LAKSHYA ADVANCED UNIT TEST (LAUT)

00 – 00		Test date : 17.07.16	
Test No : 3051341	3 Hrs.		

Hints & Solutions

PART C - MATH

SECTION I - Multiple Answer Correct

1. **a)** $y^2 = 40x$, $y = e^{-x/20}$

For $y^2 = 40x$ For $y = e^{-x/20}$

$2y\dfrac{dy}{dx} = 40$ $\dfrac{dy}{dx} = e^{-x/20} \times \dfrac{-1}{20}$

$m_1 = \dfrac{20}{y}$ $m_2 = \dfrac{-y}{20}$

$\therefore\ m_1 \times m_2 = -1$

b) $y^2 = 4ax$, $x^2 = 4ay$ **at (0, 0)**

For $y^2 = 4ax$ tangent at origin is y-axis and for

$x^2 = 4ay$ tangent at origin is x-axis.

c) $xy = a^2$, $x^2 - y^2 = b^2$

For $xy = a^2$ For $x^2 - y^2 = b^2$

$x\dfrac{dy}{dx}$ and $y = 0$ $2x - 2y\dfrac{dy}{dx} = 0$

$m_1 = \dfrac{-y}{x}$ $-y\dfrac{dy}{dx} = -x$

$m_2 = \dfrac{x}{y}$

$\therefore\ m_1 m_2 = -1$

d) $y = ax$, $x^2 + y^2 = c^2$

For $y = ax$ For $x^2 + y^2 = c^2$

$m_1 = a = \dfrac{y}{x}$ $2x + 2y\dfrac{dy}{dx} = 0$

$m_1 = -\dfrac{x}{y}$

$\therefore\ m_1 m_2 = -1$

2. **a)** $f(x)$

b) $g(x)$

d) $h(x)$

$f(x)$ is not differentiable at $x = \dfrac{1}{2}$

$g(x)$ is not continuous in [0, 1] at $x = 0$

$h(x)$ is not continuous in [0, 1] at $x = 1$

$k(x)$ is continuous and differentiable

3. **a)** ***h* is increasing when *f* is increasing**

b) ***h* is decreasing when *f* is decreasing**

$h'(x) = f'(x) - 2f(x).f'(x) + 3f^2(x)\,f'(x)$

$= f'(x)\,[3f^2(x) - 2f(x) + 1]$

$= 3f'(x)\left[f^2(x) - \dfrac{2}{3}f(x) + \dfrac{1}{3}\right]$

$= 3f'(x)\left\{\left[f(x) - \dfrac{1}{3}\right]^2 + \dfrac{2}{9}\right\}$

If $f'(x)$ is increasing then $h'(x)$ is increasing

If $f'(x)$ is decreasing then $h'(x)$ is decreasing

4. **b)** $(0, 1)$

c) $\left(\dfrac{\pi}{2}, \pi\right)$

d) $\left(0, \dfrac{\pi}{2}\right)$

$f'(x) = 100x^{99} + \cos x$

5. **a)** **$f(x)$ has horizontal tangent at $x = e$**

b) **$f(x)$ cuts x-axis only at one point**

c) **$f(x)$ is many - one function**

$f(x) = \dfrac{\ln x}{x}$

$f'(x) = \dfrac{1 - \ln x}{x^2}$

a) $f'(x) = 0$ $\ln x = 1$ $x = e$ (True)

b) $\dfrac{\ln x}{x} = 0$ $x = 1$ (True)

c) If $x \in (0, e)$, $f'(x) > 0$ $\to$ Increasing

If $x \in (e, \infty)$, $f'(x) < 0$ $\to$ Decreasing

$\therefore$ $f(x)$ is many to are

d) $f'(x) = \infty$

$$\frac{x^2}{1-\ln x} = 0$$

$x = 0$ Not in domain

6. **b)** $\lim_{x\to\infty} f(x) = 1$

d) $f(x)$ **is strictly decreasing in the inteval** $[1, \infty)$

$f(x) = x\cos\frac{1}{x},\ x \geq 1$

$f'(x) = \frac{1}{x}\sin\frac{1}{x} + \cos\frac{1}{x}$

$f''(x) = \frac{-1}{x^3}\cos\frac{1}{x}$

$\lim_{x\to\infty} f'(x) = 1$

for $x \in [1, \infty)$, $\frac{1}{x} \in (0, 1]$

$f''(x) < 0$

7. **b)** **decreases in** $\left(\frac{2}{3}, 1\right]$

c) **increases in** $\left[0, \frac{2}{3}\right)$

$g'(x) = f'\left(\frac{x}{2}\right) - f(1-x)$

If $g(x)$ is increasing $g'(x) > 0$

$\therefore\ f'\left(x/2\right) > f'(1-x)$

$\frac{x}{2} < 1 - x$

$x < \frac{2}{3}$

8. **a)** $\frac{4}{8-\sqrt{2}}$

c) $\frac{2\sqrt{2}}{4\sqrt{2}+1}$

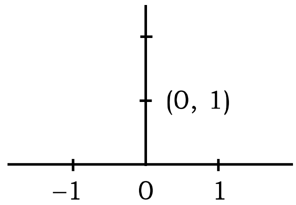
$f'(x) = 0$

$\tan x = -1$

$\therefore\ x = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$

9. **c)** **neither maximum nor minimum**

d) $f'(0)$ **does not exist**



Not differentiable

Neither maxima nor minima

10. **a)** $b - a = n\pi,\ n \in I$

b) $b - a = (2n+1)\pi,\ n \in I$

c) $b - a = 2n\pi,\ n \in I$

$f(x) = \frac{\sin(x+a)}{\sin(x+b)}$

$f'(x) =$

$$\frac{\sin(x+b)\cos(x+a) - \sin(x+a)\cos(x+b)}{\cos^2(x+b)}$$

$$= \frac{\sin(b-a)}{\sin^2(x+b)}$$

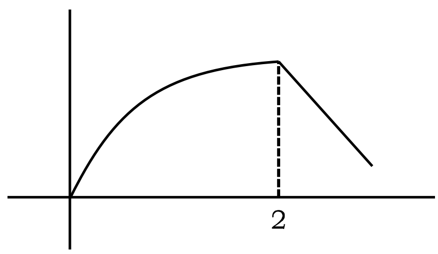
If $b - a = 0$, $f(x)$ will be constant

SECTION II (Matrix Match Type)

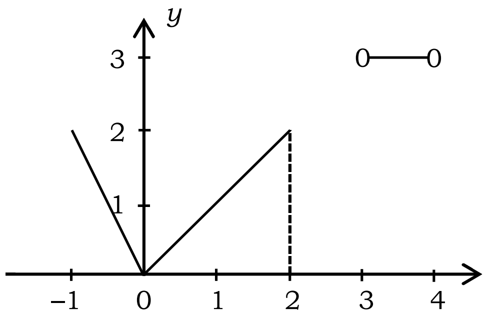
1. **A- P,R ; B- P,R,S ; C- Q,S ; D- P,S**

A) $f'(x) = 8x(x-1)(x+1) = 0$

local max. at $x = 0$

B) 

C) $f\left(\frac{1}{2}\right)^{-} > f\left(1/2\right) < f\left(1/2\right)^{+}$

D) 

2. **A- P,Q,R ; B- P,S ; C- R,S ; D- P,Q**

A) $x|x|$ is continuous, differentiable and strictly increasing in (–1, 1)

B) $\sqrt{|x|}$ is continuous in (–1, 1) and at $x = 0$

C) $x + [x]$ is strictly increasing in (–1, 1) and discontinuous at $x = 0$

D) $|x-1|+|x+1| = 2$ in (–1, 1)
The function is continuous and differentiable in (–1, 1)

SECTION III (Integer Type)

1. **3**

Let $f(x) = x^3$

According to LMVT

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$f'(c) = \frac{b^3-a^3}{b-a}$$

$3c^2 = b^2 + ab + a^2$

2. **3**

3. **2**

$f'(x) = 4x^3 - 12x^2 + 24x + 1$

$f''(x) = 12x^2 - 24x + 24$

$f''(x)$ has no real roots.

$f''(x) > 0$

$\therefore$ $f'(x)$ is strictly increasing

$f(x)$ is increasing or decreasing

$f(x)$ strictly decreasing $(-\infty\ \alpha)$ strict increasing $(\alpha.\infty)$

4. **3**

$f(x, y) = x^2 - y^2 - 4x + 6y$

Let $(x, y) \equiv (\cos\theta, \sin\theta)$, $\theta \equiv \left[0, \frac{\pi}{2}\right]$

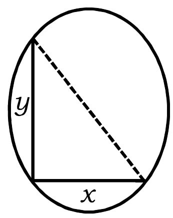
$f(x, y) = f(\theta) = -4\cos\theta + 6\sin\theta$

$f'(\theta) = 6\cos\theta + 4\sin\theta > 0 \quad \theta = \left[0, \frac{\pi}{2}\right]$

$f'(0)$ is strictly increasing

$f(\theta)_{\min} = f(0) = 1 - 4 + 0 = -3$

5. **9**



$$A(D) = \frac{1}{x}y\frac{1}{2}xy = \frac{9}{\pi} = 5$$

Area of circle

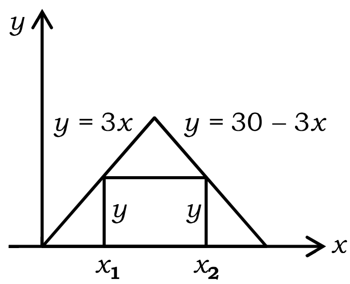
$$A = \pi\frac{x^2+y^2}{4}$$

$$= \frac{\pi}{4}\left[x^2 + \left(\frac{2s}{x}\right)^2\right]$$

$$A' = \frac{\pi x}{2} - \frac{2\pi s^2}{x^3} = 0$$

$x^2 = 2$ s

6. **5**



$A = (x_2 - x_3)y$

$$A(y) = \left(\frac{30-y}{2} - \frac{y}{3}\right)y$$

$6A(y) = (90 - 5y)\ y$

$6A'y = 90 - 10y$

$y = 9$

$A'(y) = -10 < 0$

$x = 3$ and $x^2 = \frac{4}{2}$

$A_{max} = \frac{35}{2}$

7. **8**

$f(x) = x^3 - 3x + a$

$f'(x) = 0$

$3x^2 - 3 = 0$

$x = \pm 1$

For three distinct roots

$f(1)\, f(-1) < 0$

$(a + 2)\,(a - 2) < 0$

$-2 < a < 2$

8. **2**