda - 1\right)\vec{j} + 3\lambda\vec{k}$

$= \left(3\vec{i} - \vec{j} + 0\right) + \lambda\left(\vec{i} + 2\vec{j} + 3\vec{k}\right)$

Projection of $\vec{i} + \vec{j} + \vec{k}$ on $\vec{r}$

$= \frac{\left(\vec{i}+\vec{j}+\vec{k}\right)\cdot\left(\vec{i}+2\vec{j}+3\vec{k}\right)}{\sqrt{1+4+9}}$

$= \frac{1+2+3}{\sqrt{14}} = \frac{6}{\sqrt{14}}$

68. **b)** $\frac{1}{2}(\vec{a}+\vec{b}+\vec{c})$

Given $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{d} = 0$

and $\vec{a}\times\{(\vec{r}-\vec{b})\times\vec{a}\} + \vec{b}\times\{(\vec{r}-\vec{c})\times\vec{b}\} + \vec{c}\times\{(\vec{r}-\vec{a})\times\vec{c}\} = 0$

$\Rightarrow (\vec{a}\cdot\vec{a})(\vec{r}-\vec{b}) - (\vec{a}\cdot(\vec{r}-\vec{b}))\vec{a} + (\vec{b}\cdot\vec{b})(\vec{r}-\vec{c}) - (\vec{b}\cdot(\vec{r}-\vec{c}))\vec{b} + (\vec{c}\cdot\vec{c})(\vec{r}-\vec{a}) - (\vec{c}\cdot(\vec{r}-\vec{a}))\vec{c} = 0$

$\Rightarrow \vec{r}-\vec{b}-(\vec{a}\cdot\vec{r})\vec{a}+\vec{r}-\vec{c}-(\vec{b}\cdot\vec{r})\vec{b}+\vec{r}-\vec{a}-(\vec{c}\cdot\vec{r})\vec{c} = 0$

$3\vec{r} - (\vec{a}+\vec{b}+\vec{c}) - \vec{r} = 0$

$\Rightarrow$ r $= \frac{1}{2}(\vec{a}+\vec{b}+\vec{c})$

69. **c)** $\hat{j}+\lambda(\hat{i}+2\hat{j}+\hat{k})$

Here $\vec{r}\times(\vec{i}+2\vec{j}+\vec{k}) = \vec{i}-\vec{k}$ ...(i)

Let $\vec{r} = x\vec{i}+y\vec{j}+z\vec{k}$

then $\vec{r}\times(\vec{i}+2\vec{j}+\vec{k}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ 1 & 2 & 1 \end{vmatrix}$

$= \vec{i}(y-2z)+\vec{j}(z-x)+\vec{k}(2x-y)$

From (i) $y-2z = 1$, $z-x = 0$, $2x-y = -1$

$\Rightarrow x = z = \lambda$, $y = 1+2\lambda$

$\Rightarrow \vec{r} = \lambda\vec{i}+(1+2\lambda)\vec{j}+\lambda\vec{k}$

$= \vec{j}+\lambda(\vec{i}+2\vec{j}+\vec{k})$

70. **b)** $\left(0, \frac{2\pi}{3}\right]$

$\cos\theta = ab+bc+ca$

Now $(a+b+c)^2 \geq 0$

$\Rightarrow ab+bc+ca \geq -\frac{a^2+b^2+c^2}{2}$

$= -\frac{1}{2}$

and $(a-b)^2 + (b-c)^2 + (c-a)2 > 0$

(sice a, b, c are different)

$\Rightarrow ab+bc+ca < a^2+b^2+c^2 = 1$

$\Rightarrow -\frac{1}{2} \leq \cos\theta\ 1 \Rightarrow 0 < \theta \leq \frac{2\pi}{3}$

71. **c)** $\overline{u} = 2\overline{a}$

$\vec{u} = (\vec{i}\cdot\vec{i})\,\vec{a} - (\vec{i}\cdot\vec{a})\,\vec{i} + (\vec{j}\cdot\vec{j})\,\vec{a} - (\vec{j}\cdot\vec{a})\,\vec{j} + (\vec{k}\cdot\vec{k})\vec{a} - (\vec{k}\cdot\vec{a})\,\vec{k}$

$= \vec{a}+\vec{a}+\vec{a}-\vec{a} = 2\vec{a}$

72. **d)** **none of these**

Given vectors are coplaner

$\Rightarrow \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

$\Rightarrow a^3+b^3+c^3-3abc = 0$

$= (a+b+c)(a-b)^2+(b-c)^2+(c-a)^2 = 0$

$\Rightarrow a+b+c = 0$ as $a \neq b$

$\Rightarrow \vec{v}\cdot(a\hat{i}+b\hat{j}+c\hat{k}) = \vec{v}\cdot(b\hat{i}+c\hat{j}+a\hat{k})$

$= \vec{v}\cdot(c\hat{i}+a\hat{j}+b\hat{k}) = 0$

73. **d)** **all are correct**

$\hat{\alpha}\cdot\hat{\gamma} = x(\hat{\alpha}\cdot\hat{\alpha}) + y(\hat{\alpha}\cdot\hat{\beta}) + z\hat{\alpha}\cdot(\hat{\alpha}\times\hat{\beta})$

$\Rightarrow x = \cos\theta$, similarly $y = \cos\theta$

$\hat{\gamma}\cdot(\hat{\alpha}\times\hat{\beta}) = z(\hat{\alpha}\times\hat{\beta})\cdot(\hat{\alpha}\times\hat{\beta})$

$\Rightarrow z = [\hat{\alpha}\ \hat{\beta}\ \hat{\gamma}]$

$[\hat{\alpha}\ \hat{\beta}\ \hat{\gamma}]^2 = \begin{vmatrix} 1 & 0 & \cos\theta \\ 0 & 1 & \cos\theta \\ \cos\theta & \cos\theta & 1 \end{vmatrix}$

$= 1 - 2\cos^2\theta$

74. **a)** $\frac{4}{\sqrt{13}}$

The shortest distance of point P from the plane will be projection of

$\overline{PA} = -2\hat{i}-\hat{j}$

The length of projection = $\left|\frac{\overline{PA}.\vec{\alpha}}{|\vec{\alpha}|}\right| = \frac{4}{\sqrt{13}}$

75. **d)** **all are correct**

$\vec{a} + 2\vec{b} = -3\vec{c}$

$\Rightarrow (\vec{a} + 2\vec{b}) \times \vec{a} = -3\vec{c} \times \vec{a}$

$\Rightarrow 2(\vec{b} \times \vec{a}) = -3(\vec{c} \times \vec{a})$ ...(i)

Also $(\vec{a} + 2\vec{b}) \times \vec{b} = -3\vec{c} \times \vec{b}$

$\Rightarrow \vec{a} \times \vec{b} = -3(\vec{c} \times \vec{b})$ ...(ii)

Also $(\vec{a} + 2\vec{b}) \times \vec{c} = -3\vec{c} \times \vec{c}$

$\Rightarrow \vec{a} \times \vec{c} + 2(\vec{b} \times \vec{c}) = 0$ ...(iii)

From (i), (ii) and (iii)

$\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 6(\vec{b} \times \vec{c})$

$= 2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$

76. **a)**

$\overline{AB} = \overline{OB} - \overline{OA}$

$= (\alpha - 2)\hat{i} - (5 + \beta)\hat{j} - 2\hat{k}$

$\overline{AB} = \overline{OB} - \overline{OA} = -3\hat{i} + (11 - \beta)\hat{j} + 6\hat{k}$

$\overline{AB}, \overline{AC}$ are collinear

$\Rightarrow \overline{AB} = \lambda\overline{AC}$ (For some scalar $\lambda \neq 0$)

77. **b)** **12**

$\sqrt{a^2 - 4}\ \tan A + a \tan\ B + \sqrt{a^2 + 4}$ $\tan C = 6a$

$\Rightarrow \left(\sqrt{a^2 - 4\hat{i}} + a\hat{j} + \sqrt{a^2 + 4\hat{k}}\right).$

$\left(\tan A\hat{i} + \tan B\hat{j} + \tan C\hat{k}\right) = 6a$

$\Rightarrow \sqrt{(a^2 - 4) + a^2 + (a^2 + 4)}$

$\sqrt{\tan^2 A + \tan^2 B + \tan^2 C}$ $\cos\theta = 6a$

where θ is the angle between two vectors. (since $\vec{p}.\vec{q} = |\vec{p}|\ |\vec{q}| \cos\theta$)

$\Rightarrow (\sqrt{3}\,a)\sqrt{\tan^2 A + \tan^2 B + \tan^2 C}$

$= 6a \sec\theta$

$= (3a^2)(\tan^2 A + \tan^2 B + \tan^2 C)$

$= 36a^2 \sec^2\theta$

$= \tan^2 A + \tan^2 B + \tan^2 C$

$= 12 \sec 2\theta \geq 12$ (since $\sec^2\theta \geq 1$)

78. **d)** **120^0**

since $\hat{a}$ and $\hat{b}$ is a unit vector,

$(\hat{a} + \hat{b}).(\hat{a} + \hat{b}) = 1$

$\Rightarrow \hat{a}.\hat{a} + \hat{b}\,\hat{b} + 2\hat{a}.\hat{b} = 1$

$\Rightarrow 1 + 1 + 2\hat{a}.\hat{b} = 1$

$\Rightarrow \hat{a}.\hat{b} = -1/2$

Hence the angle θ between $\hat{a}$ and $\hat{b}$ is given by

$\cos\theta = -1/2$

$\Rightarrow \theta = 120^o$.

79. **b)** ***p = q + r***

We have $3\hat{i} + 2\hat{j} - 5\hat{k} = p(2\hat{i} - \hat{j} + \hat{k})$

$+ q(\hat{i} + 3\hat{j} - 2\hat{k}) + r(-2\hat{i} + \hat{j} - 3\hat{k})$

$\Rightarrow 3p + q - 2r = 3$

$-p + 3q + r = 2,\ p - 2q - 3r = -5$

Adding all the equations, we get

$2p + 2q = 4r$ i.e $p + q = 2r$

Equation (i) gives $p = 3$

$\Rightarrow q = 1,\ r = 2$

with these values of q.p.r

80. **b)** $\sqrt{593}$

The vector representing one of the di agonals is $5\bar{a} + 2\bar{b} + \bar{a} - 3\bar{b} = 6\bar{a} - \bar{b}$

Hence the length of the diagonal

$= \sqrt{(6\bar{a} - \bar{b}).(6\bar{a} - \bar{b})}$

$= \sqrt{36|\bar{a}|^2 + |\bar{b}|^2 - 12\bar{a}.\bar{b}}$

$= \sqrt{36 \times 8 + 9 - 12 \times 3 \times 2}$

$= 15$

The other diagonal is $5\bar{a} + 2\bar{b} - \bar{a} + 3\bar{b}$

$= 4\bar{a} + 5\bar{b}$

It length $= \sqrt{16|\bar{a}|^2 + 25|\bar{b}|^2 + 40\bar{a}.\bar{b}}$

$= \sqrt{128+225+40\times2\times3} = \sqrt{593}$

81. c) $-\hat{i}-8\hat{j}+2\hat{k}$

We have $\vec{R}\times\vec{B}=\vec{C}\times\vec{B}$ and $\vec{R}.\vec{A}=0$

$\Rightarrow \overline{A}\times(\overline{R}\times\overline{B})=\overline{A}\times(\overline{C}\times\overline{B})$

$\Rightarrow (\overline{A}.\overline{B})\overline{R}-(\overline{A}.\overline{R})\overline{B}$

$=(\overline{A}.\overline{B})\overline{C}-(\overline{A}.\overline{C})\overline{B}$

$\Rightarrow (2+1)\overline{R} = 3\overline{C}-(8+7)\overline{B}$

$\Rightarrow \overline{R}=\overline{C}-5\overline{B}=\hat{i}-8\hat{j}+2\hat{k}$

82. b) **2**

The vector of unit length perpendicular to the given vectors

$= \pm\left(\frac{a\times b}{|a|\,|b|}\right)$

Hence there are two such vectors

83. c) **is equal to zero**

Sicne $\overline{r}.\overline{a}=0, \overline{r}.\overline{b}=0$ and $\overline{r}.\overline{c}=0, \overline{r}$ must be perpendicular to all the three vectors $\overline{a}, \overline{b}$ and $\overline{c}$. Hence $\overline{a}, \overline{b}$ and $\overline{c}$ must be coplanar

$\Rightarrow [\overline{a}\ \overline{b}\ \overline{c}]=0$

84. a) **π/4**

Sicne $\vec{a}$ is parallel to the line intersection of the two planes, it is par allel to both the plane $\Rightarrow \vec{a}$ is perpendicular to the normals of both the planes.

Hence $\vec{a}\left[\hat{i}\times(\hat{i}+\hat{j})\right]=0$ and

$\vec{a}\left[(\hat{i}-\hat{j})\times(\hat{j}+\hat{k})\right]=0$

$\Rightarrow a.\hat{k}=0$ and $\hat{a}.(\hat{i}+\hat{j})=0$ so that $\hat{a}$ is in the direction of $\hat{i}-\hat{j}$

Hence $\hat{a}=\pm\frac{1}{\sqrt{2}}(\hat{i}-\hat{j})$. If θ is the angle between the given vector and $\vec{a}$.

$\cos\theta = \pm\frac{1}{\sqrt{2}}(\hat{i}-\hat{j}).\frac{(\hat{i}-2\hat{j}+2\hat{k})}{3}$

$= \pm\frac{1}{\sqrt{2}} \Rightarrow \theta=\frac{\pi}{4} \text{ or } \frac{3\pi}{4}$

85. a) **0**

We have $(\vec{A}+\vec{B})\times(\vec{A}+\vec{C})$

$=\vec{A}\times\vec{C}+\vec{B}\times\vec{A}+\vec{B}\times\vec{C}$

$\Rightarrow \left[(\vec{A}+\vec{B})\times(\vec{A}+\vec{C})\right]\times(\vec{B}\times\vec{C})$

$=\left[\vec{A}.(\vec{B}\times\vec{C})\right](\vec{C}-\vec{B})$

$\Rightarrow \left[(\vec{A}+\vec{B})\times(\vec{A}+\vec{C})\right]\times(\vec{B}\times\vec{C}).(\vec{B}+\vec{C})$

$=\vec{A}.(\vec{B}\times\vec{C})\left[\vec{C}-\vec{B}\right].\left[\vec{B}+\vec{C}\right]$

$=\vec{A}.(\vec{B}\times\vec{C})\left[|\vec{C}|^2-|B|^2\right]=0$

(Given that $|\vec{B}|=|\vec{C}|$)

86. c)

$\vec{a}\times\vec{b}=\vec{c}\Rightarrow\vec{c}$ is perpendicular to both $\vec{a}$ and $\vec{b}$

$\vec{b}\times\vec{c}=\vec{a}\Rightarrow\vec{a}$ is perpendicular to both $\vec{b}$ and $\vec{c}$

This $\vec{a}\ \vec{b}\ \vec{c}$ form an orthogonal system. Taking mode of both sides of given relation

$|\vec{a}|\,|\vec{b}|\sin\frac{\pi}{2}=|\vec{c}|$ and $|\vec{b}|\,|\vec{c}|\sin\frac{\pi}{2}=|\vec{a}|$

putting for $|\vec{c}|$ we get $|\vec{b}|=1$

$\Rightarrow |\vec{a}|=|\vec{c}|=|\vec{b}|=1$

87. a) **0**

Since $\vec{a}, \vec{b}\ \vec{c}$ are unit coplanar vector

$\Rightarrow [\vec{a}, \vec{b}\ \vec{c}]=0$ so $2\vec{a}-\vec{b}, 2\vec{b}-\vec{c}-\vec{a}$ are also coplanar vectors

Hence $\left[2\vec{a}-\vec{b}\ \ 2\vec{b}-\vec{c}\ \ 2\vec{c}-\vec{a}\right]=0$

88. **a)** **0**

$\left|\vec{r}\times\vec{b}\right| = \left|\vec{r}\right|\left|\vec{b}\right|$

Hence angle between $\vec{r}$ and $\vec{b}$ is $\frac{\pi}{2}$

$\vec{r}\ \vec{a} = 0 \Rightarrow \vec{r}$ perpendicular to $\vec{a}$

$\left|\vec{r}\times\vec{c}\right| = \left|\vec{r}\right|.\left|\vec{c}\right| \Rightarrow \vec{r}$ is perpendicular to $\vec{c}$.

Thus $\vec{r}$ is perpendicular $\vec{a}, \vec{b}$ and $\vec{c}$

Hence $\vec{a}, \vec{b}, \vec{c}$ are coplanar

$\Rightarrow$ $\left[\vec{a}, \vec{b}, \vec{c}\right] = 0$

89. **b)** $\sqrt{\mathbf{306}}$

Let $\vec{a} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ $\vec{b} = \hat{i} + 5\hat{i} + 5\hat{j}$

$\vec{c} = 5\hat{i} + 5\hat{j} \Rightarrow \left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right| = \sqrt{50}$

$\Rightarrow$ origin is the circumcentre of the triangle

$\Rightarrow$ orthocentre of the triangle

$= \vec{a} + \vec{b} + \vec{c} = 9\hat{i} + 9\hat{j} + 12\hat{k}$

$\therefore$ Distance between circumcentre and the orthocentre $\left|\vec{a} + \vec{b} + \vec{c}\right|$

$= \sqrt{81 + 81 + 144} = \sqrt{306}$

90. **a)** $\mathbf{2\hat{i} + 3\hat{j} - 3\hat{k}, -2\hat{i} - \hat{j} + 5\hat{k}}$

Let $\overline{R}$ be a vector in the plane of b and c

$\Rightarrow$ $\overline{R} = \left(\hat{i} + 2\hat{j} - \hat{k}\right) + \mu\left(\hat{i} + \hat{j} - 2\hat{k}\right)$

Its projection on $\overline{a} = \frac{\overline{a}\ \overline{R}}{\left|\overline{a}\right|} = \frac{1}{\sqrt{6}}$

$[2 + 2\mu - 2 - \mu - 1 - 2\mu] = \frac{-(1+\mu)}{\sqrt{6}}$

$\Rightarrow$ $\frac{-(1+\mu)}{\sqrt{6}} = \pm\sqrt{\frac{2}{3}} \Rightarrow -1(1+\mu) = \pm 2$

$\Rightarrow$ $\mu = 1 - 3$

$\Rightarrow$ $R \equiv 2\hat{i} + 3\hat{j} - 3\hat{k}$ and $-2\hat{i} - \hat{j} + 5\hat{k}$