Single Correct

(1) $3\tan\theta \cdot \tan\phi = 1$

$$\frac{\cos(\theta-\phi)}{\cos(\theta+\phi)} = \frac{\cos\theta \cdot \cos\phi + \sin\theta \cdot \sin\phi}{\cos\theta \cdot \cos\phi - \sin\theta \cdot \sin\phi}$$

$$= \frac{1 + \tan\theta \cdot \tan\phi}{1 - \tan\theta \cdot \tan\phi}$$

$$= \underline{\underline{2}}$$

(2) $A + B + C = \frac{3\pi}{2}$

$$\cos 2A + \cos 2B + \cos 2C = 2\cos(A+B)\cos(A-B) + \cos 2C$$

$$= -2\sin C \cdot \cos(A-B) + \cos 2C$$

$$= -2\sin C \cdot \cos(A-B) + 1 - 2\sin^2 C$$

$$= 1 - 2\sin C[\cos(A-B) + \sin C]$$

$$= 1 - 2\sin C[\cos(A-B) - \cos(A+B)]$$

$$= 1 - 2\sin C[2\sin A \cdot \sin B]$$

$$= \underline{\underline{1 - 4\sin A \cdot \sin B \cdot \sin C}}$$

(3) $5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$

$$\Rightarrow 5\cos\theta + 3\left[\cos\theta \cdot \frac{1}{2} - \sin\theta \cdot \frac{\sqrt{3}}{2}\right] + 3$$

$$\Rightarrow 5\cos\theta + \frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$$

$$\Rightarrow \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$$

$\therefore$ Maximum value: $\sqrt{\frac{169}{4} + \frac{27}{4}} + 3 = \underline{\underline{10}}$

(4) $\sin\theta_1 - \sin\theta_2 = a \; ; \; \cos\theta_1 + \cos\theta_2 = b$

$$\Rightarrow a^2+b^2 = \sin^2\theta_1 + \cos^2\theta_1 + \sin^2\theta_2 + \cos^2\theta_2 - 2\sin\theta_1 \cdot \sin\theta_2 + 2\cos\theta_1 \cdot \cos\theta_2$$

$$= 2(1 - \sin\theta_1 \cdot \sin\theta_2 + \cos\theta_1 \cdot \cos\theta_2)$$

$$= 2(1 + \cos(\theta_1 - \theta_2))$$

$$\Rightarrow \underline{\underline{a^2+b^2 < 4}}$$

(5) $(a+2)\sin\alpha + (2a-1)\cos\alpha = (2a+1)$

$$\Rightarrow (a+2)\frac{2\tan(\alpha/2)}{1+\tan^2(\alpha/2)} + (2a-1)\frac{(1-\tan^2(\alpha/2))}{(1+\tan^2(\alpha/2))} = (2a+1)$$

$$\Rightarrow 2(a+2)\tan(\alpha/2) + (2a-1)(1-\tan^2(\alpha/2)) = (2a+1)(1+\tan^2(\alpha/2))$$

$$\Rightarrow 2(a+2)\tan\frac{\alpha}{2} + 2a - 2a\tan^2\frac{\alpha}{2} - 1 + \tan^2\frac{\alpha}{2} = 2a + 2a\tan^2\frac{\alpha}{2} + 1 + \tan^2\frac{\alpha}{2}$$

$$\Rightarrow 2(a+2)\tan\frac{\alpha}{2} = 4a\tan^2\frac{\alpha}{2} + 2$$

$$\Rightarrow 2a\tan^2\frac{\alpha}{2} - (a+2)\tan\frac{\alpha}{2} + 1 = 0$$

$$\Rightarrow \tan\frac{\alpha}{2} = \frac{1}{2} \text{ or } \frac{1}{a}$$

$$\Rightarrow \tan\alpha = \frac{2\tan\alpha/2}{1-\tan^2\alpha/2} = \frac{2\times\frac{1}{2}}{1-\frac{1}{4}} = \underline{\underline{\frac{4}{3}}}$$