Kinematics 1D And 2D

1. Speed V = 2 t

   Tangential acceleration $a_t = \frac{dv}{dt} = 2\,m/s^2$

   Radial acceleration $a_r = \frac{(2t)^2}{t} = 4t^2\ m/s^2$

   At $t = \frac{1}{\sqrt{2}}$, $a_t = 2m/s^2$ $a_t = a_r$

   $\vec{a}.\vec{v} = (-a_r\hat{e}_r + a_t\hat{e}_t).v\hat{e}_t = a_t v = 4t$

2. $F = \alpha t$

   $a = \frac{\alpha}{m}$

   $v = \frac{\alpha t^2}{2m}$

3. $\vec{V} = \frac{\int_0^5 V dt}{\int_0^4 V\,dt} = \frac{5}{3}\,m/s$

   $V = \frac{\left|\int_0^4 V\,dt\right| + \left|\int_4^5 V\,dt\right|}{\int_0^5 dt} = \frac{5}{3}\,m/s$

   $a = \frac{dV}{dT} = 4 - 2t$

4. A)

   $v_1 = 20 - gt$

   $v_2 = 20 - g(t-2)$

   $v_1 - v_2 = [20 - gt] - [(20-g)(t-2)]$

   or $v_1 - v_2 = -gt + g(t-2)$

   or $v_1 - v_2 = -2g$

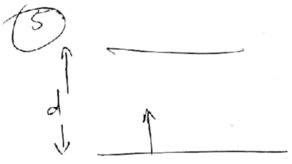
   Clearly, it is time – independent.

   B) In both the cases, the total energy remains the same.

   C) In both the cases, $|acceleration| = g$

5. AD

(5)



$t_{min} = \frac{d}{v_m} = 10$

& drift $= v_R \times t$

$120 = v_R \times 10$

$\boxed{v_R = 12}$

$v_m \sin\theta = v_R$ — (1)

$12.5 = \frac{d}{v_m \cos\theta}$

$v_m \cos\theta = d/12.5$ — (2)

$(1)^2 + (2)^2$

$v_m^2 = 12^2 + \frac{d^2}{12.5^2}$

$\Rightarrow v_m^2 = 12^2 + \frac{(10 v_m)^2}{12.5^2}$

~~(a, b, d)~~

(a, d)

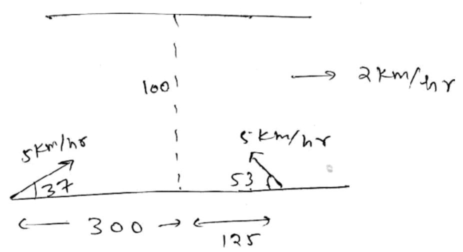
6. BCD

$v_{A,x} = 5\cos 37 + 2$

$= 6$

$v_{A,y} = 5\sin 37$

$= 3$



$v_{B,x} = 5\cos 53 - 2$

$= 1$

$v_{B,y} = 5\sin 53$

$= 4$

7. Separation between the particles will be maximum when both particles have same velocity. This situation come at t = 3 sec

Separation between the particles $x = (4 + 1 \times t) - \left(2 + \frac{t^2}{2}\right) \quad (t > 2)$

$\frac{dx}{dt} = 0 \; for \, x$ constant b = 1 sec

$x_{max} = 2 \times 2 + 1 \times 1 - \left(\frac{1}{2} \times 2 \times 2 + \frac{1}{2} \times 1 \times 1\right) = 5 - 2.5 = 2.5\,m$

8. Consider the vertical motion of the particle after entering the compartment. Let, it

Reaches the floor in time t. $3.2 = \frac{1}{2}(10)t^2 or \quad t = 0.8s$

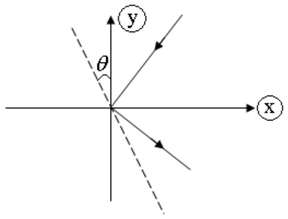
Due to velocity component u, which remains constant, it covers a distance of 2.4 m in 0.8 s

$\therefore \quad u = \frac{2.4m}{0.8s} = 3m/s$

Due to the velocity υ, point B must move forward by 16 m in 0.8 s

$$\therefore \quad \upsilon = \frac{16m}{0.8s} = 20m/s$$

9.



$2\sqrt{2}\cos(45+\theta) = \sqrt{2}\cos(45-\theta)$

$2\left[\frac{1}{\sqrt{2}}c\theta - \frac{1}{\sqrt{2}}s\theta\right] = \frac{1}{\sqrt{2}}c\theta + \frac{1}{\sqrt{2}}s\theta$

$c\theta = 3s\theta \Rightarrow \tan\theta = \frac{1}{3}$

10. Since average acceleration = change in velocity / time. So average acceleration is in the direction of change in velocity. Also if initial velocity is zero then final velocity and change in velocity will be in same direction.

11. Since the graph is a straight line, its slope is constant, it means acceleration of the particle is constant. Velocity of the particle changes from positive to negative at $t = 10s$, so particle changes direction at this time. The particle has zero displacment upto 20s, but not for the entire motion. The average speed in the interval 10s to 20s because distance covered in both time intervals is same.

12. Distance between two buses on road $= V_b T$

For A to B direction: distance = relative velocity x time

$V_b T = (V_b - V_c)T_1 \Rightarrow T_1 = \frac{V_b T}{V_b - V_c}$

For B to A direction: $V_b T = (V_b + V_c)T_2 \Rightarrow T_2 = \frac{V_b T}{V_b + V_c}$

13. $x = a\cos(pt), y = b\sin(pt)$

Equation of path in x-y plane: $\left[\frac{x}{a}\right]^2 + \left[\frac{y}{b}\right]^2 = 1$

i.e., the path of the particle is an ellipse

Position vector of a point P is: $\vec{r} = a\cos pt\hat{i} + b\sin pt\hat{j}$

$\Rightarrow \vec{v} = p(-a\sin pt\hat{i} + b\cos pt\hat{j})$ and $\vec{a} = -p^2(a\cos pt\hat{i} + b\sin pt\hat{j}) = -p^2\vec{r}$

also $\vec{v}.\vec{a} = 0$ at $t = \pi/2p$

14. (b,c,d)

$$\vec{v}_{OP} = \vec{v}_O - \vec{v}_P \quad \text{and} \quad \vec{v}_{OQ} + \vec{v}_{QP} = \vec{v}_O - \vec{v}_Q + \vec{v}_Q - \vec{v}_P = \vec{v}_O - \vec{v}_P$$

$$\vec{a}_{OP} = \vec{a}_O - \vec{a}_P \quad \text{and} \quad \vec{a}_{OQ} + \vec{a}_{QP} = \vec{a}_O - \vec{a}_Q + \vec{a}_Q - \vec{a}_P = \vec{a}_O - \vec{a}_P$$

(b,d)

15. (b,c) Use $v^2 = v^2{}_x + v^2$, differentiate to get

$\left(\frac{dv}{dt}\right) = \frac{(v_x a_x + v_y a_y)}{v} = \frac{\left(\vec{v}\,\vec{a}\right)}{v}$. Also, note that $\left(\frac{\vec{v}}{v}\right)$ is the unit vector along v.

mod-1 Matrix type

1.)

A) (P) $V = -4x + 5$

at $x = 0$ → $v = 5$ m/s, so particle is passing through origin.

Q) $a = -4v + 5$

at $x = 1$, $v = 0$, $a > 0$ so It will never pass through origin.

R) $a = -4x + 5$, $v = 0$ at $x = 1$

Now $a = 1$ at $x = 1$ so particle will go in $+x$.

$$\frac{v\,dv}{dx} = -4x + 5$$

$$\Rightarrow \int_0^v v\,dv = \int_1^0 (-4x + 5)\,dx$$

$$\Rightarrow \frac{v^2}{2} = -\frac{4}{2}(x^2)\Big|_1^0 + 5x\Big|_1^0$$

$$\Rightarrow \frac{v^2}{2} = -2(-1) + 5(0-1)$$

$\boxed{v^2 < 0}$ won't reach origin

S) $V = -4t^2 + 5 = \frac{dx}{dt}$

$$\Rightarrow \int_1^0 dx = \int_0^t (-4t^2 + 5)\,dt$$

$$\Rightarrow (-1) = -\frac{4t^3}{3} + 5t$$ ⇒ solution is possible

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