Multiple Choice Questions

(1). b,c,d $E_1 \rightarrow$ event that two I's are together in PROBABILITY.

$$P(E_1) = \frac{\frac{10!}{2!}}{\frac{11!}{2!2!}} = \frac{2}{11}.$$

$E_2 \rightarrow$ event that two B's are together in PROBABILITY.

$$P(E_2) = \frac{2}{11}.$$

$P(E_1 \cap E_2) \equiv$ ~~event~~ both B's and I's are together

$$P(E_1 \cap E_2) = \frac{9!}{\frac{11!}{2! \times 2!}} = \frac{4}{10 \times 11} = \frac{2}{55}$$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$
$$= \frac{2}{11} + \frac{2}{11} - \frac{2}{55} = \frac{18}{55}$$

$$P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{2/55}{2/11} = \frac{1}{5}.$$

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(2) (b, c) Clearly, $0 \le \frac{1+4p}{4} \le 1$; $0 \le \frac{1-p}{4} \le 1$; $0 \le \frac{1-2p}{4} \le 1$

$$0 \le 1+4p \le 4$$

$$-1/4 \le p \le 3/4 \; ; \; -3 \le p \le 1 \; ; \; -\frac{3}{2} \le p \le \frac{1}{2}.$$

$\Rightarrow p \in \left[\frac{-1}{4}, \frac{1}{2}\right]$ will be the common region.

③ (a, b)

given, $P(X=x) \propto (x+1)\left(\frac{1}{5}\right)^x$

$P(X=x) = K(x+1)\left(\frac{1}{5}\right)^x$.

$\sum_{x=0}^{\infty} P(X=x) = 1$

$K + K\times2\times\frac{1}{5} + K\times3\times\frac{1}{5^2} + \dots \infty = 1$.

$K\left[\frac{1}{1-1/5} + \frac{1/5}{(1-1/5)^2}\right] = 1$

$\Rightarrow K\left[5/4 + 5/16\right] = 1$

$K = \frac{16}{25}$.

$P(X=0) = K = \frac{16}{25}$

$P(X \geq 1) = 1 - P(X=0)$

$= 1 - \frac{16}{25} = \frac{9}{25}$.

④ (a, c, d) Given, $P(E_1) = 1/4$

$P(E_2|E_1) = \frac{1}{2}$

$\frac{P(E_2 \cap E_1)}{P(E_1)} = \frac{1}{2} \Rightarrow P(E_2 \cap E_1) = \frac{1}{8}$

$P(E_1|E_2) = \frac{1}{4}$

$\frac{P(E_2 \cap E_1)}{P(E_2)} = \frac{1}{4} \Rightarrow P(E_2) = \frac{1}{2}$.

$P(E_1 \cap E_2) = P(E_1) \times P(E_2) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

$\therefore E_1$ & E_2 are independent.