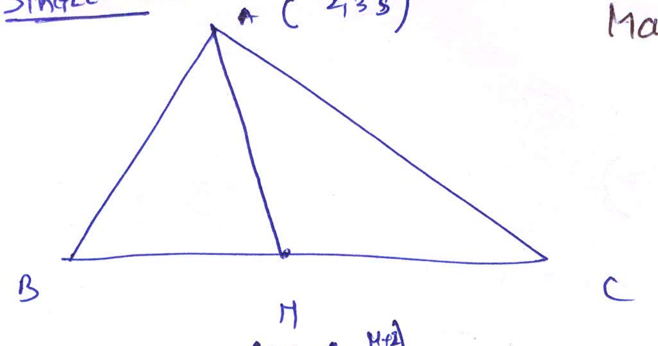
Single choice

Math-Module 20

(1)



$\left(\frac{\lambda-1}{2}, 4, \frac{\mu+2}{2}\right)$

$\overrightarrow{AM} = \left(\frac{\lambda-5}{2}, 1, \frac{\mu-8}{2}\right)$

so $\frac{\lambda-5}{2} = 1 = \frac{\mu-8}{2}$ $\Rightarrow$ $\boxed{\lambda = 7, \mu = 10}$

(2) $\vec{r} = \vec{a} + \lambda\vec{b}$, where $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 7\hat{j} - 3\hat{k}$.

(3) $\frac{x-1}{a} = \frac{y-1}{b} = \frac{z-1}{c}$ so all Three are coplanar.

i.e $\begin{vmatrix} a & b & c \\ 2 & 3 & 4 \\ 1 & 2 & 4 \end{vmatrix} = 0$

$a(4) - b(4) + c(1) = 0$

$4a - 4b + c = 0$ — (1)

Also $(1, 1, 1), (1, 2, 3)$ $(-2, +3, -3)$ $a\hat{i} + b\hat{j} + c\hat{k}$ are coplanar.

$\begin{vmatrix} a & b & c \\ 0 & 1 & 2 \\ -3 & 2 & -4 \end{vmatrix} = 0$

$-8a - b(6) + c(3) = 0$ — (2)

on solving (1) & (2) we get a, b, c ratio.

(4) $\vec{b} = \vec{n}_1 \times \vec{n}_2 = -3\hat{i} + 5\hat{j} + 4\hat{k}$.

put $z = 0$ & solve for x & y

$(x, y, z) = \left(\frac{11}{4}, -\frac{9}{4}, 0\right)$

$\frac{x - \frac{11}{4}}{-3} = \frac{y + \frac{9}{4}}{5} = \frac{z-0}{4}$.

(5) $\begin{vmatrix} x+1 & y-1 & z-1 \\ 2 & -2 & 0 \\ 1 & 2 & 2 \end{vmatrix} = 0$

Hint
$(A, B, \vec{b})$
$[\vec{PA}, \vec{AB}, \vec{b}] = 0$.

(6) $A = (-1, 3, -2)$

$\vec{b} = -3\hat{i} + 2\hat{j} + \hat{k}$

$B = (0, 7, -7)$

$[\vec{PA}, \vec{AB}, \vec{b}] = 0$.

(7) $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} - 2\hat{k}) + \mu(-\hat{i} + 0\hat{j} + 2\hat{k})$.

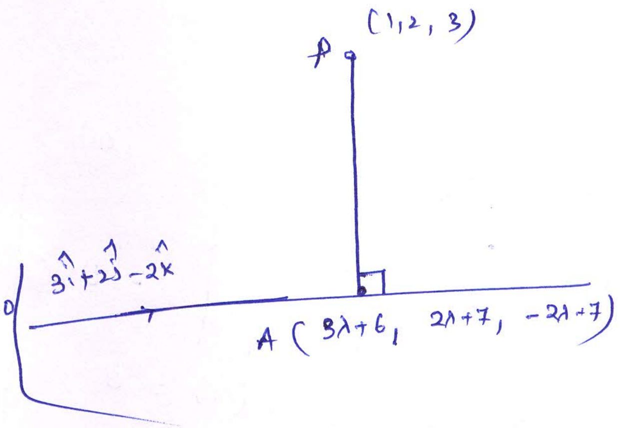
$P = (x, y, z)$

$A = (1, 2, 3)$

$\vec{b} = \hat{i} - \hat{j} - 2\hat{k}$

$\vec{c} = -\hat{i} + 0\hat{j} + 2\hat{k}$

$[\vec{AP}, \vec{b}, \vec{c}] = 0$.



(8)

$(3\lambda+5)3 + (2\lambda+5)2 + (-2\lambda+4)(-2) = 0$

$\boxed{\lambda = -1}$

$A = (3, 5, 9)$

$AP = \sqrt{4+9+36} = \sqrt{49} = 7$.

(9) $\vec{r} \cdot (\vec{a_2} \times \vec{a_1}) = a_1 \cdot (\vec{a_2} \times \vec{a_1})$

$[\vec{r}\ \ \vec{a_1}\ \ \vec{a_2}] = 0$.

(10) $\vec{r} = \vec{a_1} + \mu\vec{b}$, $\quad \vec{r} = \vec{a_2} + k\vec{b}$.

$[\vec{r} - \vec{a_1},\ \vec{a_1} - \vec{a_2}\ \ \vec{b}] = 0$.

$[\vec{r}\ \ \vec{a_1} - \vec{a_2}\ \ \vec{b}] = [\vec{a_1}\ \ \vec{a_1} - \vec{a_2}\ \ \vec{b}] = [\vec{a_1}\ \ \vec{a_1}\ \ \vec{b}] - [\vec{a_1}\ \ \vec{a_2}\ \ \vec{b}]$

$[\vec{r}\ \ \vec{a_1} - \vec{a_2}\ \ \vec{b}] = -[\vec{a_1}\ \ \vec{a_2}\ \ \vec{b}]$