MAHESH TUTORIALS SCIENCE

00 – 00		Q. Booklet Serial No: 310515	
Test No : 1201	3 Hrs.		Q. Booklet Version : 11

Hints & Solutions

PART A - PHYSICS

1. **b)** $\mathbf{\frac{R}{\sqrt{2}}}$

The electric field intensity due to an unifromly charged ring at any point an the axial line is

$$E(x) = \frac{Qx}{4\pi \in_0 (x^2 + R^2)^{3/2}}$$

Solve for $\frac{d(E(x))}{dx} = 0$ we get $x = \frac{R}{\sqrt{2}}$ (Maxima)

2. **d)** **–120v**

Since potential at any location is a scalar addition of all the x, y, z parts of it.

∴ $V_{net} = V_x + V_y$; As $V_x = -\int E_x\, dx$

$$= \left(-\int_0^3 E_x\, dx\right) + \left(-\int_0^2 E_y\, d_y\right);$$

given $Ex = 20\hat{i}$

$Ey = 30\hat{j}$

= –120v.

3. **d)** $\mathbf{8.5 \times 10^{-26}}$ **Nm**

$T = P \times E \Rightarrow T_{max} = PE$

$= (3.4 \times 10^{-30})(2.5 \times 10^4)$

$= 8.5 \times 10^{-26}$ Nm

4. **c)** $\mathbf{\frac{q \ln 2}{4\pi \in_0 x_0}}$

Potential at the origin due to all the charges is

$$V = \frac{q}{4\pi \in_0 x_0}\left(\frac{1}{x_0} - \frac{1}{2x_0} + \frac{1}{3x_0} - - -\right)$$

$$= \left(\frac{q}{4\pi \in_0 x_0}\right)(\ln 2)$$

5. **a)** $\mathbf{\frac{8q_2q_3}{4\pi \in_0}}$

$\Delta U_{net} = \Delta U_{12} + \Delta U_{23} + \Delta U_{13}$

∵ q_1 urt q_2 or q_3 doesn't charge distance,

$\Delta U_{12} = \Delta U_{13} = 0$

$$\Delta U^{net} = \frac{q_2q_3}{4\pi \in_0}\left[\frac{1}{(1/10)} - \frac{1}{(5/10)}\right]$$

$= \Delta U_{23}$

$= \frac{8q_2q_3}{4\pi \in_0}$

6. **c)** $\mathbf{180^0}$

Any object or system in a satellite a which is in a stable orbit would experience weight lessness i.e.

$9_{effective} = 0$

∴ In the absence of gravity, the charges go to the maximum possible separation.

∴ $\theta = 180^0$

7. **d)** **none of these**

The decribed system is that of a dipole and there are xO rull points in the electrostatic firld of a dipole for any finite destance.

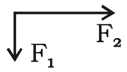
8. **c)** **A, C, B**

Let the force between,the e^- and p^+ at separation 'd' be 'F,' and at separation 'D' be 'F_2'.

The free body iopram of the e– in all 3 cares are show.

Case (A): $F_1 \rightarrow F_2 \rightarrow$ ∴ $F_{net}^{A} = F_1 + F_2$

Case (B): $F_1 \leftarrow \bullet \rightarrow F_2$ ∴ $F_{net}^{B} = F_2 - F_1$

Case (C): 

$\therefore\ F^{C}_{net} = \sqrt{F_1^2 + F_2^2}$

$\therefore\ F^{A}_{net} > F^{C}_{net} > F^{B}_{net}$

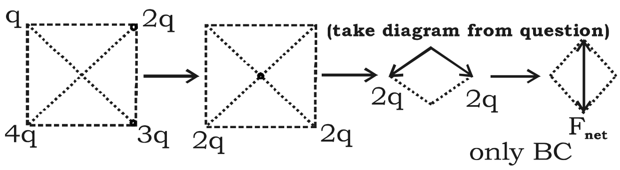
(Note : Sum of two side of a triangle is greater than the third side, which inturn is greater than the difference of the sides.)

9. **a)** $\mathbf{\frac{F}{16}}$

Let the charges be +q and –q, if 75% of charge (4) of q is given to –q, the final charges in the system are $\frac{q}{4}$ and $\frac{-q}{4}$

$\therefore\ F^1 = \frac{F}{16}$

10. **b)** **CB**



(Note : Consider the atternate charges i.e. (q,3q) and (2q, 4q) which effectively reduce t o (0, 2q) and (0, 2q) respectively. The aformentioned statement is true for fieldes not potentials.)

11. **a)** $\mathbf{k\frac{Q}{r^2}}$

From the principle of superposition.

Field due to (n – 1) charges = Field due to x charge – Field due to missing charge

$= O - \frac{kq}{r^2}$

$\therefore$ E_{net} due to (n – 1) charges is of the magnitude $\frac{kq}{r^2}$

12. **d)** **20 N/C**

The displacement along X-axis is 2 Cos 60^0 = lm

$\therefore$ W = F.S = (qE) S (Cos O^0)

= (qE) (lm)

$\therefore$ 4 = (0.2) E

$\Rightarrow$ E = 20N/C

13. **b)** $\mathbf{\frac{E^2q^2t^2}{2m}}$

Force felt in uniform electric field = qE

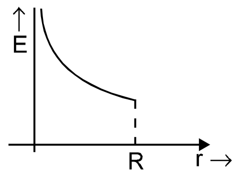
$\therefore$ Acceleration = $\frac{qE}{m}$

$\therefore$ Instantaneous velocity at any time 't' is

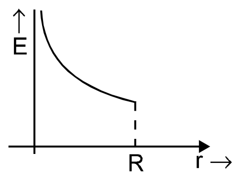
V = u + at

$V = O + \left(\frac{qE}{m}\right) t$

$\therefore\ KE = \frac{1}{2} mv^2 = \frac{q^2E^2t^2}{2m}$

14. **a)** 

Inside any conductor, the contribution to the field by any external charge is zero.

15. **a)** 

$\therefore$ The field inside is only due to the charge placed at the centre i.e. q. which is governed by the inverse square law.

16. **b)** **A, B and C**

One can assume the two pairs of charges provided (_+q, –v) and the other (+q, –q) as two dipoles.

The potential due to a dipole is zero on its perpendicular bisector i.e. ABC.

17. **a)** **8 V**

Constant field ⇔ line only varying potential.

$F = qE \quad \therefore \quad E = 400$ N/C.

$\therefore \quad \Delta V = \int_{a}^{a+2} E.dx$

$= E \int_{a}^{a+2} dr$

$= (400)\left(\frac{2}{100}\right) = 8$ V.

18. **d)** **-1.52×10^5 V**

Clearly $OB = OC = \frac{1}{2}$ m $OA = \frac{\sqrt{3}}{2}$ m

$\therefore \quad QA = -6\mu C; QB = -2\mu C; Q_C = -3\mu C$

$\therefore \quad Vat\ 0 = \frac{K(Q_A)}{OA} + \frac{K(Q_B)}{OB} + \frac{K(Q_C)}{OC}$

$= -1.5 \times 10^5 V$

19. **c)** $\mathbf{\frac{q}{2\pi\varepsilon_0}}$

$\therefore \quad Vat\ 0 = Kq\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \ldots\right)$

$= \frac{q}{2\pi \epsilon_0}$

20. **d)** **0**

Notice that $(5_a, 0)$ and $(-3_a, 4_a)$ lie on a circle with origin as center and radius 5a. This circle is an equipotential surface for the given 20mC charge placed at its center.

$\therefore \quad \Delta V = 0$

21. **d)** **F / 8**

The field at the end on position is inbersely praportional to r^3.

$\therefore \quad F^1 = \frac{F}{8}$

22. **d)** **2pE**

W electric field $= -\bar{p}.\bar{E}$

$\therefore$ W external source $= +\bar{p}.\bar{E}$

$= pE$ (for the first 90^0)

and pE for the next 90^0 as well

$\therefore \quad W_{net} = 2pE$

(Note : This is the work done by extrnal souece Not the field)

23. **a)** **a force and a torque**

Consider the dipole, there is a field E_1 making an angle of θ_1 with the dipole at +q and field E_2 making an angle of θ_2 at –q.

$\therefore \quad F_{net} = \overline{F_1} + \overline{F_2} = q\overline{F_1} + q\overline{F_2} \neq 0.$

$\left(\text{as } \overline{E_1} \neq \overline{E_2}\right)$

$Tnet = \left(\frac{P}{2}\right) \times \overline{E_1} + \frac{P}{2} \times \overline{E_2}$

$\neq 0.$

24. **b)** **3.71×10^5V/m**

$q = 6.75\mu C; F = 2.5 N = qE$

as $E = \frac{\partial V}{\partial r} = \frac{-\Delta V}{\Delta r}$

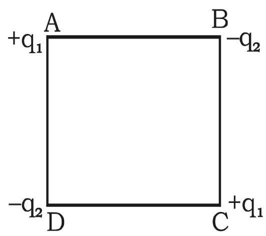
(not always, only in special cases. as in $\frac{\Delta y}{\Delta x} = \frac{dy}{dx}$ if $y = mx + C$.)

$\therefore \quad E = 3.71 \times 10^5$V/m

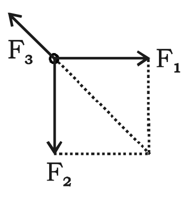
25. **c)** **constant throughout the region**

$E = \frac{\partial V}{\partial r}$; if $E = 0 \Rightarrow$ V(r) is constant

26. **d)** $\mathbf{\frac{2\sqrt{2}}{1}}$



free body diagram of the charge at A is.



for equilibrium

$\overline{E_1} + \overline{E_2} + \overline{E_3} = 0$ also $qE_i = E_i$

$\therefore \quad \frac{-Kq_2}{a^2}\frac{-Kq_2}{a^2} + \frac{-Kq_1}{\left(\sqrt{2_a}\right)^2} = 0$

$\therefore \quad \frac{-Kq_2}{a^2}\left(\sqrt{2}\right) + \frac{Kq_1}{2_a{}^2} = 0$

$\therefore \quad \frac{q_1}{q_2} = \frac{2\sqrt{2}}{1}$

27. **a)** $\mathbf{\frac{V}{6r}}$

Let the holow sphere have a charge $\overline{q}^{1}$

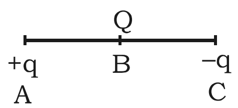
$\therefore \quad \frac{Kq}{3r} - \frac{Kq}{r} = \Delta V$

$\therefore \quad |\Delta V| = \frac{Kq.(2)}{(3r)} = V$

$\therefore \quad$ E at 3r $= \frac{Kq}{9r^2} = \frac{V}{6r}$

28. **b)** **1 : 2**

let the equal separation be 'd'



U_{net} at A = $U_{AB} + U_{AC}$.

$= \frac{KqQ}{d} - \frac{Kqq}{2d}$

given Unet at +q = 0

$\therefore \quad Q = \frac{q}{2}$

$\therefore \quad$ or $\frac{q}{Q} = \frac{2}{1}$ $i.e. \frac{Q}{V} = \frac{1}{2}$

29. **a)** $\mathbf{\frac{k}{343}\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right) V/m}$

$V = \frac{K}{r}$ is of the from $\frac{Kq}{r}$

$\therefore \quad E = \frac{-\partial V}{\partial r} = \frac{-\partial\left(\frac{V}{r}\right)}{dr} = \frac{V}{r^2}\left(\hat{r}\right)$

$\therefore \quad$ given $\overline{r} = 2\hat{i} + 3\hat{j} + 6\hat{k} \Rightarrow \left|\overline{r}\right| = 7$

$\therefore \quad E = \frac{K}{(|r|)^2}\hat{r}$ also $\hat{r} = \frac{1}{7}\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$

$= \frac{K}{343}\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right) V/m$

30. **c)** **neutral**

2, 3 are simples charges

4, 5 are simples charges

2, 4 attract

$\therefore$ 2 and 4 are oppositrly charged.

and 1 is attracted by both 2 and 4.

$\therefore$ 1 must be neutral

PART C - MATHS

61. **c)** **6**

$2\sin^{-1}(x) + 3\,\text{Sin}^{-1}(y) = \frac{5\pi}{2}$

holds only when $x = y = 1$

Substituting in $y = \text{K}\,x - 5$

$1 = \text{K} - 5$

$\text{K} = 6$

62. **c)** **17/6**

$\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)$

$\Rightarrow \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{2}{3}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}\right)$

$= \tan(\tan^{-1}(17/6))$

63. **c)** π

$\tan^{-1}\left(\sqrt{\frac{(a\text{k})}{bc}}\right) + \tan^{-1}\left(\sqrt{\frac{(b\text{k})}{ac}}\right)$

$= \tan^{-1}\left(\frac{\sqrt{\frac{\text{k}}{abc}}(a+b)}{1-\frac{\text{k}}{c}}\right)$

$\Rightarrow \pi - \tan^{-1}\left(\sqrt{\frac{\text{k}c}{ab}}\right)$

As $\tan^{-1}(a) + \tan^{-1}(b) = \pi + \tan^{-1}\left(\frac{a+b}{1-ab}\right)$ $(ab > 1)$

$\Rightarrow \tan^{-1}\left(\sqrt{\frac{a\text{k}}{bc}}\right) + \tan^{-1}\left(\sqrt{\frac{b\text{k}}{ac}}\right) + \tan^{-1}\left(\sqrt{\frac{\text{k}c}{ab}}\right) = \pi$

64. **d)** **11**

$(\tan(\sec^{-1}(2)))^2 + (\cot(\text{cosec}^{-1}(3))^2$

$= (\tan(\tan^{-1}(\sqrt{3})))^2 + (\cot(\cot^{-1}(\sqrt{8})))^2$

$= 3 + 8$

$= 11$

65. **d)** $3\pi/4$

$\cos^{-1}(\cos(2\cot^{-1}(\sqrt{2}-1)))$

$= \cos^{-1}(\cos(2\tan^{-1}(\sqrt{2}+1)))$

$2\tan^{-1}\left(\sqrt{2}+1\right) = \pi + \tan^{-1}\left(\frac{2\left(\sqrt{2}+1\right)}{1-\left(\sqrt{2}+1\right)^2}\right)$

$= \pi + \tan^{-1}\left(\frac{2\left(\sqrt{2}+1\right)}{-2-2\sqrt{2}}\right)$

$= \pi - \frac{\pi}{4}$

$= \frac{3\pi}{4}$

$= \cos^{-1}\left(\cos\left(\frac{3\pi}{4}\right)\right)$

$= \frac{3\pi}{4}$

66. **c)** **3**

$\cos^{-1}(\alpha) + \cos^{-1}(\beta) + \cos^{-1}(\nu) = 3\pi$

$\Rightarrow \alpha = \beta = \nu = -1$

$\alpha\beta + \beta\nu + \nu\alpha = 3$

67. **b)** $\pi/3$

$\sin^{-1}(x) + \sin^{-1}(y) = \frac{2\pi}{3}$

$\cos^{-1}(x) + \cos^{-1}(y) = \pi - (\sin^{-1}(x) + \sin^{-1}(y))$

$= \pi - \frac{2\pi}{3}$

$= \frac{\pi}{3}$

68. **d)** **3**

$\sin^{-1}\left(\frac{x}{5}\right) + \cos^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$

$\sin^{-1}\left(\frac{x}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right) = \frac{\pi}{2}$

$\sin^{-1}\left(\frac{x}{5}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{2}$

$\Rightarrow x = 3$

69. **c)** **$x = 0$**

$\tan^{-1}(1 + x) + \tan^{-1}(1 - x) = \frac{\pi}{2}$

$\tan^{-1}(1 + x) + \cot^{-1}\left(\frac{1}{1-x}\right) = \frac{\pi}{2}$

$\Rightarrow \quad 1 + x = \frac{1}{1-x}$

$\Rightarrow \quad 1 - x^2 = 1$

$\Rightarrow \quad x^2 = 0$

$\Rightarrow \quad x = 0$

70. **b)** **0.96**

$2\sin^{-1}\left(\frac{4}{5}\right) = \sin^{-1}\left(2 \times \frac{4}{5} \times \frac{3}{5}\right)$

$= \sin^{-1}(0.96)$

$= \sin(\sin^{-1}(0.96))$

$= 0.96$

71. **b)** **unique solution**

$\sin^{-1}(x) - \cos^{-1}(x) = \frac{\pi}{6}$

$\sin^{-1}(x) - \cos^{-1}(x) = \frac{\pi}{2}$

$\Rightarrow \quad x = \frac{\sqrt{3}}{2}$

72. **d)** **$2\pi - x$**

$\pi \le x \le 2\pi$

$\cos^{-1}(\cos x) = \cos^{-1}(\cos(2\pi - x))$

$= 2\pi - x$

73. **c)** **$\tan^{-1}(0.3)$**

$\tan(x + y) = 33$

$\tan x = 3$

$\frac{\tan x + \tan y}{1 - \tan x \tan y} = 33$

$\frac{3 + \tan y}{1 - 3\tan x} = 33$

$\Rightarrow \quad \tan y = 0.3$

$y = \tan^{-1}(0.3)$

74. **b)** **$3\pi/4$**

$\cot^{-1}(2) + \cot^{-1}(3) = \frac{\pi}{4}$

Third angle $= \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

75. **a)** **0**

$x = y = z = 1$

$x^{100} + y^{100} + z^{100} - \frac{9}{x^{101} + y^{101} + z^{101}}$

$\Rightarrow \quad 3 - \frac{9}{3}$

$\Rightarrow \quad 3 - 3 = 0$

76. **d)** ∞

$\lim_{x\to 0} \frac{\sin x^{13}}{(\sin x)^{17}}$

$= \lim_{x\to 0} \frac{\frac{\sin x^{13}}{x^{13}}}{\left(\frac{\sin x}{x}\right)^{13} \sin^4 x}$

$\Rightarrow \quad \lim_{x\to 0} \frac{1}{\sin^4 x} = \infty$

77. **b)** **is -1/3**

$x \to \infty \quad \frac{x \sin\left(\frac{1}{x}\right) - 2}{\frac{1}{x^2} + 3}$

$\Rightarrow \quad \lim_{x\to\infty} \frac{\frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} - 2}{\frac{1}{x^2} + 3} \quad \Rightarrow \quad \frac{1-2}{3} = \frac{-1}{3}$

78. **d)** **None of these**

L.H.L $= \lim_{x\to 0} \frac{\sin[x]}{[x]} = \sin 1$

R.H.L is not in the domain

$= \sin 1$

79. **b)** π

$$\lim_{x\to 0}\frac{\sin\left(\pi\cos^2 x\right)}{x^2}$$

Applying L. Hospital

$\Rightarrow$ $$\lim_{x\to 0}\frac{\cos\left(\pi\cos^2 x\right)\pi(2\cos x)(-\sin x)}{2x}$$

$\Rightarrow$ π

80. **d)** **None of these**

L.H.L = $\lim_{x\to 0}\frac{e^{\{x\}}-1}{x}=\infty$

R.H.L = $\lim_{x\to 0^+}\frac{e^{\{x\}}-1}{x}=\lim_{x\to 0^+}\frac{e^{x}-1}{x}=1$

Limit does not exist.

81. **a)** $\mathbf{\frac{10}{7},\frac{4}{7}}$

$$\lim_{x\to 0}\frac{\tan 3x}{\sin 7x} = \lim_{x\to 0}\frac{\frac{\tan 3x}{x}}{\frac{\sin 7x}{x}}$$

$$= \frac{3}{7}$$

$$|k-1| = \frac{3}{7}$$

$\Rightarrow$ $k = \frac{10}{7},\frac{4}{7}$

82. **c)** $\mathbf{e^{ab}}$

$$\lim_{x\to 0}(\cos x + a\sin bx)^{\frac{1}{x}}$$

$\Rightarrow$ $e^{\lim_{x\to 0}\frac{\cos x-1+a\sin bx}{x}}$

$\Rightarrow$ $e^{\lim_{x\to 0}\frac{-\sin x+ab\cos bx}{1}}$

$\Rightarrow$ e^{ab}

83. **c)** **does not exists**

$$\lim_{x\to 0}\frac{\sqrt{\frac{1-\cos 2x}{2}}}{x}$$

$\Rightarrow$ $$\lim_{x\to 0}\frac{|\sin x|}{x}$$

L.H.L = –1

R.H.L = 1

Limit does not exist

84. **d)** **None of these**

$$\lim_{x\to 0}\left(\sqrt{x^2+1+x}-x\right) = 1$$

85. **c)** $-8\sqrt{3}$

$$\lim_{x\to\frac{\pi}{3}}\frac{\tan^2 x-3}{\cos\left(\pi+\frac{\pi}{6}\right)}$$ [Applying L.Hospital rule]

$\Rightarrow$ $$\lim_{x\to\frac{\pi}{3}}\frac{2\tan x\sec^2 x}{-\sin\left(x+\frac{\pi}{6}\right)}$$

$\Rightarrow$ $-8\sqrt{3}$

86. **b)** $\mathbf{\frac{1}{\sqrt{3}}}$

$$\lim_{x\to\frac{\pi}{3}}\frac{\sin\left(\frac{\pi}{3}-x\right)}{2\cos x-1}$$ [Applying L.Hospital rule]

$\Rightarrow$ $$\lim_{x\to\frac{\pi}{3}}\frac{\cos\left(\frac{\pi}{3}-x\right)(-1)}{-2\sin x}$$

$\Rightarrow$ $\frac{1}{\sqrt{3}}$

87. **d)** **None of these**

$$\lim_{x\to 1}\frac{1+\log x-x}{1-2x+x^2}$$ [Applying L.Hospital rule]

$\Rightarrow$ $$\lim_{x\to 1}\frac{\frac{1}{x}-1}{-2+2x}$$

$\Rightarrow$ $$\lim_{x\to 1}\frac{1-x}{-2(1-x)x}=\frac{-1}{2}$$

88. **c)** **2 log 2**

$$\lim_{x\to 0}\frac{x\,2^x-x}{1-\cos x}$$

$$\Rightarrow \quad \lim_{x\to 0} \frac{x\left(2^x - 1\right)}{2\sin^2\left(\frac{x}{2}\right)}$$

$$\Rightarrow \quad \lim_{x\to 0} \frac{\frac{2^x - 1}{x}}{2\frac{\sin^2\left(\frac{x}{2}\right)}{x^2}}$$

$\Rightarrow \quad 2 \ln 2$

89. **d)** **None of these**

$$\lim_{x\to 0} \left(\frac{\cos 4x - \cos 6x}{\sin^2 5x}\right)$$

$$\Rightarrow \quad \lim_{x\to 0} \frac{2\sin 5x \sin x}{\sin^2 5x}$$

$$\Rightarrow \quad \lim_{x\to 0} \frac{2\sin x}{\sin 5x}$$

$$\Rightarrow \quad \frac{2}{5}$$

90. **a)** **1**

$\lim_{x\to\infty} a^{1/x}$ $\qquad (0 < a < 1)$

$\Rightarrow \quad 1$ $\qquad$ (As $a^0 = 1$)