LAKSHYA ADVANCED UNIT TEST (LAUT)

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Test No : 3451311	3 Hrs.		

Hints & Solutions

PART C - MATH

SECTION I - MULTIPLE ANSWER CORRECT TYPE

1. **b)** $h = \frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right)$

d) $\frac{b^2 - 4ac}{a^2} = \frac{q^2 - 4pr}{p^2}$

$\alpha - \beta = \alpha_1 - \beta_1$

$(\alpha + \beta)^2 - 4\alpha\beta = (\alpha_1 + \beta_1)^2 - 4\alpha_1\beta_1$

$\frac{b^2}{a^2} - \frac{4c}{a} = \frac{q^2}{p^2} - \frac{4r}{p}$

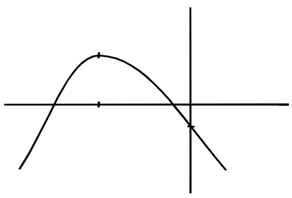
$\alpha + \beta = -\frac{b}{a}$

$\alpha + \beta + 2h = \frac{-q}{p}$

$\frac{-b}{a} + 2h = \frac{-q}{p}$

$2h = \frac{b}{a} - \frac{q}{p}$

2.



a) $b^2 - 4ac > 0$

b) $b < 0$

d) $c < 0$

$a < 0$

$D < 0$

$c < 0$

$\frac{-b}{2a} < 0 \Rightarrow \frac{b}{2a} > 0 \Rightarrow b < 0$

3. **a)** $1 \le x \le 3$

c) $-\frac{1}{3} \le y \le \frac{1}{3}$

$x^2 - 4x + 4 + 9y^2 - 1 = 0$

$(x - 2)^2 + 9y^2 = 1 \; (x - 2)^2 = 1 - 9y^2$

$\Rightarrow 1 - 9y^2 \ge 0 \Rightarrow \frac{-1}{3} \le y \le \frac{1}{3}$

Also, $1 - (x - 2)^2 \ge 0 \Rightarrow (x - 2)^2 \le 1$

$\Rightarrow -1 \le x - 2 \le 1$

$\Rightarrow 1 \le x \le 3$

4. **a)** $x^2 = \left(\frac{\sqrt{5} - 1}{2}\right)$

c) $\sin(\cos^{-1} x) = \left(\frac{\sqrt{5} - 1}{2}\right)$

$\cos^{-1} x = \text{Tan}^{-1} x \Rightarrow x > 0 \Rightarrow x \in (0,1]$

Let $\cos^{-1} x = A \Rightarrow x = \cos A \Rightarrow \tan A = \sqrt{\frac{1 - x^2}{x}}$

$\Rightarrow \sqrt{\frac{1 - x^2}{x}}$

$\Rightarrow \sqrt{1 - x^2} = x^2 \Rightarrow 1 - x^2 = x^4$

$\Rightarrow x^4 + x^2 - 1 = 0 \Rightarrow x^2 = \frac{-1 + \sqrt{5}}{2}$

$\sin \cos^1 x = \sin \sin^{-1} \sqrt{1 - x^2} = \sqrt{1 - x^2} =$

$\sqrt{1 - \left(\frac{-1}{2} + \frac{\sqrt{5}}{2}\right)} = \sqrt{\frac{3 - \sqrt{5}}{2}} = \frac{\sqrt{5} - 1}{2}$

$\tan \cos^{-1} x = \tan \tan^{-1} \sqrt{\frac{1 - x^2}{x}} = \sqrt{\frac{1 - x^2}{x}} =$

$x = \sqrt{\frac{\sqrt{5} - 1}{2}}$

5. **a)** $\tan^{-1} 2 + \tan^{-1} 3$
d) $\sec^{-1}\left(-\sqrt{2}\right)$

$\tan^{-1} \dfrac{4n}{1+(n^2-1)^2}$

$= \tan^{-1} \dfrac{4n}{1+(n+1)^2}$

$(n-1)^2 = \tan^{-1} \dfrac{(n+1)^2-(n-1)^2}{1+(n+1)^2(n-1)^2}$

$= \tan^{-1}(n+1)^2 - \tan^{-1}(n-1)^2$

put $n = 1, 2, 3, \text{-----} \infty$

$= \dfrac{\pi}{2} + \dfrac{\pi}{2} - \dfrac{\pi}{4} = \dfrac{3\pi}{4} = \tan^{-1} 1 + \tan^{-1} 2$

$= \sec^{-1}(-\sqrt{2})$

6. **a)** Graph of f is symmetrical about the line $x = 1$

$f(x) = \ln(2x - x^2) + \sin\dfrac{\pi}{2}x$

$= \ln(1+(1-x)^2) + \sin\dfrac{\pi x}{2}$

If line is symmetric about $x = 1$

$\Rightarrow f(1+x) = f(1-x)$

$f(1+x) = \ln(1+x^2) + \dfrac{\pi x}{2}$

$f(1-x) = \ln(1+x^2) + \cos\dfrac{\pi x}{2}$

If line is symmetric about $x = 2$

$f(2+x) = f(2-x)$

7. **a)** $f(x) = x - [x]$, where $[x]$ denotes the greatest integer less than or equal to the real number x

$f(x) = \{x\}$ is periodic

$f(x) = \sin\dfrac{1}{x} \to \sin\dfrac{1}{x+T} = \sin\dfrac{1}{x} \to$

Not periodic

$f(x) = x\cos \to (x+T)\cos(x+T) = x\cos \to$

Not

8. **a)** domain of f is $[1, 2)$

c) range of f is $\left\{\log\dfrac{\pi}{2}\right\}$

$f(x) = \sin^{-1} \ln[x] \quad + \quad \ln \sin^{-1}[x]$

$\downarrow \qquad \downarrow$

$-1 \ \ln[x] \le 1 \cap 0 < \sin^{-1}[x] \le \pi/2$

$\downarrow \qquad \downarrow$

$\dfrac{1}{e} \le [x] \le e \quad \cap 0 < [x] \le 1$

$\downarrow \qquad \downarrow$

$[x] = 1, 2 \quad \cap [x] = 1$

$[x] = 1 \quad \Rightarrow x \in [1,2)$

put $[x] = 1$

$f(x) = 0 + \ln\dfrac{\pi}{2}$

SECTION II - PARAGRAPH TYPE

9. **b)** $\sin^{-1}\left(\dfrac{17}{5\sqrt{13}}\right)$

$\alpha = \cos^{-1}\dfrac{4}{5}, \ \beta = \tan^{-1}\dfrac{2}{3}$

$\alpha + \beta = \tan^{-1}\dfrac{3}{4} + \tan^{-1}\dfrac{2}{3} = \tan^{-1}\dfrac{\frac{2}{5}+\frac{3}{4}}{1-\frac{3}{4}\frac{2}{3}}$

$= \tan^{-1}\dfrac{17}{6} = \sin^{-1}\dfrac{17}{5\sqrt{13}}$

10. **b)** $\cos\beta$

$\sin\cot^{-1}\tan\cos^{1}\dfrac{3}{\sqrt{13}}$

$= \sin\cot^{-1}\dfrac{2}{3}$

$= \dfrac{3}{\sqrt{13}} = \cos\beta$

11. **a)** 1

$f(x) = k\,|\cos x| + k^2\,|\sin x|$

$\pi \qquad \pi$

Period will be $\dfrac{\pi}{2}$ if $k = k^2 \Rightarrow k = 1$

12. **a)** 4

$$f(x) = \{3x + 3\} + \sin\frac{\pi x}{2}$$

$$\frac{1}{3} \qquad 4$$

$$\text{LCM}\left(\frac{1}{3},\frac{4}{1}\right) = \frac{\text{LCM}(1,4)}{\text{HCF}(1,3)} = \frac{4}{1} = 4$$

SECTION III - INTEGER TYPE

1. **3**

$7x - 3 > (x + 1)^2 > 5x - 1$

$\Rightarrow \; 5x - 1 < x^2 + x^2 + 2x + 1 < 7x - 3$

$\Rightarrow \; x^2 - 3x + 2 > 0 \quad \& \; x^2 - 5x + 4 < 0$

$(x - 1)(x - 2) > 0 \;\&\; (x - 1)(x - 4) < 0$

$(-\infty, 1) \cup (2, \infty) \quad \cap (1, 4)$

$\Rightarrow \; (2, 4)$

Integer value of $x = 3$

2. **2**

$3x^2 + px + 3 = 0$

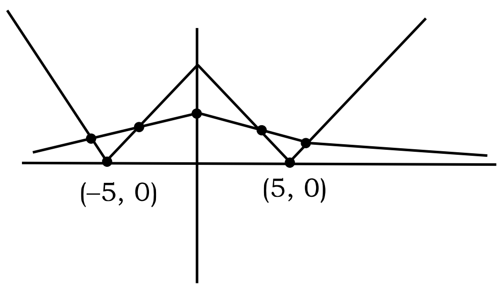
$\alpha + \alpha^2 = {}^{-p}/_{3}, \; \alpha^3 = 1$ → $\alpha = 1$ or $\alpha + \alpha^2 = -1$

$$\left.\begin{array}{l}\text{If } \alpha = 1 \Rightarrow p = -6 \\ \hline \text{If } \alpha + \alpha^2 = -1 \Rightarrow p = 3\end{array}\right] \; 2 \text{ Values}$$

3. **4**

$7^{|x|}\left(|5 - |x||\right) = 1$

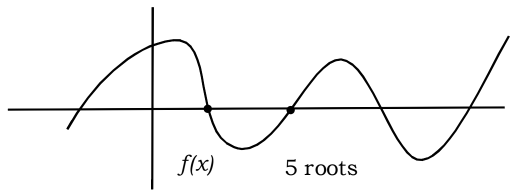
$||x| - 5| = 7^{-|x|}$

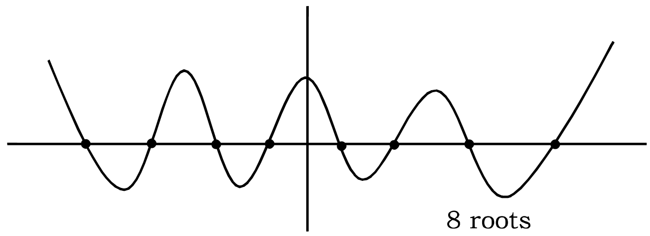


4. **5**

$f(|x|) = 0 \rightarrow 8$ real roots

$f(x) \; ax^5 + bx^4 + cx^3 + dx^2 + ex + f = 0$





5. **3**

$$\sin^{-1}\frac{3\sin 2\theta}{5 + 4\cos 2\theta} = \frac{\pi}{2}$$

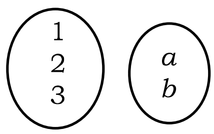
$$\Rightarrow \frac{3\sin 2\theta}{5 + 4\cos 2\theta} = 1$$

$$\Rightarrow 3\left(\frac{2\tan\theta}{1 + \tan^2\theta}\right) = 5 + 4\,\frac{(1 - \tan^2\theta)}{1 + \tan^2\theta}$$

$\Rightarrow \; 6\tan\theta = 9 + \tan^2\theta$

$\tan\theta = 3$

6. **6**



Total $F^n = 2^3$

Info = 2

onto = $2^3 - 2 = 6$

7. **4**

$$x - f(x) = 19\left[\frac{x}{19}\right] - 90\left[\frac{f(x)}{90}\right]$$

$x = 1990$

$$1990 - f(1990) = 19\left[\frac{1990}{19}\right] - 90\left[\frac{f(1990)}{90}\right]$$

$$\frac{1900}{90} < \frac{f(1900)}{90} < \frac{2000}{90} \Rightarrow 21 < \frac{f(1900)}{90}$$

≤ 22

$$\left[\frac{f(1990)}{90}\right] = 22 \text{ or } 21$$

Also $\left[\frac{1900}{19}\right] = 104$

$\Rightarrow$ 1990 $-f(1990) = 19(104) - 90$ (22 or 21)

$f(1990) = 1990 - 19(104) + 90$ (22 or 21)

$f(1990) = 14 + 1980$ or 1890

$f(1990) = 1994$ or 1904

8. **6**

$f(x) + f\left(\frac{1}{x}\right) = f(x)\, f\left(\frac{1}{x}\right)$

$\Rightarrow f(x) = 1 \pm x^n$

$f(4) = 1 \pm 4^n = 65$

$\Rightarrow f(x) = 1 + x^3$

$f(6) = 1 + 6^3 = 217 = abc$

$\Rightarrow a = 2,\ b = 1,\ c = 7$

$c + b - a = 7 + 1 - 2 = 6$