MAHESH TUTORIALS SCIENCE

00 – 00		Q. Booklet Serial No: 170515	
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Hints & Solutions

PART A - PHYSICS

1. **d)** **Coolest** **Z** **Hottest** **Y**

For a mole of an ideal gas, the equation of state is PV = RT

or $T = \frac{PV}{R}$

which is proportional to the product pV

At x, pV = $(4 \times 10^5)(1 \times 10^{-4})$ = 40 Nm

At y, pV = $(1 \times 10^5)(5 \times 10^{-4})$ = 50 Nm

At z, pV = $(1 \times 10^5)(1 \times 10^{-4})$ = 10 Nm

Thus, T is maximum at y since pV is the highest and T is minimum at z since pV is the smallest PV = RT

2. **a)** **–5 J**

$\Delta W_{AB} = P\Delta V = (10)(2-1) = 10\ J$

$\Delta W_{BC} = 0$

From first law of thermodynamics

$\Delta Q = \Delta W + \Delta U$

$\Delta U = 0$ (process ABCA is cyclic)

$\therefore \Delta Q = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CA}$

$\therefore \Delta W_{CA} = \Delta Q - \Delta W_{AB}$

$= 5 - 10 - 0$

$= -5\ J$

3. **d)** $\mathbf{4\frac{P}{T}, 3.5\frac{P}{T}, 2.5\frac{P}{T}}$

For adiabatic process :

Slope : $\frac{dP}{dT} = \left(\frac{\gamma}{\gamma - 1}\right)\frac{P}{T}$

$\gamma = \frac{5}{3}\Big|\frac{dP}{dT} = \left(\frac{5/3}{5/3 - 1}\right)\frac{P}{T}$

$= \frac{5/3}{2/3}\frac{P}{T} = 2.5\frac{P}{T}$

$\gamma = \frac{7}{5}\Big|\frac{dP}{dT} = \left(\frac{7/5}{2/5}\right)\frac{P}{T} = 3.5\frac{P}{T}$

$\gamma = \frac{4}{3}\Big|\frac{dP}{dT} = \left(\frac{4/3}{1/3}\right)\frac{P}{T} = 4\frac{P}{T}$

4. **b)** $\mathbf{\frac{7}{2}R}$

$U = U_0V \Rightarrow nC_vt = U_0V$

$\Rightarrow$ T ∝ V isobaric process

So at constant pressure

$C = C_v + R = \frac{5}{2}R + R$

$= \frac{7}{2}R$

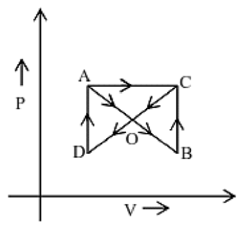
5. **b)** $\mathbf{\frac{11R}{6}}$

$\frac{n_1C_{V_1} + n_1C_{V_2}}{n_1 + n_2} = (C_V)_{mix}$

$= \frac{2 \times \frac{3}{2}R + 1 \times \frac{5}{2}R}{2+1} = \frac{3R + \frac{5}{2}R}{3}$

$= \frac{11R}{6}$

6. **b)** **20 J**



Net work done = Area of rectangle formed by AC with volume axis

= (200 K × 100cc) J

= $200 \times 10^3 \times 100 \times 10^{-6}$

= 20 J

7. **b)** **$16\ P_0$**

$$PV^{\gamma} = \text{constant}$$

$$P_0V^{4/3} = P'\left(\frac{V}{8}\right)^{4/3}$$

$$P' = (2)^4P_0 = 16P_0$$

8. **c)** **7 : 5 : 2**

$$\Delta Q = nC_P\Delta T = \frac{7}{2}nR\Delta T \quad \left(C_P = \frac{7}{2}R\right)$$

$$\Delta U = nC_V\Delta T = \frac{5}{2}nR\Delta T \quad \left(C_V = \frac{5}{2}R\right)$$

and $\Delta W = \Delta Q - \Delta U = nR\Delta T$

$\therefore$ $\Delta Q : \Delta U : \Delta W = 7 : 5 : 2$

9. **d)** **none of these**

$\rho = \frac{PM}{RT}$ and $\rho \propto \frac{1}{V}$

During AB, ρ and hence V is constant
Therefore, work done is zero.
During BC, $P \propto \rho$ i.e., T = constant
and hence, U is constant.

10. **a)** **$\Delta U_1 > \Delta U_2 > \Delta U_3$**

Process 2 is an isothermal process
Hence, $\Delta U_2 = 0$
Process 1 is an isobaric (P = constant) expansion.
Hence, temperature of the gas will increase
or ΔU_1 = positive
Process 3 is an adiabatic expansion. Hence, temperature will decrease
or ΔU_3 = negative
therefore, $\Delta U_1 > \Delta U_2 > \Delta U_3$ is the correct option.

11. **a)** **253 K**

Temperature of source $T_1 = 100 + 273 = 373$ K
Temperature of sink $T_2 = 0 + 273 = 273$ K

Efficiency of carnot engine

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{273}{373}$$

$$\eta = \frac{100}{373}$$

To increase $\eta' = \frac{100}{373} + \frac{100}{373} \times \frac{1}{5} = \frac{120}{373}$

$$\eta' = 1 - \frac{T_2}{373} = \frac{120}{373}$$

or $T_2 = \frac{373 - 120}{373} \times 373 = 253$ K

New sink temp = 253 K

12. **b)** **$\Delta T_a = \Delta T_b = \Delta T_c$**

All are in series therefore current remains same. Hence temperature difference = (current × thermal resistance) are equal for every case.

13. **b)** **$\frac{K_1 + 8K_2}{9}$**

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{K_{eq}(9\pi R^2)}{L} = \frac{K_1\pi R^2}{L} + \frac{K_2 8\pi R^2}{L}$$

$$K_{eq} = \frac{K_1 + 8K_2}{9}$$

14. **c)** **Temperature difference across AB is less than that of across CD**

Rate of flow of heat $\frac{dQ}{dt}$ or H is equal throughout the rod.
Temperature difference = (H) (thermal resistance) or Temperature dif ference $\propto$ termal resistance (R)

where $R = \frac{1}{KA}$ or $R \propto \frac{1}{A}$

Area across CD is less. Therefore, temperature difference across CD will be more.

15. **b)** $\mathbf{\frac{7}{2}}$

As no heat flows in rod AB

Therefore $T_A = T_B = 20^0C$

∴ Rate of heat flow throug CB and BD are same.

Hence, $\frac{90-20}{\frac{BC}{KA}} = \frac{20-0}{\frac{BD}{KA}}$

$\frac{7}{BC} = \frac{2}{BD}$

$\frac{BD}{BC} = \frac{7}{2}$

16. **d)** **48 s**

$t \propto \frac{1}{A}, t' \propto \frac{2l}{A/2}$

17. **a)** **40^{0}C**

$\frac{K_P A[100-\theta]}{1} = \frac{K_Q.A[\theta]}{1}$

∴ $\frac{K_P}{K_Q} = \frac{2}{3} = \frac{\theta}{100-\theta}$ OR ($\theta = 40^0C$)

18. **b)** **0.2**

For first rod $\frac{\Delta Q}{\Delta t} = \frac{\Delta m}{\Delta t} L = \frac{KA(T_1 - T_2)}{L}$

$0.1 \times L = \frac{KA(T_1 - T_2)}{L}$ ...(i)

For second rod

$\frac{\Delta m}{\Delta t} L = \frac{0.25K \times 4A(T_1 - T_2)}{L/2}$ ...(ii)

∴ $\frac{\Delta m}{\Delta t} = 0.2 g/s$

19. **b)** **Q_B is maximum**

$Q \propto AT^4$ and λ_m T = constant. Hence,

$Q \propto \frac{A}{(\lambda_m)^4}$ or $Q \propto \frac{r^2}{(\lambda_m)^4}$

$Q_A : Q_B : Q_C = \frac{(2)^2}{(3)^4} : \frac{(4)^2}{(4)^4} : \frac{(6)^2}{(5)^4}$

$= \frac{4}{81} : \frac{1}{16} : \frac{36}{625}$

= 0.05 : 0.0625 : 0.0576

i.e., Q_B is maximum.

20. **b)** **3025 K**

From Wien's law

λ_{max} T = b where b = Wien's constant.

b = 2.89 × 10^{-3} m K.

Let For Sum λ_{m1} = 4753 Å and T_1 = 6050 K.

For Starb λ_{m2} = 9506 Å and T_2 = ?

or $\lambda_{m2} T_2 = \lambda_{m1} T_1$

$T_2 = \frac{4753}{9506} \times 6050 = 3025$ K.

Temperature of Star = 3025 K.

21. **b)** $\mathbf{\frac{2\, ln2}{K}}$

Let θ_0 = Temperature of environment

θ_1 = Initial temperature of body

θ = Temperature of body at any instant t

∴ Maximum heat body can lose

$= ms(\theta_1 - \theta_0)$

[m = mass of body s = specific heat of body]

$\frac{d\theta}{dt} = -K(\theta - \theta_0)$

$\frac{\theta - \theta_0}{\theta_1 - \theta_0} = e^{-Kt}$

$t = \frac{1}{K} \ln\left(\frac{\theta_1 - \theta_0}{\theta - \theta_0}\right)$

Body will lose $\frac{3}{4}$ its maximum heat when

$\theta = \frac{\theta_1 + 3\theta_0}{4}$

∴ $t = \frac{2 ln2}{K}$

22. **a)** **1 : 1**

$$\frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^4 \times \left(\frac{r_1}{r_2}\right)^2$$

23. **a)** **2 : 1**

$$\frac{E_1}{E_2} = \frac{T_1^4 - T_0^4}{T_2^4 - T_0^4} = \frac{(600)^4 - (300)^4}{(500)^4 - (300)^4}$$

$$\frac{E_1}{E_2} = \frac{6^4 - 3^4}{5^4 - 3^4} = \frac{36^2 - 9^2}{25^2 - 9^2} = \frac{1296 - 81}{625 - 81}$$

$$\frac{E_1}{E_2} = \frac{1215}{544} = 2.23$$

$$\frac{E_1}{E_2} \approx 2 : 1$$

24. **a)** $\mathbf{\frac{81}{16}}$

Area enclosed is the total radiation emitted

$$\frac{A_2}{A_1} = \frac{E_2}{E_1} = \left(\frac{T_2}{T_1}\right)^4$$

25. **b)** **256/81**

$$\lambda_0 T_1 = \frac{3\lambda_0}{4} T_2 \Rightarrow T_2 = \frac{4T_1}{3}$$

Power = Energy per unit time and $E \propto T^4$

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^4 = \left(\frac{4}{3}\right)^4 = \frac{256}{81}$$

26. **b)** **RP $\propto$ r**

Rate of radiation per unit area is proportional to (T^4)

$\therefore$ $P \propto AT^4$

$\Rightarrow$ $P \propto T^2$

Also $ms\frac{dT}{dt} = \propto AT^4$

$\therefore$ $\frac{dT}{dt} = R \propto \frac{1}{r}$

(because $m = (V\rho) \propto r^3$ and $A \propto r^2$)

$\therefore$ $PR \propto r$

27. **b)** **6 minutes**

In first case:

$$\frac{61 - 59}{4} = K\left[\frac{61 + 59}{2}\right]$$

$\therefore$ $\frac{1}{2} = K \times 30 \Rightarrow K = \frac{1}{60}$

In second case

$$\frac{51 - 49}{t} = \frac{1}{60}\left[\frac{51 + 49}{2} - 30\right]$$

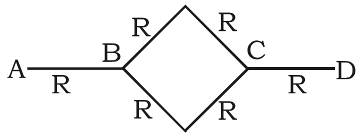
$\therefore$ $\frac{2}{t} = \frac{1}{60} \times 20 \quad \Rightarrow \quad t = 60$ min.

28. **d)** **3000 Å**

$$\lambda = \frac{b}{t}; \frac{\lambda_2}{\lambda_1} = \frac{T_1}{T_2} = \frac{1500}{2500} = \frac{3}{5}$$

$$\lambda = \frac{3}{5}\lambda_1 = 3000\ \text{Å}$$

29. **c)** **140°C**



$\Rightarrow$

$$\left(\frac{dQ}{dt}\right)_1 = \left(\frac{dQ}{dt}\right)_2$$

$$\frac{kA(200 - \theta)}{l} = \frac{kA(\theta - 20)}{2l}$$

$400 - 2\theta = \theta - 20;\ \theta = \frac{400}{3}\ 140°C$

30. **a)** $\mathbf{\sqrt{\frac{\pi}{6}} : 1}$

$S_s = S_c \Rightarrow m_s > m_c;\ \frac{dQ}{dt} = \frac{4\sigma A T_0^3}{ms}\ \Delta T$

Given $6a^2 = 4\pi r^2$ or $\frac{a}{r} = \sqrt{\frac{4\pi}{6}} = \sqrt{\frac{2\pi}{3}}$

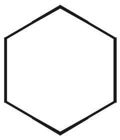
$$\frac{dQ}{dt} \propto \frac{A}{m}; \frac{\left(\frac{dQ}{dt}\right)_s}{\left(\frac{dQ}{dt}\right)_c} = \frac{\frac{4\pi r^2}{\frac{4}{3}\pi r^3 \sigma}}{\frac{6a^2}{a^3\sigma}} = \frac{3/r}{6/a} = \frac{a}{2r}$$

$$= \frac{1}{2}\sqrt{\frac{2\pi}{3}} = \sqrt{\frac{\pi}{6}} : 1$$

PART B - CHEMISTRY

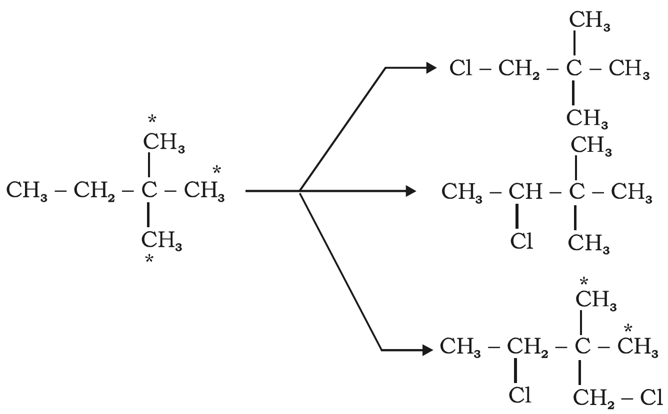
31. **a) chlorine**
Order of reactivity is
Cl > Br > I

32. **d)**



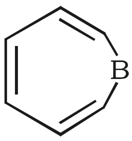
Cyclo hexane is stable among the given compound.

33. **b) $CH_3CH_2C(CH_3)_2CH_3$**



*** equivalent Carbonn**

34. **d)**



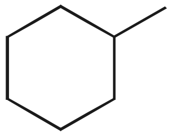
'B' involves in conjugation with empty p-orbital.

35. **c) HC ≡ C – C ≡ CH**
Alkyne 'C – C' bond always shorter.

36. **a)** C_2H_6**(excess) +** $Cl_2 \xrightarrow{\text{UV light}}$

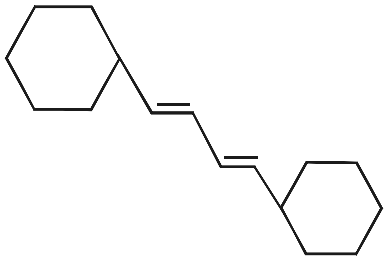
Conceptual

37. **d) None of these**



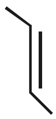
The actual product of hydrogenation.

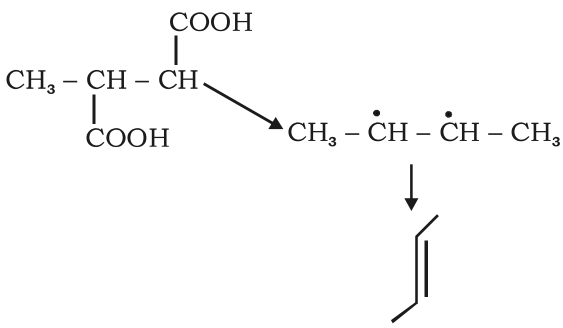
38. **b)**



Given reaction is a trans reduction (Birch reduction)

39. **c)**





Since trans alkenes are more stable.

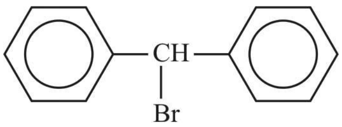
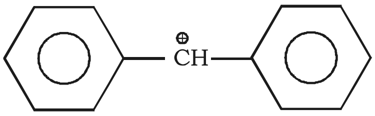
40. **c) both (a) and (b)**
Both can attack electrophilic centre with oxygen's lone pairs.

41. **a)** $\overset{\ominus}{C}H_3 > \overset{\ominus}{N}H_2 > \overset{\ominus}{O}H > \overset{\ominus}{F}$
Nucleophilicity decreases in period as E.N. of element increases.

42. **d)** $ClCH_2—CH=CH_2$
$CH_2 = CH – CH_2 – Cl \rightarrow$ forms allyl carbocation (stable)

43. **a)** $CH_3Br + OH^{\ominus} \longrightarrow CH_3OH + Br^{\ominus}$
1⁰ alkyl halide is more reactive in SN^2.

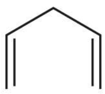
44. **c)** **II > III > I > IV**
Depends on the stability of carbocation.

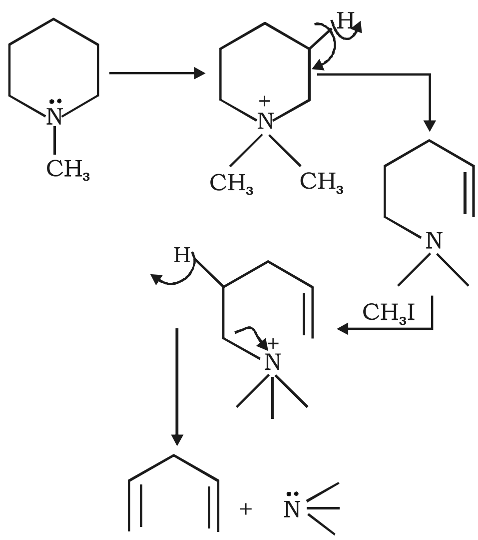
45. **a)** 
Forms stable carbocation.
 (Di phenyl methyl carbocation)

46. **d)** **R > Q > P > S**
1⁰ R–X are more reactive and smaller alkyl halides are more reactive.

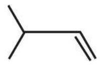
47. **d)** **P > Q > R > S**
Carbocation can be stabilized by Hyper conjugation.

48. **c)** $CH_3-\underset{}{\overset{CH_3}{\overset{|}{C}}}=CH_2$
E_2 - elimination

49. **b)** 



50. **c)** $H_3C-\overset{CH_3}{\underset{CH_3}{C}}-O^{\ominus}$
If the leaving group is bulky hoffman product predominates.

51. **b)** 
Hoffman elimination

52. **d)** $H_2C=CH_2$
Hoffman elimination

53. **c)** **both a and b**
Conceptual

54. **d)** **E2**
Conceptual

55. **a)** 
Hoffman elimination

56. **a)** **Water**
Polar protic solvent is suitable for solvolysis, water have high dielectric constant value.

57. **d)** **all of these**
Conceptual

58. **a)** **V > IV > III > II >I**
Conceptual

59. **a)** $CH_3CD_2CH=CHCD_3$
Kinetic isotopic effect

60. **c)** **1-bromo-3, 3-dimethylbutane**
1⁰ alkyl halide is more reactive in E_2 - elimination.

PART C - MATHS

61. **b)** $\sin^{-1}(\sin\frac{7\pi}{5})$

a) $\cos^{-1}\cos\frac{7\pi}{5} = \cos^{-1}\cos(2\pi - \frac{7\pi}{5})$

$= \cos^{-1}\cos\frac{3\pi}{5} = \frac{3\pi}{5}$

b) $\sin^{-1}\sin\frac{7\pi}{5} = \sin^{-1}\sin(\pi - \frac{7\pi}{5})$

$= \sin^{-1}\sin\left(-\frac{2\pi}{5}\right) = -\frac{2\pi}{5}$

62. **d)** $-1 \le x < \frac{1}{\sqrt{2}}$

$(\cos^{-1}x - \sin^{-1}x)\ \frac{\pi}{2} > 0$

$\frac{\pi}{2} - 2\sin^{-1}x > 0 \Rightarrow \frac{\pi}{4} > \sin^{-1}x$

$\Rightarrow -\frac{\pi}{2} \le \sin^{-1}x < \frac{\pi}{4} \Rightarrow -1 \le x < \frac{1}{\sqrt{2}}$

63. **d)** $-\cos 2$

$\tan^{-1}\tan 2 = \tan^{-1}\tan(2-\pi) = 2-\pi$

$\cos(2-\pi) = \cos(\pi-2) = -\cos 2$

64. **c)** $\frac{2\pi}{3}$

$\cos\theta = \frac{(\alpha-\beta)^2 + (\alpha+\beta)^2 - (3\alpha^2+\beta^2)}{2(\alpha^2-\beta^2)}$

$= \frac{2\alpha^2 - 2\beta^2 - 3\alpha^2 - \beta^2}{2(\alpha^2-\beta^2)} = \frac{\beta^2-\alpha^2}{2(\alpha^2-\beta^2)} = \frac{-1}{2}$

$\theta = 120^0 = \frac{2\pi}{3}$

65. **c)** ***a*** **cos A +** ***b*** **cos B +** ***c*** **cos C = 4R sin A sin B sin C**

$a\cos A + b\cos B + c\cos C$

$= 2R\sin A\cos A + 2R\sin B\cos B + 2R\sin C\cos C$

$= R(\sin 2A + \sin 2B + \sin 2C)$

$= R(4\sin A\sin B\sin C)$

66. **b)** 60^0

$\frac{a+b+c}{a+c} + \frac{a+b+c}{b+c} = 3$

$\frac{b}{a+c} + \frac{a}{b+c} = 1$

$\Rightarrow$ $b^2 + bc + a^2 + ac$

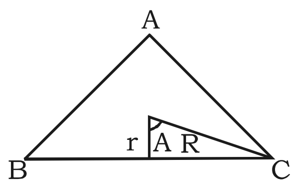
$ab + ac + bc + c^2$

$\Rightarrow \frac{a^2+b^2-c^2}{2ab} = \frac{1}{2}$

$\Rightarrow \cos C = \frac{1}{2}$

$C = \frac{\pi}{3}$

67. **b)** 1



$\cos A = \frac{\pi}{R}$

$\cos A + \cos B + \cos C = 1 + \frac{\pi}{R}$

$\Rightarrow \cos B + \cos C = 1$

68. **b)** **Isosceles Triangle**

$2R\sin A\cos B = 2R\sin B\cos A$

$\sin(A-B) = 0 \quad \Rightarrow \quad A = B$

69. **a)** ***a***

$(s-b)\tan\frac{B}{2}\cot\frac{B}{2} + (s-C)$

$\tan\frac{C}{2}\cot\frac{C}{2}$

$= 2s(b+c) = a$

70. **a)** $1+\frac{r}{R}$

$\cos A + \cos B + \cos C = 1+\frac{r}{R}$

71. **b)** **2**

$$\left(1-\frac{s-b}{s-a}\right)\left(1-\frac{s-c}{s-a}\right)$$

$$=\frac{(b-a)(c-a)}{(s-a)^2}=\frac{bc-ab-ac+a^2}{b^2+c^2+a^2+2bc-2ab-2ab}4$$

Also, A = 90 $\Rightarrow$ $b^2+c^2=a^2$

$$=\frac{(a^2+bc-ab-ac)4}{2a^2+2bc-2ca-2ab}$$

$$=\frac{4}{2}=2$$

72. **a)** **3**

$$\sin^{-1}\frac{3}{\sqrt{10}}=\text{Tan}^{-1}3$$

$\Rightarrow$ $x = 3$

73. **d)** $\boldsymbol{\frac{\pi}{2}}$

$$\cos^{-1}\frac{1}{x}=\sin^{-1}\frac{1}{y}=\frac{\pi}{2}-\cos^{-1}\frac{1}{y}$$

$$\cos^{-1}\frac{1}{x}+\cos^{-1}\frac{1}{y}=\frac{\pi}{2}$$

74. **b)** π

$\text{Tan}^{-1}1+\text{Tan}^{-1}2+\text{Tan}^{-1}3=\pi$

75. **a)** $\boldsymbol{\frac{\pi}{2}}$

$$\cos^{-1}\cos\frac{5\pi}{3}=\cos^{-1}\cos\left(2\pi-\frac{5\pi}{3}\right)=\frac{\pi}{3}$$

$$\sin^{-1}\cos\frac{5\pi}{3}=\frac{\pi}{2}-\frac{\pi}{3}$$

$\Rightarrow$ $\cos^{-1}X+\sin^{-1}X=\frac{\pi}{2}$ where $X=\cos\frac{5\pi}{3}$

76. **a)** **No solution**

$$\frac{\pi}{2}+\cos^{-1}X=\frac{11\pi}{6}$$

$$\cos^{-1}X=\frac{11\pi}{6}-\frac{\pi}{2}=\frac{11\pi-3\pi}{6}$$

$$=\frac{8\pi}{6}=\frac{4\pi}{3}$$

No solution as $\cos^{-1}x\in[0,\pi]$

77. **b)** **1**

$$x-\frac{x^2}{2}+\frac{x^3}{4}\text{ ——— }=x^2-\frac{x^4}{2}+\frac{x^6}{4}\text{ ——— }$$

$$\frac{x}{1+\frac{x}{2}}=\frac{x^2}{1+\frac{x^2}{2}}$$

$\Rightarrow$ $x+\frac{x^3}{2}=x^2+\frac{x^3}{2}$ $\Rightarrow$ $x=x^2$

$\Rightarrow$ $x = 0, 1$

78. **c)** **Two**

$x(x+1)\geq 0$ & $x^2+x+1\leq 1$

$\Rightarrow$ $x(x+1)\geq 0$ & $x(x+1)\leq 0$

$\Rightarrow$ $x(x+1)=0$ $\Rightarrow$ $x=0,-1$

2 solution

79. **c)** π

$$\text{Tan}^{-1}\frac{12}{5}+\text{Tan}^{-1}\frac{3}{4}+\text{Tan}^{-1}\frac{63}{16}$$

(Answer is Positive)

$$\text{Tan}^{-1}\frac{\frac{12}{5}+\frac{3}{4}}{1-\frac{36}{20}}+\text{Tan}^{-1}\frac{63}{16}$$

$$=\text{Tan}^{-1}\frac{63}{-16}+\text{Tan}^{-1}\frac{63}{16}=\pi$$

80. **d)** **None of these**

$\text{Sin}^{-1}X\in\left[\frac{\pi}{2},\frac{\pi}{2}\right]\longrightarrow$ no solution

81. **a)** $\mathbf{\sin^2\alpha}$

$$\frac{xy}{ab}-\sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}=\cos\alpha$$

$$\left(\frac{xy}{ab}\cos\alpha\right)^2=1-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{x^2y^2}{a^2b^2}$$

$$\frac{x^2y^2}{a^2b^2}+\cos^2\alpha-\frac{2xy}{ab}$$

$$\cos\alpha=1-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{x^2y^2}{a^2b^2}$$

$\Rightarrow \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\alpha = \sin^2\alpha$

82. **c)** $\frac{\pi}{3}$

$\frac{a^2 + c^2 - b^2}{2ac} = \frac{1}{2} = \cos B \Rightarrow B = 60^0$

83. **c)** **3*abc***

$ab^2 \cos A + ba^2 \cos B + ac^2 \cos A + ca^2 \cos C + bc^2 \cos B + cb^2 \cos C$

$\Rightarrow$ $ab(b \cos A + a \cos B)$ + ______

$= abc + abc + abc$

$= 3abc$

84. **a)** **A.P.**

$a\left(\frac{1 + \cos C}{2}\right) + c\left(\frac{1 + \cos A}{2}\right) = \frac{3b}{2}$

$a + c + a\cos C + c\cos A = 3b$

$a + c + b = 3b$

$\Rightarrow \quad a + c = 2b$

A.P.

85. **a)** $\tan^{-1}\left(\frac{5}{6}\right)$

$\text{Tan}^{-1}\frac{1}{3} + \text{Tan}^{-1}\frac{1}{7} + \text{Tan}^{-1}\frac{1}{13} +$ ______

$= \text{Tan}^{-1}\frac{1}{1+2} + \text{Tan}^{-1}\frac{1}{1+6} + \text{Tan}^{-1}\frac{1}{1+12}$

$= \text{Tan}^{-1}\frac{1}{1+1.2} + \text{Tan}^{-1}\frac{1}{1+2.3} + \text{Tan}^{-1}\frac{1}{1+3.4}$

$= \text{Tan}^{-1}\frac{2-1}{1+1.2} + \text{Tan}^{-1}\frac{3-2}{1+2.3} +$ ______

$= \text{Tan}^{-1} 2 - \text{Tan}^{-1} 1 + \text{Tan}^{-1} 3 - \text{Tan}^{-1} 2 +$ ______ $\text{Tan}^{-1} 11 - \text{Tan}^{-1} 10$

$= \text{Tan}^{-1} 11 - \text{Tan}^{-1} 1$

$= \text{Tan}^{-1}\frac{10}{12} = \text{Tan}^{-1}\frac{5}{6}$

86. **c)** $\frac{2\pi}{3}$

$K = \sin 3x = \sin 3A = \sin 3B$

$\Rightarrow \quad 3A = \pi - 3B$

$\Rightarrow \quad A + B = \frac{\pi}{3}$

$\Rightarrow \quad C = \frac{2\pi}{3}$

87. **b)** **38**

$2(4^n) + \lambda = 133$

$2(5^n) + \lambda = 255$

$\Rightarrow \quad 5^n - 4^n = 61 \quad \Rightarrow \quad n = 3$

$\Rightarrow \quad 2(64) + \lambda = 133 \quad \Rightarrow \quad \lambda = 5$

$f(3) - f(2) = 2(3^n - 2^n) = 2(27 - 8) = 38$

88. **d)** $x = 2, y = \pi/2$

$(x - 2)^2 + 1 = \sin y$

as $(x - 2)^2 \geq 0$

& $\sin y \leq 1$

$\Rightarrow \quad x = 2, \sin y = 1 \Rightarrow y = \frac{\pi}{2}$

89. **c)** $\pi/2$

$\sin^4(\pi/2 + X) + \cos^4(\pi/2 + X)$

$= \cos^4 X + \sin^4 X$

Period $= \pi/2$

Also $\sin^4 X + \cos^4 X = 1 - 2\sin^2 \times \cos^2 X$

$= 1 - \frac{1}{2}\sin^2 2x$

$\sin^2 2x \longrightarrow$ Time period $= \pi/2$

90. **c)** $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

$\sin^{-1} X + \cos^{-1} X + \text{Tan}^{-1} X$

Domain $= [-1, 1]$

$= \frac{\pi}{2} + \text{Tan}^{-1} X$

range $= \left[\frac{\pi}{2} - \frac{\pi}{4}, \frac{\pi}{2} + \frac{\pi}{4}\right]$

$= \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$