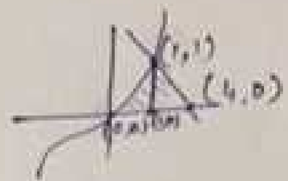
Single -choice.

1) $y = x^3$, normal at $(1,1)$



$y' = 3x^2 \Rightarrow y'_{(1,1)} = 3$

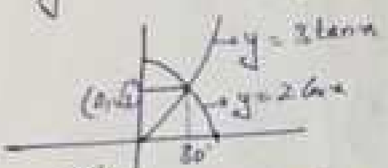
$m_N = -\frac{1}{3} \Rightarrow y - 1 = -\frac{1}{3}(x-1)$

$3y - 3 = -x + 1 \Rightarrow 3y + x = 4$

$\text{Area} = \int_0^1 x^3\,dx + \frac{1}{2} \times 3 \times 1$

$= \frac{1}{4} + \frac{3}{2} = \frac{7}{4}$

2) $y = 2\cos x$, $y = 3\tan x$, y-axis



$3\tan x = 2\cos x$

$\Rightarrow 3\sin x = 2\cos^2 x$

$\Rightarrow 3\sin x = 2 - 2\sin^2 x$

$\Rightarrow 2\sin^2 x + 3\sin x - 2 = 0$

$\Rightarrow 2\sin^2 x + 4\sin x - \sin x - 2 = 0$

$\Rightarrow (2\sin x - 1)(\sin x + 2) = 0$

$\Rightarrow \sin x = \frac{1}{2} \Rightarrow x = 30^\circ$

$\text{Area} = \int_0^{\pi/6} (2\cos x - 3\tan x)\,dx$

$= \left[2\sin x - 3\ln|\sec x|\right]_0^{\pi/6}$

$= 2 \times \frac{1}{2} - 3\ln\left(\frac{2}{\sqrt{3}}\right)$

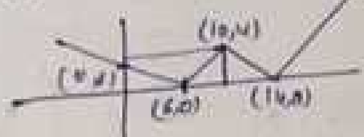
$= 1 - 3\ln 2/\sqrt{3}$

3) $y = |4 - |10-x|| = ||10-x| - 4|$

$y = |10-x|$

$\Rightarrow y = |10-x| - 4$

$y = ||10-x| - 4| \equiv$



$\text{Area} = \frac{1}{2} \times 8 \times 4 = 16$

4) $\dfrac{dy}{dx} = 1-2x \Rightarrow y = x - x^2 + K$

$-2 = 2 - 4 + K \Rightarrow K = 0$

$\therefore y = x - x^2$

$y = ax$

$\therefore x - x^2 = ax \Rightarrow x = 0$ or $1 - x = a \Rightarrow x = 1-a$

$$\text{Area} = \left| \int_0^{1-a} \left[(x - x^2) - ax\right] dx \right| = \frac{32}{3}$$

$$= \left| \left[\frac{x^2}{2} - \frac{x^3}{3} - \frac{ax^2}{2} \right]_0^{1-a} \right| = \frac{32}{3}$$

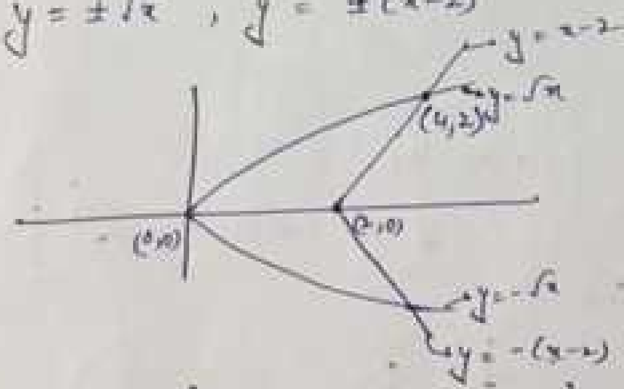
$$= \left| (1-a)^2 \left[\frac{1}{2} - \frac{1-a}{3} - \frac{a}{2} \right] \right| = \frac{32}{3}$$

$$= \left| (1-a)^2 \left(\frac{3 - 2 + 2a - 3a}{\cancel{6}\ 2} \right) \right| = \frac{32}{\cancel{3}}$$

$$\Rightarrow \left| (1-a)^3 \right| = 64 \Rightarrow 1 - a = 4 \Rightarrow a = -3$$

$$1 - a = -4 \Rightarrow a = 5$$

5) $y = \pm\sqrt{x}$, $y = \pm(x-2)$



$x - 2 = \sqrt{x}$

$x^2 - 4x + 4 = x$

$x^2 - 5x + 4 = 0$

$x^2 - 4x - x + 4 = 0$

$(x-4)(x-1) = 0$

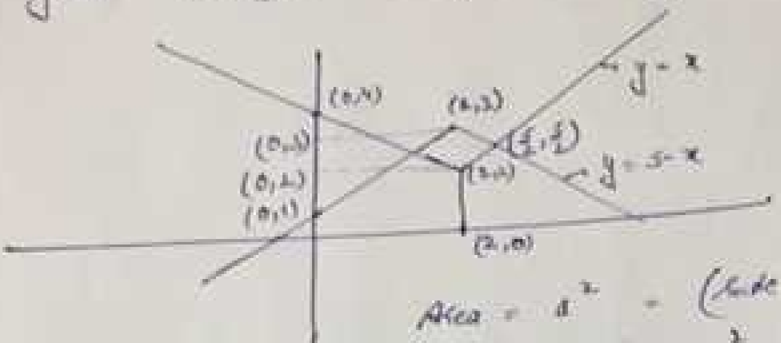
$x = 4, 1$

$$\text{Area} = 2 \int_0^2 (x_2 - x_1)\, dy = 2 \int_0^2 \left[(y+2) - y^2 \right] dy$$

$$\frac{A}{2} = \left[\frac{y^2}{2} + 2y - \frac{y^3}{3} \right]_0^2 = 2 + 4 - \frac{8}{3} = \frac{18-8}{3} = \frac{10}{3} \Rightarrow A = \frac{20}{3}$$

9) $f(x) = \max\{2+|x-2|, 3-|x-2|\}$

$g(x) = \min\{2+|x-2|, 3-|x-2|\}$



Enclosed figure is a square

Area $= a^2 = (\text{side length})^2$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2}$

10) $x = \sqrt{\ln y}$, $x = 0$, $y = e$, $y = e^4$

$\Rightarrow y = e^{x^2}$

Area $= \int_e^{e^4} (x_2 - x_1)\, dy = \int_e^{e^4} \sqrt{\ln y}\, dy$

$\ln y = t^2 \Rightarrow \frac{1}{y} dy = 2t\,dt \Rightarrow dy = e^{t^2} \cdot 2t\,dt$

Area $= \int_1^2 e^{t^2} \cdot 2t^2\,dt = \int_1^2 t \cdot (e^{t^2} \cdot 2t)\,dt$

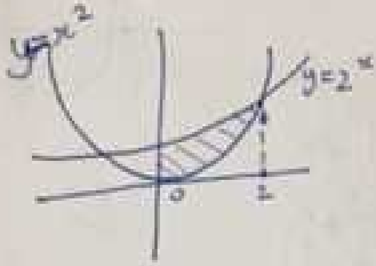
$I_1 = \int e^{t^2} 2t\,dt$, $t^2 = y \Rightarrow 2t\,dt = dy$

$= \int e^y dy = e^y = e^{t^2}$

Area $= t \cdot e^{t^2} \Big|_1^2 - \int_1^2 e^{t^2} dt$

$= 2e^4 - e - \beta$

9)



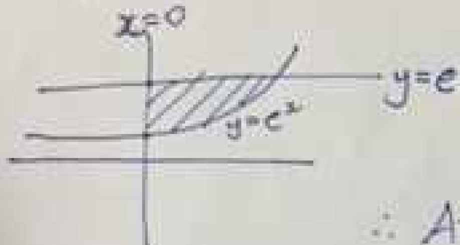
$y = 2^x$, $y = x^2$

$\therefore 2^x = x^2$

$\therefore x = 2$ satisfies the eq's in 1st quadrant.

$\therefore$ Area $= \int_0^2 (2^x - x^2)\,dx$.

7)



$y = e^x$ $\quad x(y-e) = 0$

$\therefore x = \ln y$. $\quad \therefore y = e$, $x = 0$

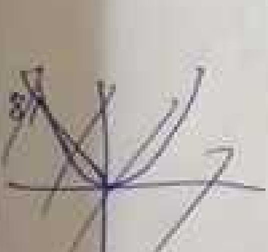
$\therefore$ Area $= \left|\int_1^e \ln y \, dy\right|$.

$= \left|(y\ln y - y)\Big|_1^e\right|$

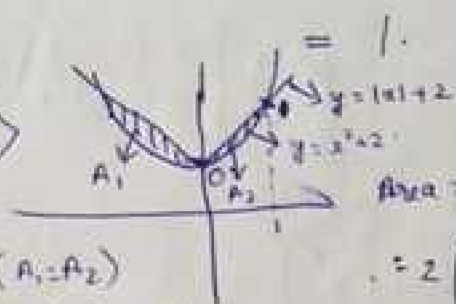
$= [e\ln e - e - (1\ln 1 - 1)]$

$= e - e - 0 + 1$

$= 1$.



8)



$(A_1 = A_2)$

Area(A) = $2A_2$

Area $= 2\left(\int_0^1 (|x|+2) - \int_0^1 (x^2+2)\right)$

$= 2\left(\left[\frac{x^2}{2} + 2x\right]_0^1 - \left[\frac{x^3}{3} + 2x\right]_0^1\right)$

$= 2\left(\frac{1}{2} + \cancel{2} - \frac{1}{3} - \cancel{2}\right)$

$= 1 - \frac{2}{3}$

$= \frac{1}{3}$

2) $y = 2\cos x$, $y = 3\tan x$, y axis.

Area (ABD) = Area ADC − Area DBC

$$= \left|\int_0^{\pi/2} \cos x\right| - \left(\left|\int_0^{\pi/6} \tan x\right| + \left|\int_{\pi/6}^{\pi/2} \cos x\right|\right)$$

$$= 1 - \left(\left|[\ln|\sec x|]_0^{\pi/6}\right| + \left|[\sin x]_{\pi/6}^{\pi/2}\right|\right)$$

$$= 1 - \left(\left|\ln\frac{2}{\sqrt{3}} - \cancel{\ln 1}\right| + \left|1 - \tfrac{1}{2}\right|\right)$$

$$= 1 - \ln\frac{2}{\sqrt{3}} + \frac{1}{2}$$

$$= \frac{3}{2} - \ln\frac{2}{\sqrt{3}}$$

$3\tan x = 2\cos x$

$\frac{3\sin x}{\cos x} = 2\cos x$

$3\sin x = 2 - 2\sin^2 x$

$2\sin^2 x + 3\sin x - 2 = 0$

$\sin x = \frac{1}{2}$ or -2 not poss

$x = \pi/6$

11) $\frac{dy}{dx} = y - \ln|x| + \frac{1}{x}$

$\frac{dy}{dx} + (-y) = -\ln|x| + \frac{1}{x}$

$IF = e^{-x}$

$$e^{-x} y = -\int e^{-x} \ln|x|\, dx + \int e^{-x}\left(\frac{1}{x}\right) dx$$

$$e^{-x} y = -\left[\ln|x|(-e^{-x}) - \cancel{\int e^{-x}\frac{1}{x}}\right] + \cancel{\int e^{-x}\left(\frac{1}{x}\right) dx} + c$$

$$e^{-x} y = +\ln|x|\, e^{-x} + c$$

$$\boxed{y = \ln|x| + c e^{x}}$$

(12) $\frac{1}{y^3}\frac{dy}{dx} + \frac{1}{xy^2} = 1$

$\frac{1}{y^2} = t \Rightarrow -\frac{2}{y^3}dy = dt$

$-\frac{dt}{2dx} + \frac{t}{x} = 1 \Rightarrow \frac{dt}{dx} - \frac{2t}{x} = -2$

$I.F = e^{-\int\frac{2}{x}} = \frac{1}{x^2}$

$\int d\left(t\cdot\frac{1}{x^2}\right) = \int -\frac{2}{x^2}\,dx$

$\frac{t}{x^2} = \frac{2}{x} + K \Rightarrow \frac{1}{y^2x^2} = \frac{2}{x} + K$

$(1,1) \Rightarrow K = -1$

$\frac{1}{y^2x^2} = \frac{2}{x} - 1 \Rightarrow 1 = 2xy^2 - x^2y^2$

(13) $\frac{x}{2b} + \frac{y}{b} = 1 \Rightarrow \boxed{x + 2y = 2b}$

$\frac{dy}{dx} = -\frac{1}{2} \Rightarrow 2y = -x + K$

(13) $(0, y-mx)$ b $2b$ $\left(-\frac{y}{m}+x, 0\right)$

$Y - y = m(X - x)$

$2|y - mx| = \left|-\frac{y}{m} + x\right|$

$2|y-mx| = \left|\frac{-y+mx}{m}\right| \Rightarrow |y-mx|\left(2 - \frac{1}{|m|}\right) = 0$

$\Rightarrow y = mx \Rightarrow \frac{dy}{dx} = \frac{y}{x} \Rightarrow \ln|y| = \ln|x| + K$

$f(1) = 1 \Rightarrow \boxed{y = \pm x}$

... condition given condition

13) By options only (2) satisfies
$x\text{-int} = 3, \quad y\text{-int} = 3/2$

14.) $y = (a\sin x + (b+c)\cos x)e^{x+d}$
$= (a\sin x + K_1\cos x)e^x.e^d$
$= (a.e^d\sin x + K_1 e^d\cos x)e^x$
$= (K_2\sin x + K_3\cos x)e^x$

order $= 2$

15) $x\,dy = y(dx + y\,dy)$
$x = y\frac{dx}{dy} + y^2$
$\frac{dx}{dy} + y = \frac{x}{y} \Rightarrow \frac{dx}{dy} - \frac{x}{y} = -y$
$\frac{y\,dx - x\,dy}{y} = -y\,dy \Rightarrow \frac{y\,dx - x\,dy}{y^2} = -dy$
$\int d(x/y) = -\int dy \Rightarrow \frac{x}{y} = -y + K$
$y(1) = 1 \Rightarrow K = 2$

$\frac{x}{y} = -y + 2$, $y(-3) = ?$
$-3 = -y^2 + 2y \Rightarrow y^2 - 2y - 3 = 0$
$y^2 - 3y + y - 3 = 0 \Rightarrow y = 3, -1$

16)

$Y - y = m(X - x)$

$x = \dfrac{x - \frac{y}{m} + 0}{2} \Rightarrow 2x = x - \frac{y}{m}$

$\Rightarrow x = -\frac{y}{m} \Rightarrow \frac{dy}{dx} = -\frac{y}{x} \Rightarrow \frac{dy}{y} = -\frac{dx}{x}$

$\ln|y| = -\ln|x| + K$

$\ln|xy| = K \quad , (1,1) \Rightarrow K = 0$

$\therefore xy = 1$

17) $\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right) + y = 0$

$\frac{dy}{dx} = \dfrac{x \pm \sqrt{x^2 - 4y}}{2}$

$x^2 - 4y = t^2 \Rightarrow 2x - 4\frac{dy}{dx} = 2t\frac{dt}{dx}$

$\dfrac{2x - 2t\frac{dt}{dx}}{4} = \dfrac{x \pm t}{2}$

$x - t\frac{dt}{dx} = x \pm t$

$dt = \pm dx \Rightarrow t = \pm x + K$

$\Rightarrow \sqrt{x^2 - 4y} = \pm x + K$

$\cancel{x^2} - 4y = \cancel{x^2} \pm 2Kx + K^2$

$\boxed{4y = \pm 2Kx - K^2} \rightarrow \boxed{K = 4 \Rightarrow y = \pm 2x - 4}$

$\cancel{4y' + 2x = y' \Rightarrow -3y' = 2x \Rightarrow y' = -\frac{2}{3}x}$

$\cancel{\therefore m_{\perp} = \frac{3}{2x} \Rightarrow \frac{dy}{dx} = \frac{3}{2x}}$

$\cancel{\Rightarrow \int 2\,dy = 3\int \frac{dx}{x} \Rightarrow 2y = 3\ln|x| + K}$

18.) $2y^2 + x^2 = y + C \Rightarrow 4yy' + 2x = y'$

$\Rightarrow y'(1-4y) = 2x \Rightarrow \frac{dy}{dx} = \frac{2x}{1-4y}$

$\therefore$ for orthogonal trajectory;

$\frac{dy}{dx} = \frac{4y-1}{2x} \Rightarrow \frac{\ln|4y-1|}{4} = \frac{\ln|x|}{2} + K$

$\ln|4y-1| = 2\ln|x| + K$

$\ln\frac{|4y-1|}{x^2} = K \Rightarrow \boxed{4y-1 = Kx^2}$

19.) $(1-e^x)\sec^2 y\,dy + 3e^x \tan y\,dx = 0$

$\tan y = t \Rightarrow \sec^2 y\,dy = dt$; $1-e^x = p \Rightarrow -e^x dx = dp$

$p\,dt - 3t\,dp = 0 \Rightarrow p\,dt = 3t\,dp$

$\int \frac{dt}{t} = \int 3\frac{dp}{p} \Rightarrow \ln|t| = 3\ln|p| + K$

$\frac{t}{p^3} = K \Rightarrow \frac{\tan y}{(1-e^x)^3} = K$

$y(\ln 2) = \frac{3}{4} \Rightarrow \frac{1}{-1} = K \Rightarrow K = -1 \Rightarrow \tan y = (e^x-1)^3$

$y(\ln 3) \Rightarrow \tan y = 2^3 = 8 \Rightarrow \boxed{y = \tan^{-1} 8}$

21.) $f(x) = x + \int_1^x \frac{f(t)}{t}\,dt \Rightarrow f'(x) = 1 + \frac{f(x)}{x}$

$\Rightarrow \frac{dy}{dx} = 1 + \frac{y}{x} \Rightarrow \frac{dy}{dx} - \frac{y}{x} = 1 \Rightarrow \frac{x\,dy - y\,dx}{x^2} = \frac{dx}{x}$

$\Rightarrow \int d\left(\frac{y}{x}\right) = \int \frac{dx}{x} \Rightarrow \frac{y}{x} = \ln|x| + K$

$f(1) = 1 + \int_1^1 \frac{f(t)}{t}\,dt = 1 \Rightarrow K = 1$

$\therefore\ y = x(\ln|x| + 1)$

$f(x) = x(\ln|x| + 1) \Rightarrow f(\sin\theta) = \sin\theta(\ln|\sin\theta| + 1)$

$\cancel{\frac{f(\sin\theta - \theta)}{\sin\theta} = \frac{(\sin\theta - \theta)[\ln|\sin\theta - \theta| + 1]}{\sin\theta}}$

$\cancel{I = \int_0^\pi \ln|\sin\theta - \theta| + 1 - \frac{\theta[\ln|\sin\theta - \theta| + 1]}{\sin\theta}\,d\theta}$

$\cancel{I = \int_0^\pi \ln|\sin\theta - \pi + \theta| + 1 - \frac{(\pi - \theta)[\ln|\sin\theta - \pi + \theta| + 1]}{\sin\theta}\,d\theta}$

$I = \int_0^\pi \frac{f(\sin\theta) - \sin\theta}{\sin\theta}\,d\theta = \int_0^\pi \ln|\sin\theta|\,d\theta$

$I = 2\int_0^{\pi/2} \ln|\sin\theta|\,d\theta = -2 \times \frac{\pi}{2}\ln 2 = -\pi\ln 2$

22.) $$y = ax^2 + 2 \Rightarrow y' = 2ax \Rightarrow 2a = y'/x$$

$$y = \frac{y'}{2\cancel{x}} \times x^{\cancel{2}} + 2 \Rightarrow 2y = y'x + 4$$

$$\Rightarrow y' = \frac{2y-4}{x}$$

$$\because \text{orthogonal} \Rightarrow \frac{dy}{dx} = -\frac{x}{2y-4} = \frac{x}{4-2y}$$

$$\int (4-2y)\,dy = \int x\,dx \Rightarrow 4y - y^2 = \frac{x^2}{2} + K$$

$$(2,2) \Rightarrow 8 - 4 = 2 + K \Rightarrow K = 2$$

$$\therefore 4y - y^2 = \frac{x^2}{2} + 2 \Rightarrow 8y - 2y^2 = x^2 + 4$$

$$2y^2 - 8y + x^2 + 4 = 0$$

$$2[y^2 - 4y] + x^2 + 4 = 0$$

$$2[(y-2)^2 - 4] + x^2 + 4 = 0$$

$$2(y-2)^2 + x^2 = 4$$

$$(y-2)^2 + \frac{x^2}{2} = 2 \Rightarrow \frac{(y-2)^2}{2} + \frac{x^2}{4} = 1$$

23) $\frac{dy}{dx} = \sqrt[3]{y+x} + \sqrt{y+x} - 1$

$x + y = t^6 \Rightarrow 1 + \frac{dy}{dx} = 6t^5 \frac{dt}{dx}$

$\Rightarrow 6t^5 \frac{dt}{dx} - 1 = t^2 + t^3 - 1$

$\Rightarrow 6t^5 \frac{dt}{dx} = t^2(1+t) \Rightarrow 6t^3 \frac{dt}{dx} = 1+t$

$\Rightarrow \frac{6t^3}{(1+t)} dt = dx \Rightarrow \int \frac{t^3+1-1}{1+t} \cdot dt = \int \frac{dx}{6}$

$\int \left[(t^2 - t + 1) - \frac{1}{1+t} \right] \cdot dt = \int \frac{dx}{6}$

$\frac{t^3}{3} - \frac{t^2}{2} + t - \ln|1+t| = \frac{x}{6} + K$

$\frac{\sqrt{x+y}}{3} - \frac{\sqrt[3]{x+y}}{2} + \sqrt[6]{x+y} - \ln|1 + \sqrt[6]{x+y}| = \frac{x}{6} + K$

$(0,0) \Rightarrow K = 0$

$\because x + y = 64 \Rightarrow \frac{8}{3} - \cancel{\frac{4}{2}} + \cancel{2} - \ln|1+2| = \frac{x}{6}$

$x = 6\left(\frac{8}{3} - \ln 3\right) = 16 - 6\ln 3$

24) Problem in question.

25) $\frac{dy}{dx} = e^{x}y + p(x).$

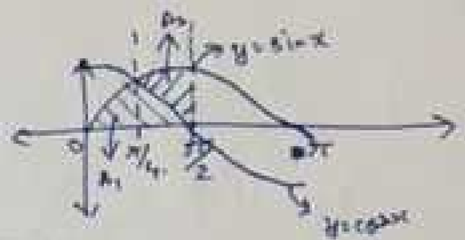
$\frac{dy}{dx} + (-e^{x})y = p(x)$

$I.F = e^{-e^{x}}.$

so, $e^{-e^{x}}y = p(x)\left(e^{-e^{x}}\right) + \lambda$

$y = p(x) + \lambda^{e e^{x}}$

27)



$$\therefore \frac{A_1}{A_2} = \frac{\int_0^{\pi/4} \sin x\,dx \int_{\pi/4}^{\pi/2} \cos x\,dx}{\int_{\pi/4}^{\pi/2} (\sin x - \cos x)\,dx}$$

$$= \frac{-\cos x\Big|_0^{\pi/4} + \sin x\Big|_{\pi/4}^{\pi/2}}{-\cos x\Big|_{\pi/4}^{\pi/2} - \sin x\Big|_{\pi/4}^{\pi/2}}$$

$$= \frac{\frac{-1}{\sqrt{2}} + 1 - \frac{1}{\sqrt{2}}}{-0 + \frac{1}{\sqrt{2}} - \left(1 + \frac{1}{\sqrt{2}}\right)} = \frac{|1-\sqrt{2}|}{|-1+\sqrt{2}|}$$

$= 1:1.$

26) $\frac{dy}{dx} = \frac{1}{2}\left(\frac{y}{x} + \frac{1}{y}\sec\left(\frac{y^2}{x}\right)\right)$

$2\frac{dy}{dx} - \frac{y}{x} = \frac{1}{y}\sec\left(\frac{y^2}{x}\right)$

$2y\frac{dy}{dx} - \frac{y^2}{x} = \sec\left(\frac{y^2}{x}\right)$; $y^2 = t \Rightarrow 2y\,dy = dt$

$\frac{dt}{dx} - \frac{t}{x} = \sec\left(\frac{t}{x}\right)$; $t = px$

$\frac{dt}{dx} = p + x\frac{dp}{dx}$

$\therefore \cancel{p} + x\frac{dp}{dx} - \cancel{p} = \sec p$

$\frac{dp}{\sec p} = \frac{dx}{x} \Rightarrow \int \cos p\, dp = \int \frac{dx}{x}$

$\sin p = \ln|x| + K$

$\Rightarrow \sin\left(\frac{t}{x}\right) = \ln|x| + K \Rightarrow \sin\left(\frac{y^2}{x}\right) = \ln|x| + K$

$(1,0) \Rightarrow K = 0 \Rightarrow \sin\left(\frac{y^2}{x}\right) = \ln|x|$

$\int_1^{e^2} \sin\left(\frac{y^2}{x}\right) \cdot dx = \int_1^{e^2} \ln x\, dx = \ln x \cdot x \Big|_1^{e^2} - \int_1^{e^2} dx$

$= 2e^2 - (e^2 - 1) \Rightarrow e^2 + 1$

Q 29 & Q 30 are same.

30) between At $x=1$ and $x=2$, x^3 is above x^2.

$\therefore$ Area $= \int_1^2 (x^3 - x^2)\,dx$

$= \left(\frac{x^4}{4} - \frac{x^3}{3}\right)\Big|_1^2$

$= \frac{2^4}{4} - \frac{1}{4} + \frac{1}{3} - \frac{2^3}{3}$

$= 4 - \frac{1}{4} + \frac{1}{3} - \frac{8}{3}$

$= \frac{17}{12}$.

28) $2x = y^2 - 1$

At $x = 0$, $y = \pm 1$.

$\therefore x = \frac{y^2}{2} - \frac{1}{2}$

$\therefore$ ~~x~~ Area $= \left|\int_{-1}^{+1} \left(\frac{y^2}{2} - \frac{1}{2}\right) dx\right|$

$= \left(\frac{y^3}{3} - \frac{1}{2}x\right)\Big|_{-1}^{+1}$

$= \left|\frac{1}{6} + \frac{1}{6} - \frac{1}{2} - \frac{1}{2}\right|$

$= \frac{2}{3}$.

20) $\frac{(x-\alpha)^2}{a^2} + \frac{(y-\beta)^2}{b^2} = 1$

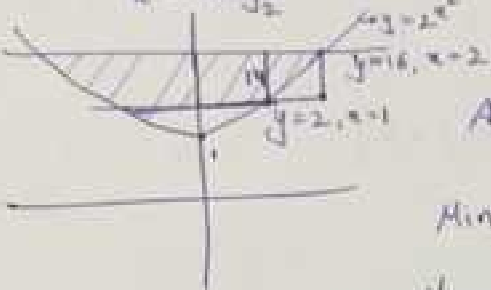
$\therefore$ as the no. of arbitary constants are 4,
the order of D.E is 4.

MCQ - Multicorrect

1)

$y = 2^{x^2}$, $y^2 - 18y + 32 = 0 \Rightarrow y^2 - 16y - 2y + 32 = 0$

$x^2 = \log_2 y$ $\qquad \Rightarrow y = 16, 2$



$\text{Area} = \int_2^{16} x\, dy = \int_2^{16} \sqrt{\log_2 y}\, dy$

Min Area $= 2 \times 14 = 28$

Max Area $= 4 \times 14 = 56$

2) $A_n = \int_0^{\pi/4} (\tan x)^n dx$, $A_{n+2} = \int_0^{\pi/4} (\tan x)^{n+2} \cdot dx$

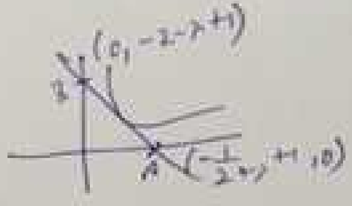
$A_n + A_{n+2} = \int_0^{\pi/4} \tan^n x \cdot \sec^2 x \, dx$

$\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$A_n + A_{n+2} = \int_0^1 t^n dt = \frac{t^{n+1}}{n+1}\Big|_0^1 = \frac{1}{n+1}$

$A_1 = \int_0^{\pi/4} \tan x\, dx = \ln|\cos x|\Big|_0^{\pi/4} = \ln\left(\frac{1}{\sqrt{2}}\right)$

3.)

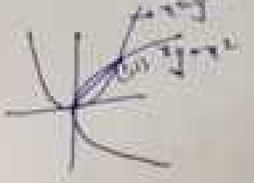


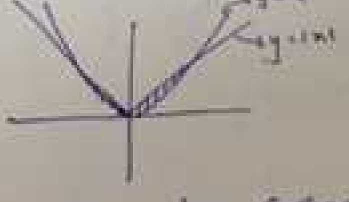
$f(1) = 1 \Rightarrow 1 = 1 + \lambda - \lambda$

$f'(x) = 2x + \lambda \Rightarrow f'(1) = 2 + \lambda$

$y - 1 = (2+\lambda)(x-1)$

4)





Area enclosed between two curves are identical.

5) $y = x - x^2$, $y = mx$

$mx = x - x^2 \Rightarrow x = 0$, $x = 1 - m$

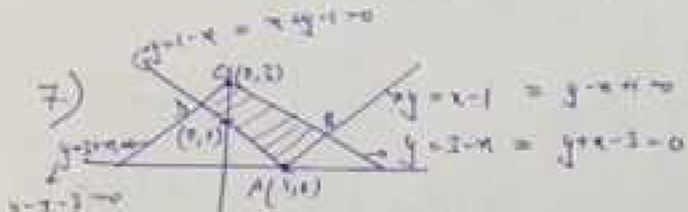
$$\text{Area} = \left| \int_0^{1-m} (x - x^2 - mx)\,dx \right| = \frac{9}{2}$$

6.) 

$$\text{Area} = \int_1^3 \ln x \, dx$$

$$= \left[\ln x \cdot x - x\right]_1^3$$

$$= 3\ln 3 - 3 + 1 = 3\ln 3 - 2$$

7.) 

Area = Area of Rectangle ABCD

$AB = \frac{2}{\sqrt{2}} = \sqrt{2}$, $AD = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

Area = 4

8) 

$\frac{BP}{AP} = \frac{3}{1}$

$y - y_1 = m(x - x_1)$

$$x_1 = \frac{3\left(x_1 - \frac{y_1}{m}\right)}{4} \Rightarrow 4x_1 = 3x_1 - \frac{4y_1}{m}$$

$$\Rightarrow x = -\frac{y}{m} \times 4 \Rightarrow \frac{dy}{dx} = -\frac{4y}{x}$$

$$\Rightarrow \ln|y| = -4\ln|x| + K$$

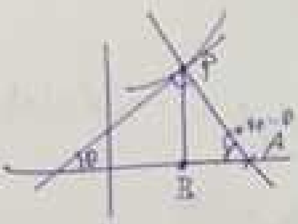
9.) $\int \frac{2y\,dy}{2y^2\sqrt{y^2-1}} = \int \frac{2x\,dx}{2x^2\sqrt{x^2-1}}$; $y^2-1 = u \Rightarrow 2y\,dy = du$

$x^2-1 = v \Rightarrow 2x\,dx = dv$

$\Rightarrow \int \frac{du}{(u+1)u} = \int \frac{dv}{(v+1)v} \Rightarrow \ln\left|\frac{u}{u+1}\right| = \ln\left|\frac{v}{v+1}\right| + K$

$\Rightarrow \ln\left|\frac{u(v+1)}{v(u+1)}\right| = K \Rightarrow \ln\left|\frac{(y^2-1)x^2}{(x^2-1)y^2}\right| = K$

10.)



$\tan(90-\theta) = \frac{BP}{AB}$

$\cot\theta = \frac{|K|}{\text{length of s.n.}}$

length of s.n $= |K|\ m = 2yK^2$

$\frac{dy}{dx} = \pm 2y \Rightarrow \ln|y| = \pm 2x + K$

$f(0) = 1 \Rightarrow K = 0$

$|y| = e^{\pm 2x}$; $y = e^{2x}$; $y = e^{-2x}$ is decreasing hence rejected

11) $y = f(x) \Rightarrow \frac{dy}{dx} = f'(x) \Rightarrow \frac{dx}{dy} = \frac{1}{f'(x)} \Rightarrow \frac{d^2x}{dy^2} = \frac{-1}{(f'(x))^2} f''(x)\frac{dx}{dy}$

$\Rightarrow \frac{d^2x}{dy^2} = \frac{-f''(x)}{(f'(x))^3}$

$\frac{d^2y}{dx^2} = \frac{x^3}{y^3}\left(\frac{-f''(x)}{(f'(x))^3}\right) \Rightarrow f'(x) = -\frac{x}{y} \Rightarrow y^2 + x^2 = K$

$x = 0, y = 2 \Rightarrow \boxed{x^2 + y^2 = 4}$ Domain $\in [-2, 2]$, Range $\in [0, 2]$

& $\frac{d^2y}{dx^2} + \frac{8y^3}{x^3}\frac{d^2x}{dy^2} = 0 \Rightarrow \frac{d^2y}{dx^2} + \frac{8y^3}{x^3}\left(\frac{-y''}{(y')^3}\right) = 0$

$+\frac{8y^3}{x^3(y')^3} = 1 \Rightarrow y' = +\frac{2y}{x}$

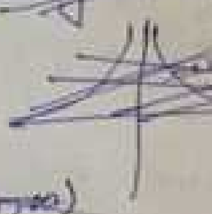
$\Rightarrow \frac{dy}{y} = +\frac{2}{x}dx \Rightarrow \ln|y| = +2\ln|x| + K$

$x = 1, y = \frac{1}{3} \Rightarrow -\ln 3 = K$

$\cancel{\Rightarrow \ln|y| = -\ln 3x^2}$

$\ln|y| = +\ln x^2 - \ln 3$

$\cancel{\Rightarrow \boxed{|y| = \frac{1}{-3x^2}}}$

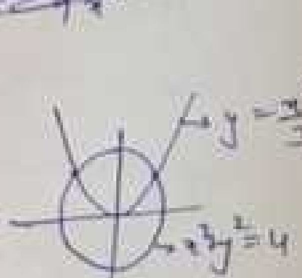


~~Domain $\in R - \{0\}$, Range $\in (0, \infty)$~~

$\cancel{x^2 + x^4}$

$|y| = x^2/3 \Rightarrow y = x^2/3$

Domain $\in R$, Range $\in [0, \infty)$



$x^2 + \frac{x^4}{9} = 4$

$x^4 + 9x^2 - 36 = 0$

$x^4 + 12x^2 - 3x^2 - 36 = 0$

$x^2 = 3 \Rightarrow x = \pm\sqrt{3}$

$\therefore$ Common domain $\in [-\sqrt{3}, \sqrt{3}]$, Range $\in [0, 2]$

12) $y = e^{-x}\sin x \Rightarrow ye^{x} = \sin x$

$\Rightarrow y'e^{x} + ye^{x} = \cos x$

$\Rightarrow y''e^{x} + y'e^{x} + y'e^{x} + ye^{x} = -\sin x$

$\Rightarrow y'' + 2y' + y = -e^{-x}\sin x \Rightarrow y'' + 2y' + 2y = 0$

$\Rightarrow y''' + 2y'' + 2y' = 0$

$\Rightarrow y_n + 2y_{n-1} + 2y_{n-2} = 0$

12) $y = e^{-x}\sin x$

$y' = e^{-x}\cos x - e^{-x}\sin x$

$y'' = -e^{-x}\sin x - e^{-x}\cos x + e^{-x}\sin x - e^{-x}\cos x$

$y'' = -2e^{-x}\cos x \Rightarrow y''' = +2e^{-x}\sin x + 2e^{-x}\cos x$

$\Rightarrow y'''' = 2e^{-x}\cos x - 2e^{-x}\sin x - 2e^{-x}\sin x - 2e^{-x}\cos x$

$\Rightarrow y'''' = -4e^{-x}\sin x \Rightarrow y_4 = -4y$

$\Rightarrow y_4 + 4y = 0 \Rightarrow a_4 = 4$

13) $y = c_1e^{x+c_2} + c_2e^{-x+c_3} + c_1e^{-x-c_3}$

$y = c_1e^{c_2}.e^{x} + c_2.e^{c_3}.e^{-x} + c_1e^{-c_3}.e^{-x}$

$y = k_1e^{x} + k_2e^{-x}$

order $= 2$, degree $= 1$

14.) $\int_0^x f(x)\,dx = K(f(x))^3$

$\Rightarrow f(x) = K3f^2(x)\cdot f'(x)$

$\Rightarrow f(x) = 0$ or $\frac{3\,dy}{dx} = \frac{1}{Ky}$ $\Rightarrow K3y\,dy = dx$

$\Rightarrow K.\frac{3y^2}{2} = x + K'$ $\Rightarrow f(0) = 0$ $\Rightarrow K' = 0$

$\cancel{3y^2 = 2x}$ $\quad f(1) = 3 \Rightarrow K\times\frac{27}{2} = 1$

$\cancel{y = \frac{2x}{3}}$ $\quad \Rightarrow K = 2/27$

$\frac{\cancel{2}}{\cancel{27}_{9}}\times\frac{\cancel{3}}{\cancel{2}}y^2 = x \Rightarrow y^2 = 9x$

$\Rightarrow y = \pm 3\sqrt{x}$

b) $f(\ln(2^x-3)) = 3\sqrt{\ln(2^x-3)}$

$2^x - 3 > 0$, $\ln(2^x-3) \geq 0$

$x > \log_2^3$, $2^x - 3 \geq 1 \Rightarrow x \geq 2$

c) $f(\sec^2 x) = 3|\sec x|$

Range $\in [3, \infty)$

d) Area $= \int_1^2 x\,dy = \int_1^2 \frac{y^2}{9}\,dy = \frac{y^3}{27}\Big|_1^2$

$= \frac{7}{27}$

a) $f(x) = 3\sqrt{\sin^2 x} = 3|\sin x|$

Range $\in [0, 3]$

Paragraph - 1

1) $f(x) = x^n \tan^{-1} x$

$$A_n = \int_0^1 x^n \tan^{-1} x \, dx$$

$$A_{n+2} = \int_0^1 x^{n+2} \tan^{-1} x \, dx$$

$$(n+1) A_n + (n+3) A_{n+2} \cancel{= \int_0^1 x^n \tan^{-1} x \left[(n+1) + (n+3) x^2\right] dx}$$

$$\Rightarrow (n+1) \int_0^1 x^n \tan^{-1} x \, dx + \int_0^1 (n+3) x^{n+2} \tan^{-1} x \, dx$$

$$\Rightarrow (n+1) \tan^{-1} x \cdot \frac{x^{n+1}}{n+1} \Big|_0^1 - \int_0^1 \frac{1}{(1+x^2)} \cdot x^{n+1} dx + \tan^{-1} x \, x^{n+3} \Big|_0^1$$

$$- \int_0^1 \frac{1}{1+x^2} \cdot x^{n+3} dx$$

$$\Rightarrow \frac{\pi}{4} + \frac{\pi}{4} - \int_0^1 x^{n+1} dx \Rightarrow \frac{\pi}{2} - \frac{1}{n+2}$$

2) $\sum_{k=1}^{4} (k+1) A_k = 2A_1 + 3A_2 + 4A_3 + 5A_4$

$\because (n+1) A_n + (n+3) A_{n+2} = \frac{\pi}{2} - \frac{1}{n+2}$

$2A_1 + 4A_3 = \frac{\pi}{2} - \frac{1}{3}$ $\rightarrow$ By $n = 1$

$3A_2 + 5A_4 = \frac{\pi}{2} - \frac{1}{4}$ $\rightarrow$ By $n = 2$

$LHS = \pi - \frac{7}{12}$

3) $A_4 = \int_0^1 x^4 \tan^{-1}x \, dx = \tan^{-1}x \cdot \frac{x^5}{5}\Big|_0^1 - \int_0^1 \frac{x^5}{5(1+x^2)} dx$

$A_4 = \frac{\pi}{20} - \frac{1}{5}\int_0^1 \frac{x^5}{1+x^2} dx$; Let; $1+x^2 = t$
$2x\,dx = dt$

$A_4 = \frac{\pi}{20} - \frac{1}{5}\times\frac{1}{2}\int_1^2 \frac{(t-1)^2}{t} dt = \frac{\pi}{20} - \frac{1}{10}\int_1^2 \left(t - 2 + \frac{1}{t}\right) dt$

$A_4 = \frac{\pi}{20} - \frac{1}{10}\left[\frac{t^2}{2} - 2t + \ln t\right]_1^2$

$= \frac{\pi}{20} - \frac{1}{10}\left[\cancel{2} - 4 + \ln 2 - \frac{1}{2} + \cancel{2}\right]$

$= \frac{\pi}{20} - \frac{\ln 2}{10} + \frac{1}{20} = \frac{\pi - \ln 4 + 1}{20}$

Paragraph - 2

$(1+x^2)\frac{dy}{dx} + 2yx = 4x^2$

$\Rightarrow \frac{dy}{dx} + \frac{2yx}{1+x^2} = \frac{4x^2}{1+x^2}$

$\text{I.F} = e^{\int \frac{2x}{1+x^2} dx} = 1+x^2$

$\Rightarrow \int d\left(y(1+x^2)\right) = \int 4x^2 dx$

$\Rightarrow y(1+x^2) = \frac{4}{3}x^3 + C$, Origin $(0,0)$
$\Rightarrow C = 0$

$\Rightarrow y = \frac{4x^3}{3(1+x^2)}$

4.) $f(x) = \dfrac{4x^3}{3(1+x^4)} \Rightarrow \dfrac{3}{4} f'(x) = \dfrac{(1+x^2)3x^2 - 2x^4}{(1+x^2)^2}$

$\dfrac{3}{4} f'(x) = \dfrac{3x^2 + x^4}{(1+x^2)^2}$

$\therefore$ $f(x)$ is strictly increasing.

5.) $\text{Area} = \displaystyle\int_0^{2/3} \frac{4x^3}{3(1+x^4)} dx$

$1 + x^2 = t \Rightarrow 2x\,dx = dt$

$\text{Area} = \displaystyle\int_1^{13/9} \frac{2(t-1)\,dt}{3t}$

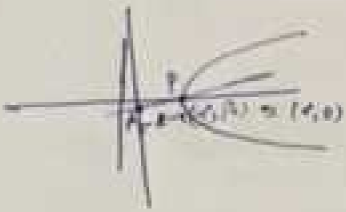
$\dfrac{3}{2} \text{Area} = \displaystyle\int_1^{13/9} dt - \int_1^{13/9} \frac{1}{t} dt$

$= \dfrac{4}{9} - \ln 13/9$

$\text{Area} = \dfrac{8}{27} - \dfrac{2}{3} \ln 13/9$

Matrix - Match

2.) A)



Axis : $y-\beta = m(x-\alpha)$

$y = m(x-\alpha) \Rightarrow \boxed{y - mx + \alpha m = 0}$

$A(\alpha + a\cos\theta,\ 0 + a\sin\theta)$

where , $m = \tan\theta$

Tangent at vertex :

$y - 0 = -\frac{1}{m}(x-\alpha)$

$-my = x - \alpha \Rightarrow my + x - \alpha = 0$

$\therefore$ Directrix : $y - a\sin\theta = -\frac{1}{m}(x - \alpha - a\cos\theta)$

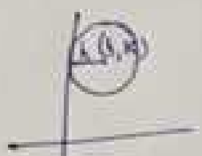
$\Rightarrow my - am\sin\theta = -x + \alpha + a\cos\theta$

$\Rightarrow \boxed{my + x - am\sin\theta - \alpha - a\cos\theta = 0}$

Eq^n of Parabola : $(\text{Distance from Axis})^2 = (\text{length of latus rectum}) \times (\text{Distance from Tangent at vertex})$

$\therefore$ Eq^n of Parabola has two arbitrary constants a and m ; hence order = 2

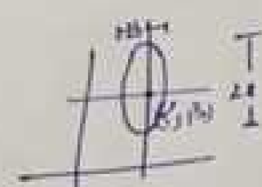
B)



$(x-\alpha)^2 + (y-\alpha)^2 = \alpha^2$

$\because$ two arbitrary ~~constants~~ Constants hence degree = 2

c.)



$\frac{(x-\alpha)^2}{b^2} + \frac{(y-\beta)^2}{a^2} = 1$

$\therefore$ 4 arbitrary Constants ; hence degree = 4

D) $(x-\alpha)^2 + (y-\beta)^2 = a^2$

$\because$ 3 arbitrary Constants ; hence degree = 3

Matrix

1) B) $f(x) = x(\sin x+\cos x)^2 = x(1+\sin 2x) = x + x\sin 2x$

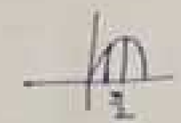
$$\int_0^1 f(x)\,dx = \int_0^1 (x + x\sin 2x)\,dx = \frac{x^2}{2}\Big|_0^1 + \int_0^1 x\sin 2x\,dx$$

$$A = \frac{1}{2} + \left[-x\frac{\cos 2x}{2}\right]_0^1 + \int_0^1 \frac{\cos 2x}{2}\,dx$$

$$= \frac{1}{2} - \frac{\cos 2}{2} + \frac{\sin 2x}{4}\Big|_0^1$$

$$= \frac{1}{2} - \frac{\cos 2}{2} + \frac{\sin 2}{4} = \sin^2 1 + \frac{\sin 1\cos 1}{2}$$

$$A = \sin^2 1 + \frac{\sin 2}{4}$$



$\cancel{f(x) = 2x\sin^2(x+\frac{\pi}{4})}$ $\cancel{\Rightarrow f'(x) = 2\sin^2(x+\frac{\pi}{4}) + 4x\sin(x+\frac{\pi}{4})\cos(x+\frac{\pi}{4})}$

$\cancel{2\sin(x+\frac{\pi}{4})\left[\sin(x+\frac{\pi}{4}) + 2x\cos(x+\frac{\pi}{4})\right]}$ →

$\cancel{0 \le f(x) \le}$

$$A\,(\text{approx}) = \frac{3}{4} + \frac{\sqrt{3}}{8} = \frac{6+\sqrt{3}}{8}$$

c) $f(x) = \frac{1}{\sqrt{4-x^2-x^2}} < \frac{1}{\sqrt{3-x^2}}$

$$\int_0^1 \frac{1}{\sqrt{4-x^2}}\,dx < A < \int_0^1 \frac{1}{\sqrt{3-x^2}}\,dx = \frac{1}{\sqrt{3}}\sin^{-1}\frac{x}{\sqrt{3}}\Big|_0^1$$

$$\frac{1}{2}\sin^{-1}\frac{x}{2}\Big|_0^1 < A < \frac{1}{\sqrt{3}}\sin^{-1}\frac{1}{\sqrt{3}}$$

$$\frac{1}{2}\sin^{-1}\frac{1}{2} < A < \frac{1}{\sqrt{3}}\sin^{-1}\frac{1}{\sqrt{3}}$$

$$\frac{\pi}{6} < A < \frac{1}{\sqrt{3}}\sin^{-1}\frac{1}{\sqrt{3}}$$

D) $f(x) = \frac{1}{\sqrt{x^6+1}} \quad \forall x \in [0,1]$

$f(x) > \frac{1}{\sqrt{x^2+1}}$

$A > \int_0^1 \frac{1}{\sqrt{x^2+1}} \cdot dx = \ln(1+\sqrt{2})$

$f(x) < \frac{1}{\sqrt{1-x^2}} \Rightarrow A < \int_0^1 \frac{1}{\sqrt{1-x^2}} = \frac{\pi}{2}$

A) $f(x) = \sqrt{x^3+2} \quad \forall x \in [0,1]$

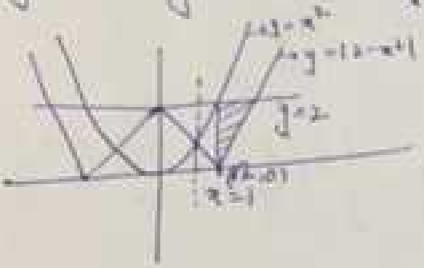
$\sqrt{2} \le f(x) \le \sqrt{3}$

$\therefore \sqrt{2} \le A \le \sqrt{3}$

-egce type

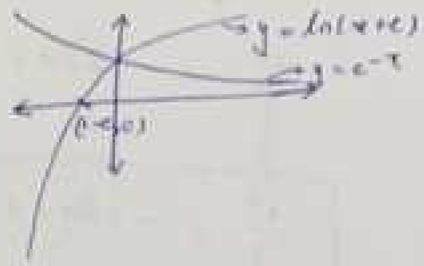
1) $y = x^2$, $y = |2-x^2|$, $y = 2$

$$y = |2-x^2| = \begin{cases} 2-x^2 ; & 1 \le x \le \sqrt{2} \\ x^2-2 , & x > \sqrt{2} \end{cases}$$



$$A = \int_1^{\sqrt{2}} \left(x^2 - (2-x^2)\right) dx + \text{Area of shaded triangle}$$

2) $y = \ln(x+e)$, $y = e^{-x}$



$$\text{Area} = \int_{1-e}^{0} \ln(x+e)\, dx + \int_0^{\infty} e^{-x}\, dx$$

3) $y = \frac{|x|}{x} = \begin{cases} 1 & ; x > 0 \\ -1 & ; x < 0 \end{cases}$, $y = 0$, $x = 0$, $x = 3$

Area $= 1 \times 3 = 3$

4) 

$y - y_1 = -\frac{1}{m}(x - x_1)$; $y = 0 \Rightarrow my_1 + x_1 = x$

$A(my_1 + x_1, 0)$, $B\left(0, y_1 + \frac{x_1}{m}\right)$

$$\frac{1}{my_1 + x_1} + \frac{1 \cdot m}{my_1 + x_1} = 1 \Rightarrow 1 + m = my + x$$

$$\Rightarrow m(y-1) = 1 - x \Rightarrow \frac{dy}{dx} = \frac{1-x}{y-1}$$

$$\frac{y^2}{2} - y = x - \frac{x^2}{2} + K$$

5) $\frac{dx}{dy} = \frac{x^2 - 2y - 1}{2xy}$; Let $x^2 = t \Rightarrow 2x\,dx = dt$

$$\frac{dt}{dy} = \frac{t - 2y - 1}{y} \Rightarrow \frac{dt}{dy} - \frac{t}{y} = -\frac{(2y+1)}{y}$$

$$\frac{y\,dt - t\,dy}{y^2} = -\frac{(2y+1)}{y^2}dy \Rightarrow \int d\left(\frac{t}{y}\right) = -\int\left(\frac{2}{y} + \frac{1}{y^2}\right)dy$$

$$\frac{t}{y} = -2\ln y + \frac{1}{y} + K$$

$$\boxed{\frac{x^2}{y} = -2\ln y + \frac{1}{y} + K}$$

6) $\cos y \frac{dy}{dx} = \frac{\cos x \sin y + \tan^2 x}{\sin x}$; $\sin y = t \Rightarrow \cos y\, dy = dt$

$$\frac{dt}{dx} = \frac{\cos x\, t + \tan^2 x}{\sin x} \Rightarrow \frac{dt}{dx} - t\cot x = \frac{\tan^2 x}{\sin x}$$

$$\text{I.F.} = e^{-\int \cot x\, dx} = e^{-\ln \sin x} = \frac{1}{\sin x}$$

$$\int d\left(t \times \frac{1}{\sin x}\right) = \int \frac{1}{\cos^2 x} \cdot dx$$

$$\frac{t}{\sin x} = \tan x + K \Rightarrow \frac{\sin y}{\sin x} = \tan x + K$$

7) $\frac{dy}{dx} = \frac{(x+y-2)^2 - (y-1)^2}{(x+3)^2} = \frac{(x-1)(x+2y-3)}{(x+3)^2}$

$y-1 = v, \quad x+3 = u$

$dy = dv \;; \quad dx = du$

$\frac{dv}{du} = \frac{(u+v)^2 - v^2}{u^2} = \frac{u^2 + 2uv}{u^2} = 1 + \frac{2v}{u}$

$v = ut \rightarrow \frac{dv}{du} = t + u\frac{dt}{du}$

$t + u\frac{dt}{du} = 1 + 2t \Rightarrow u\frac{dt}{du} = 1 + t$

$\cancel{\ln t} \; \ln|1+t| = \ln|u| + K$

$\ln\left|1 + \frac{v}{u}\right| = \ln|u| + K$

$\ln\left|1 + \frac{y-1}{x+3}\right| = \ln|x+3| + K$

$K = -\ln 3$

$f(3) = \cancel{\ln}\,? \Rightarrow \ln\left|1 + \frac{y-1}{6}\right| = \ln|6| - \ln 3$

$\Rightarrow 1 + \frac{y-1}{6} = 2 \Rightarrow y-1 = 6$

$\rightarrow \boxed{y = 7}$

8) $\frac{dy}{x \cdot 2dx} + \frac{y}{x^2} = \frac{\ln^{-1}x}{2x^2} \Rightarrow \frac{dy}{dx} + \frac{2y}{x} = \frac{\ln^{-1}x}{x}$

$I.F = e^{\int \frac{2}{x}dx} = x^2$

$\int d(y \cdot x^2) = \int x \cdot \ln^{-1}x \, dx$

9) $y' < 2y \Rightarrow$

$$y' < 2y$$

$$y'e^{-2x} - 2y \cdot e^{-2x} < 0$$

$$\Rightarrow d(y \cdot e^{-2x}) < 0$$

$\therefore$ $y \cdot e^{-2x}$ is decreasing function $\forall x \in [\frac{1}{2}, 1]$

Let ; $g(x) = y \cdot e^{-2x}$

$$g(\tfrac{1}{2}) = 2e \times e^{-1} = 2$$

$$g(\ln 2) = y \cdot e^{-2\ln 2} = \frac{y}{4}$$

$$g(\tfrac{1}{2}) \geq g(\ln 2) \Rightarrow 2 \geq \frac{y}{4} \Rightarrow y \leq 8$$

$\therefore$ Max. value $= 8$

10) $c = \dfrac{x^2 - y^2}{x^2 + y^2} \Rightarrow 0 = \dfrac{(x^2+y^2)(2x - 2yy') - (x^2-y^2)(2x+2yy')}{(x^2+y^2)^2}$

$$\Rightarrow (x^2+y^2)(x - yy') = (x^2-y^2)(x + yy')$$

$$\cancel{x^3} - x^2yy' + y^2x - \cancel{y^3y'} = \cancel{x^3} + x^2yy' - y^2x - \cancel{y^3y'}$$

$$y^2x = x^2yy' \Rightarrow y' = \frac{y}{x}$$

$$\frac{y}{x} = \frac{x(xy^2 - x^2)}{3x^2 + xy^2} \Rightarrow 3x^2y + xy^3 = x \cdot x^2y^2 - x^4$$