MAHESH TUTORIALS SCIENCE

00 – 00		Q. Booklet Serial No: 050415	
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Hints & Solutions

PART A - PHYSICS

1. **b)** $\mathbf{\frac{2\pi a_0}{3}}$

$V_{wave} = \frac{2\pi v}{(2\pi / \lambda)} = v\lambda$

Velocity of particle,

$v_p = \frac{\partial y}{\partial t} = 2\pi v a_0 \left[\sin 2\pi\left(vt - \frac{x}{\lambda}\right)\right]$

Now, $(v_p)_{max} = 3V_{wave}$

$2\pi v a_0 = 3(v\lambda)$

$\therefore \quad \lambda = \frac{2\pi a_0}{3}$

2. **a)** $\mathbf{y = A\cos\frac{n\pi x}{l}\cos\left(\frac{n\pi ct}{l} + \phi\right)}$

Since x = 0 & L have to be antinodes

∴ particles

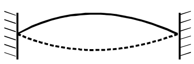
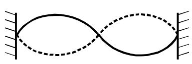
∴ at x = 0 & L should have max amplitudes

∴ $A\cos\frac{n\pi x}{L}$ must be there out of given options.

Hence A & D

Note : No information about position of particle at any times is given. Hence we can not decide between A & D.

3. **c) Statement-1 is true, statement-2 is false.**

4. **c)**  

$f = \frac{nv}{2l}$

$v_I > v_{II}$

∴ for $f_I = f_{II}$

$n_I < n_{II}$

5. **a) 20 m/s**

y_{max} when $x - vt = 0$

$y_{max} = \frac{6.4\times10^{-6}}{1.6\times10^{-3}}$

$= 4 \times 10^{-3}$

$\frac{4\times10^{-3}}{2} = \frac{6.4\times10^{-6}}{1.6\times10^{-3} + v^2\times4\pi\times10^{-6}}$

taking x = 0

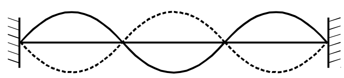
$3.2 \times 10^{-6} + 8 \times 10^{-9} v^2 \times 6.4 \times 10^{-6}$

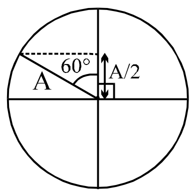
$8 \times 10^{-9} v^2 = 3.2 \times 10^{-6}$

$v^2 = \frac{32\times10^2}{8}$

$v = 20$ m/s

6. **c)** $\mathbf{A_0 \sin\left(\omega t + \frac{5\pi}{6}\right)}$





$3\frac{\lambda}{2} = 3L \Rightarrow \lambda = 2L$

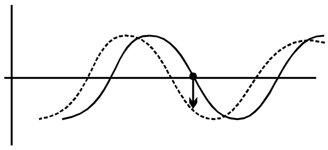
$y = A_0 \sin kx \sin\left(\omega t + \frac{\pi}{2} + \frac{\pi}{3}\right)$

$= A_0 \sin\left(\frac{2\pi}{2L}\right)\times\left(\frac{L}{2}\right)\sin\left(\omega t + \frac{5\pi}{6}\right)$

$= A_0 \sin\left(\omega t + \frac{5\pi}{6}\right)$

7. **b)** ↓

$v_p = -v_w \dfrac{\partial y}{\partial x}$; as $\dfrac{\partial y}{\partial x} < 0$; $v_w < 0$



$v_p < 0$; particle moves down

8. **c) all particles between consecutive nodes vibrate in the same phase**

9. **c)** $[(x-1)/(x+1)]^2$

$\dfrac{A_1 + A_2}{A_2 - A_2} = x$

$\dfrac{A_2}{A_1} = \dfrac{x-1}{x+1}$

Energy $\propto A^2 \Rightarrow \left(\dfrac{x-1}{x+1}\right)^2$

10. **d) is unchanged while the wavelength is halved**

The wave speed depends on factors of tension in the string and its composition. Hence, since the wave speed is unchanged, with an increased frequency, there is a decreased wavelength ($v = f\lambda$)

11. **b) 4/9**

$$\frac{E_t}{E_i} = \left(\frac{A_t}{A_i}\right)^2 = \left(\frac{2V_2}{V_1 + V_2}\right)^2 = \left[\frac{2}{V_1/V_2 + 1}\right]^2$$

$$= \left[\frac{2}{\sqrt{(\mu_1/\mu_2)} + 1}\right]^2 = 4/9$$

12. **a) the fundamental frequency of the A string in violin 1 is less than that for violin 2**

On increasing f slowly, $|f_1 - f_2|$ decreases means $f_1 < f_2$

13. **a)** $\mathbf{0.5\ ms^{-1}}$

14. **b)** $\dfrac{1}{\sqrt{2}}L$

The speed of a wave on a string is found as $v = \sqrt{\dfrac{T}{\mu}}$ where μ is the mass per unit length.

For string A, to be vibrating at the fundamental frequency, one writes $v = f\lambda$ $\rightarrow \sqrt{\dfrac{T}{\mu}} = f(2L)$.

For string B, one writes $v = f\lambda \rightarrow \sqrt{\dfrac{T}{2\mu}} = f(2L_B)$ as the mass per unit length of the string is double by it having twice the density.

As a result, by taking the ratio of these two expressions, one has

$$\frac{1/\sqrt{2\mu}}{1/\sqrt{\mu}} = \frac{L_B}{L} \Rightarrow L_B = \frac{1}{\sqrt{2}}L$$

15. **d) 64 : 225**

The intensity of a wave is proportional to square of amplitude as well as square of frequency. The amplitudes are in the ratio 2 : 5 whereas the frequencies are in the ratio 4 : 3.

16. **b)** $\mathbf{f_A = 4f_B}$

Note that the frequency of the vibrating string $f \propto \dfrac{1}{lr}\sqrt{\dfrac{T}{\rho}}$ where the symbols have their usual meanings

17. **a)** $\mathbf{Bs_0k}$

From the equation of the sound wave, we get $\dfrac{\partial s}{\partial x} = ks_0\cos(kx - \omega t)$. Now use the expression $\Delta p = -B\dfrac{\partial s}{\partial x}$ where B is the bulk modulus. This gives an expression for the pessure change $\Delta p = -Bs_0k\sin[(kx - \omega t) - \pi/2]$, also indicating a phase lag of $\pi/2$ with respect to displacement.

18. **d) $3f/4$**

$$\frac{f = \frac{v}{2l}}{f' = \frac{v}{4\left(\frac{2l}{3}\right)}}$$

$$\frac{f}{f'} = \frac{4}{3}$$

$$f' = \frac{3f}{4}$$

19. **a) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.**

20. **d) Statement-1 is false, statement-2 is true.**

21. **d) 4L, 4L/3, 4L/5**

For n^{th} over tone

$$n\frac{\lambda}{2} + \frac{\lambda}{4} = L \quad \Rightarrow \lambda = \frac{4L}{2n+1}$$

As n increases frequency increases.

3 wavelengths, n = 0, n = 1, n = 2

$$\lambda = 4L, \frac{4L}{3}, \frac{4L}{5}$$

22. **b) $\sqrt{2}$ mm**

$$\frac{4L}{5} = \lambda = \lambda = 8\text{cm}$$

thus 2 cm corresponds to $\Delta\phi = z/2$

1 cm corresponds to $\Delta\phi = z/4$

So $y = A\text{sm}\ \pi/4,\ 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}$

23. **a) 1880 m/s**

On increasing f slowly, $|f_1 - f_2|$ decreases means $f_1 < f_2$.

24. **d) 3600 Hz**

$f = (2n + 1)f_0$

(f is fundamental frequency n = 0, 1, 2, 3)

$= (2n + 1)\ 300$

$\Rightarrow (2n + 1) = \frac{f}{300}$ = odd integer

Not satisfied for f = 3600 Hz.

25. **a) f**

The distance between source and receiver is not changing, so there is no change in frequency.

26. **d) 30 cm**

The intensity of a wave is proportional to square of amplitude as well as square of frequency. The amplitudes are in the ratio 2 : 5 whereas the frequencies are in the ratio 4 : 3.

27. **c) 600 Hz**

Since the standing wave mode has a displacement antinode at the opening, there is a displacement node at the water-air interface. By increasing the height of the air column, to go from one harmonic to the next, an additional length equal to 1/2 wavelength is required.

Hence

$$\frac{\lambda}{2} = (0.38 - 0.12)\text{ m}$$

$\Rightarrow \lambda = 0.52$ m. Finally, from $v = f\lambda$, we find that $\lambda = v/f = 312/0.52 = 600$ Hz. If one checks, this problem deals with the 1st and 3rd harmonics.

28. **c) is 100 times greater**

The decibel scale is logarithmic β dB = 10 log $\left(\frac{I}{I_o}\right)$. Each increase in intensity by a power of ten increases the decibel reading by 10 units. Hence, to increases the decibel reading by 20, there needs to be an increase in the intensity of 10 × 10 = 100

29. **a) zero**

Frequency observed by man is same as "observed" by wall and it reflects the same and as man and wall are relatively at rest, hence man observers same frequency of reflected sound. Hence no beat frequency.

30. **a) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.**

S-1 : $I \propto A^2$ and $I \propto f^2$

$$\frac{I_1}{I_2} = \left(\frac{A_1}{A_2}\right)^2 \left(\frac{f_1}{f_2}\right)^2 = \left(\frac{2}{1}\right)^2 = 1$$

S-2 : Velocity amplitude = $A\,w \propto \sqrt{I}$

Therefore if velocity amplitude is same then intensity will also be same

PART C - MATHS

61. **b)** $\mathbf{1 - \alpha^2 - \beta\gamma = 0}$

$\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \alpha^2 + \beta\gamma = 1$

$\Rightarrow 1 - \alpha^2 - \beta\gamma = 0$

62. **c)** $\mathbf{\pm 3}$

$\therefore |A|^3 = 125$

$\Rightarrow |A|^3 = 125$

$\Rightarrow |A| = 5$

$\Rightarrow \alpha^2 - 4 = 5$

$\Rightarrow \alpha = \pm 3$

63. **a)** $\begin{bmatrix} \mathbf{1} & \mathbf{2007} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$

$P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

$Q = PAP^T$

Here $P.P^T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = I,$

Now $Q = PAP^T$

Also $x = P^TQ^{2007}P = P(PAP^T)^{2007}$

$P = P^T(PAP^T)(PAP^T)^{2006},P$

$= (A)PT\ (PAP^T)2006.$

$P = AP^T(PAP^T)(PAP^T)^{2005}P.$

$= \ldots\ldots\ldots\ldots$

$\therefore x = A^{2006}P^T.(PAP^T)\,P = A^{2006}(A)$

$\therefore x = A^{2007} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix}$

64. **a)** **infinitely many solutions**

This is the case of non-trivial solution and system of equations has infinite many solutions.

65. **a)** $\mathbf{(14)^4}$

We know that adj(adj A) $= |A|^{n-2} A$ if $|A| \neq 0$

66. **b)** **1**

Since the system possess infinitely many solutions, we must have

$\frac{K+1}{K} = \frac{8}{K+3} = \frac{4K}{3K-1} \Rightarrow K = 1$

67. **b)** $\mathbf{(2006)^2}$

$|A| = r^2 - (r-1)^2 = 2r - 1$

$\Rightarrow |A_1| + |A_2| + \ldots + |A_{2006}|$

$= \sum_{r=1}^{2006} (2r-1) = (2006)^2$

68. **b)** **nilpotent**

$A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$

Hence nilpotent

69. **d)** **finitely many solutions**

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \quad \ldots(i)$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad \ldots(ii)$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad \ldots(iii)$

By adding (i)(ii) equations, $\frac{2x^2}{a^2} = 2$

$\Rightarrow x = \neq a$

and $\frac{y^2}{b^2} = \frac{z^2}{c^2}$

from equation (iii), $1 + \frac{2y^2}{b^2} = 2$

$\Rightarrow y = 0$

$x = \neq a,\ y = 0 \quad \Rightarrow z = 0$

solutions are $(a, 0, 0)$ and $(-a, 0, 0)$

70. **c)** A^T

Find A^T, then $2A^T$ and finally $A + 2A^T$

71. **c)** **Zero**

$$\begin{vmatrix} 103 & 115 & 114 \\ 111 & 108 & 106 \\ 104 & 113 & 116 \end{vmatrix}$$

$C_1 \leftrightarrow C_3$ $\qquad$ $C_1 \leftrightarrow C_2$

$$\Rightarrow -\begin{vmatrix} 114 & 115 & 103 \\ 106 & 108 & 111 \\ 116 & 113 & 104 \end{vmatrix} \Rightarrow \begin{vmatrix} 115 & 114 & 103 \\ 108 & 106 & 111 \\ 113 & 116 & 104 \end{vmatrix}$$

$R_1 \leftrightarrow R_3$

$$\Rightarrow -\begin{vmatrix} 113 & 116 & 104 \\ 108 & 106 & 111 \\ 115 & 114 & 103 \end{vmatrix}$$

72. **d)** **None of these**

$$\Delta = \frac{1}{abc}\begin{vmatrix} ab^2c^2 & abc & ab+ac \\ bc^2a^2 & abc & bc+ab \\ a^2b^2c & abc & ac+bc \end{vmatrix}$$

$$\Delta = \frac{a^2b^2c^2}{abc}\begin{vmatrix} bc & 1 & ab+ac \\ ca & 1 & bc+ab \\ ab & 1 & ca+bc \end{vmatrix}$$

$R_1 \rightarrow R_3 + R_1$

$$\Delta = abc\begin{vmatrix} ab+bc+ca & 1 & ab+ac \\ ab+bc+ca & 1 & ba+ab \\ ab+bc+ca & 1 & ca+bc \end{vmatrix}$$

$\Delta = 0$

73. **d)** **0**

$$\begin{vmatrix} x^2+y^2 & ax+by & px+qy \\ ax+by & a^2+b^2 & ap+bq \\ px+qy & ap+bq & p^2+q^2 \end{vmatrix}$$

$$= \begin{vmatrix} x & y & 0 \\ a & b & 0 \\ p & q & 0 \end{vmatrix}\begin{vmatrix} x & a & p \\ y & b & q \\ 0 & 0 & 0 \end{vmatrix}$$

$= 0$

74. **b)** $(a-b)^2(b-c)^2(c-a)^2$

$$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}^2$$

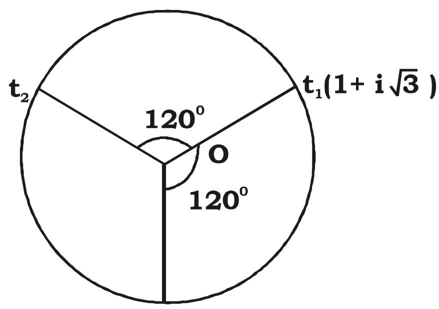
$= (a-b)^2(b-c)^2(c-a)^2$

75. **a)** **0**

R_1, R_2, R_3 are A.P's

$\Rightarrow \Delta = 0$

76. **c)** $-2,\ 1 - i\sqrt{3}$



$t_2 = t_1 e^{i2\pi/3}$

$t_3 = t_1 e^{-i2\pi/3}$

77. **a)** $(a^2+b^2)(|z_1|^2+|z_2|^2)$

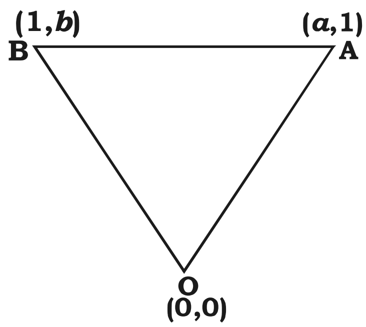
$|az_1 - bz_2|^2 + |bz_1 - az_2|^2$

$= (az_1 - bz_2)(a\bar{z}_1 - b\bar{z}_2) + (bz_1 - az_2)(b\bar{z}_1 - a\bar{z}_2)$

$= a^2|z_1|^2 + b^2|z_2|^2 + b^2|z_1|^2 + a^2|z_1|^2$

$= (a^2+b^2)(|z_1|^2+|z_2|^2)$

78. **b)** $a = b = 2 - \sqrt{3}$



$OA^2 = OB^2 \Rightarrow a = b$

$OA^2 = AB^2$

$\Rightarrow a^2 + 1 = (a-1)^2 + (a+1)^2$

$a^2 + 1 = 2a^2 - 4a + 2$

$\Rightarrow a^2 - 4a + 1 = 0$

$\Rightarrow a = 2 - \sqrt{3}$

79. **a)** **n**

$x^n - 1 = (x-1)(x-\alpha_1)(x-\alpha_2) \ldots\ldots (x-\alpha_{n-1})$

$\frac{x^n - 1}{x-1} = (x-\alpha_1)(x-\alpha_2) \ldots\ldots (x-\alpha_{n-1})$

$\Rightarrow (x-\alpha_1)(x-\alpha_2) \ldots\ldots (x-\alpha_{n-1})$

$= 1 + x + x^2 + x^3 + \ldots. x^{n-1}$

(By G.P formula)

substituting $x = 1$ on both sides

$\Rightarrow (x-\alpha_1)(x-\alpha_2) \ldots\ldots (x-\alpha_{n-1})$

$= n$

80. **c)** $\boldsymbol{x^2 + y^2}$

Taking magnitude on both sides and squaring 2.5.10 ......$(1 + n^2)$

$= x^2 + y^2$

81. **a)** **rectangular Hyperbola**

if $z = x + iy$

$z^2 = x^2 - y^2 + 2ixy$

$\text{Im}(z^2) = 2xy = \lambda$

$\Rightarrow xy = \frac{\lambda}{2}$

82. **a)** **circle**

$|z| = 1$

$\Rightarrow z = \cos\theta + i\sin\theta$

$\frac{2}{z} = 2(\cos\theta + i\sin\theta) = h + ki$

$h = 2\cos\theta \qquad K = -2\sin\theta$

$\Rightarrow h^2 + k^2 = 4$

83. **b)** **0**

$f(x) = x^{3p} + x^{3q+1} + x^{3r+2}$

$f(\omega) = \omega^{3p} + \omega^{3q+1} + \omega^{3r+2}$

$= 1 + \omega + \omega^2$

$= 0$

84. **a)** ω

$t^3 + 2t^2 + 2t + 1 = 0$

$t^3 + 1 + 2t^2 + 2t = 0$

$(t+1)(t^2 - t + 1) + 2t(t+1) = 0$

$(t+1)(t^2 + t + 1) = 0$

$t = -1, \omega, \omega^2$

$t^{1985} + t^{100} + 1 = 0$ satisfies ω, ω^2

85. **d)** **none of these**

arg(–ve real number) = π

86. **a)** **2**

$|z|^2 + 4z = 0$

$\Rightarrow x^2 + y^2 + 4x - 4iy = 0$

$\Rightarrow y = 0 \;\; x^2 + y^2 + 4x = 0$

on solving we get $x = 0$, $x = -4$

solving are (0,0) (–4,0)

87. **d)** **4**

$(a - ib)^3 = x + iy$

$a^3 + ib^3 - 3a^2ib - 3ab^2 = x + iy$

$x = a^3 - 3ab^2$, $y = b^3 - 3a^2 b$

$\frac{x}{y} - \frac{y}{b} = 4(a^2 - b^2)$

88. **b)** **3**

$|z_1| = 1, |z_2| = 2$

$|z_1 + z_2|^2 + |z_1 - z_2|^2$

$= 2(|z_1|^2 + |z_2|^2)$

$= 2(1 + 4)$

$= 10$

89. **b)** **–1**

$x + \frac{1}{x} = 1$

$\Rightarrow x^2 + 1 = x$

$\Rightarrow x^2 - x + 1 = 0$

$\Rightarrow x = -\omega, -\omega^2$

90. **b)** **1**

$\tan\left(\left\{\omega^{100} + \omega^{101}\right\}\pi + \frac{\pi}{4}\right)$

$= \tan\left(-\pi + \frac{\pi}{4}\right)$

$= \tan\left(\frac{-3\pi}{4}\right) = 1$