XI - PHYSICS - SOLUTIONS

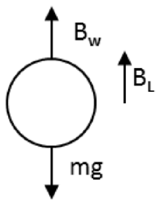
1. **b)** A and B have equal volumes
Archimedes principle

2. **c)** It will sink
Basic concept

3. **a)** $v_1 = v_2$
$v_1 = v_2 = \sqrt{2gh}$ (Independent of density)

4. **c)** It will fall
Basic concept

5. **c)** 3.5×10^3



$B_w + B_L = mg$

$\frac{4}{5}v\times10^3\times g + \frac{V}{5}\times13.5\times10^3 g$

$= v\rho g$

or $\rho = 3.5\times10^3 kg/m^3$

6. **b)** 3 litre/sec

We know $AV_1 = Aa\sqrt{\frac{2(P_1 - P_2)}{A^2 - a^2}}$

$A = \pi r_1^2 = \pi\left(\frac{10}{2}\right)^2 = 25\pi cm^2$

$a = \pi r_1^2 = \pi\left(\frac{6}{2}\right)^2 = 9\pi cm^2$, $p_1 - p_2 = 5$cm of water column =

$5\times1\times980 dyne/cm^3$

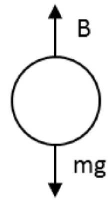
$\therefore AV_1 = 25\pi\times9\pi\sqrt{\frac{2\times5\times1\times980}{(25\pi)^2-(9\pi)^2}}$

= 3000 cc = 3 litre/sec

7. **b)** 10 cm
$B = m_1g + m_2g$
$l^3\times\rho_l\times g = 0.2\times g + l^3\times\rho\times g$
$\Rightarrow l^3(\rho_l - \rho) = 0.2$
$\Rightarrow l^3(10^3 - \rho) = 0.2$ ... (i)
When mass is removed
$l^3\times\rho\times g = l^2(l - 2\times10^{-2})\times\rho_l\times g$
$\rho_l = (l - 2)\times10^3$ ... (ii)
On solving equation (i) and (ii)
We get $l = 10$ cm

8. **a)** $\sqrt{\left(\frac{2h_1}{g}\right)}\times\frac{d_1}{d_2 - d_1}$

Velocity achieved by body
$v = \sqrt{2gh_1}$



B - mg = ma
$V\times d_2\times g - Vd_1g = V\times d_1\times a$
$\Rightarrow a = \frac{2}{d_1}(d_2 - d_1)$
v = u + at
$0 = \sqrt{2gh_1} - \frac{g}{d_1}(d_2 - d_1)t$
$\Rightarrow t = \sqrt{\frac{2R_1}{g}}\left(\frac{d_1}{d_2 - d_1}\right)$

9. **d)** m (1/y – 1/x)
Initial volume = $\frac{m}{x}$
Final volume = $\frac{m}{y}$
Change in volume = $m\left(\frac{1}{y}-\frac{1}{x}\right)$

10. **a)** $\sqrt{[4h(H-h)]}$
Horizontal distance of both jets are same
So, $x = v_1t_1 = v_2t_2$
$x = \sqrt{2gh}\times\sqrt{\frac{2(H-h)}{g}} = \sqrt{2g(H-h)}\times\sqrt{\frac{2h}{g}}$
or $x = \sqrt{[4h(H-h)]}$

11. **d)** 2m
$A_1v_1 = A_2v_2$
$v_2 = \left(\frac{A_1}{A_2}\right)v_1 = \frac{\pi r_1^2}{\pi r_2^2}\times0.25$

$= 4 m/s$

$x = v \times t$

$= 4 \times \sqrt{\frac{2h}{g}}$

$= 4 \times 0.5 = 2m$

12. c) 0m/s

Frictional force = μmg

$= 0.4 \times 100 = 40$ N

Force by water $f = \frac{dp}{dt} = \frac{d}{dt}(m\nu)$

$= v\left(\frac{dm}{dt}\right)$

$= 8 \times 2 = 16$ N

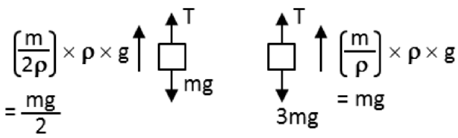
which is less than frictional force.

So, block will not move

13. b) 3g/8

Volume of mass m

$V_1 = \frac{m}{2\rho}$



And volume of mass 3m

$V_2 = \frac{3m}{3\rho} = \frac{m}{\rho}$

$3mg - (T + mg) = 3ma$

or $2mg - T = 3ma$ ... (i)

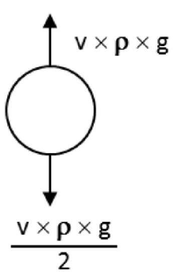
$T + \frac{mg}{2} - mg = ma$

$T - \frac{mg}{2} = ma$ ... (ii)

On solving equation (i) and (ii)

We get $a = \frac{3g}{8}$

14. a) 19.6m



$\nu = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 19.6} = 19.6 m/s$

$v^2 = 4^2 + 2ay$

$mg - B = ma$

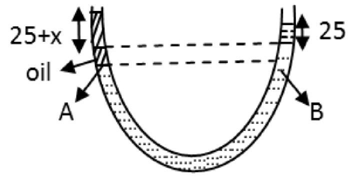
$v \times \frac{\rho}{2} g - v\rho g = v \times \frac{\rho}{2} \times a$

$a = -\mathbf{g}$

$v^2 = u^2 + 2ay$

$\Rightarrow y = 19.6m$

15. b) 12.50 cm



$P_A = P_B$

$(50 + x)0.8g = 50(1)g$

$x = 12.5$

16. d) 12/17

In first liquid.

$\frac{2}{3} v \times \rho_1 \times g = v \times \rho_\lambda \times g$

$\Rightarrow \rho_1 = \frac{3}{2}\rho_\lambda$

In second liquid

$B' = mg$

$\frac{3}{4} v \times \rho_2 \times g = v \times \rho_s \times g$

$\Rightarrow \rho_2 = \frac{4}{3}\rho_s$

In mixture, $\rho = \frac{m}{v} = \frac{\frac{3}{2}\rho_s \times v + \frac{4}{3}\rho_s \times v}{2v}$

or $\rho = \frac{17}{12}\rho_s$

When solid is in mixture.

$B^1 = mg$

$v' \times \frac{17}{12}\rho_s g = v \times \rho_s \times g$

$\Rightarrow \frac{v'}{v} = \frac{12}{17}$

17. a) 2

Weight = 60 N

$v \times \rho \times g = 60$ ....(i)

Buoyancy force = 30

$v \times \rho_l \times g = 30$ ....(ii)

(i)/(ii)

$\frac{\rho}{\rho_l} = 2$

18. d) none of these

B = 400 × 10^{-6} × 10^3 × 10 = 4N
Weight in water = 100 - 4 = 96

19. d) 3/5
Buoyancy force = weight

20. c) 80 kg
Buoyancy force concept

21. b) $\left(\frac{\sigma}{\rho}-1\right)h$
B – mg = ma
$v\sigma g - v\rho g = v\rho a$
$a = (\sigma - \rho)\frac{g}{\rho}$
$v^2 = u^2 + 2ah$
$v = \sqrt{2a(\sigma - \rho)\frac{gh}{\rho}}$
When ball reaches at surface, onwards motion will be motion under gravity.
$v^2 = u^2 + 2gh'$
$0 = 2a(\sigma - \rho)\frac{gh}{\rho} - 2g \times h'$
$\Rightarrow h' = \left(\frac{\sigma}{\rho} - 1\right)h$

22. a) 60 kg
B = mg
3 × 2 × 10^{-2} × 10^3 × g = mg
$\Rightarrow m = 60kg$

23. c) 1.54 N
$F = \pi r^2 \upsilon^2 p, \upsilon = \sqrt{2gh} = \sqrt{20}h$
= 3.14 × $(0.5 \times 10^{-2})^2$ × 20 × 10^3
= 1.54 N

24. d) zero
Basic concept

25. a) $\sqrt{2}t$
$\upsilon = \sqrt{2gh}$
Volume coming out of orifice in time dt
= Area × velocity × dt
$= a \times \sqrt{2gh} \times dt$
and $d\upsilon = -Adh$
$\int dt = -\int \frac{Adh}{a\sqrt{2gh}}$

26. b) 0.02 m
$y = \frac{w^2 r^2}{2g} = \frac{(2\pi f)^2 r^2}{2g}$
$= \frac{(2 \times 3.14 \times 2)^2 \times (5 \times 10^{-2})^2}{2 \times 10}$
= 0.02 m

27. a) 36.64cm
The flow of water will be stopped by the tube dipping in the stream and facing the flows. Therefore the loss of kinetic energy per unit mass of water is $\frac{\upsilon^2}{2}$. This will cause the gain in pressure energy. i.e. $\frac{p}{\rho}$
$\therefore \frac{p}{\rho} = \frac{\upsilon^2}{2} \text{ or } p = \frac{1}{2}\rho\upsilon^2$
Let h be the height to which water rises in the tube
$p = h\rho g \text{ or } h\rho g = \frac{\upsilon^2 \rho}{2}$
$h = \frac{\upsilon^2}{2g} = \frac{(2.68)^2}{2 \times 9.8} = 0.3664m = 36.64cm$

28. a) increase by $hV(\rho - \rho')g$
Work done by gravity + work done by Buoyancy force = Charge in potential energy

29. a) h = r
Equate forces

30. a) Pascal's principle
Pascal's law

31. **c)** 9.5
Conc. Salt formed = 1M
$p^x = 7 + \frac{1}{2}(\log s + 5) = 9.5$
$\left[p^x = 7 + \frac{1}{2}\left[\log c + p^{Ka}\right]\right]$

32. **c)** 6×10^{-6} M $CaCl_2$ and 3×10^{-6} M $(NH_4)_2SO_4$
For pp+ $Q_{sp} > K_{sp}$
$Q_{sp}(CaSO_4) = \left[\frac{6\times10^{-6}}{2}\right]\left[\frac{3\times10^{-6}}{2}\right]$
$= 4.5\times10^{-12}$

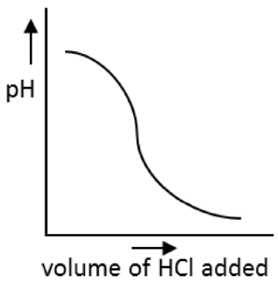
33. **d)** 0.200 mol of aniline and 0.100 mol of HCl
$C_6H_5NH_4 + HCl \longrightarrow C_6H_5NH_3 + Cl^-$
0.2 0.1 0.1
0.1 0
mix of weak base and its conjugate Acid

34. **c)** $HSO_4^-, H_2PO_4^-, H_2PO_3^-$
Conceptual

35. **d)** 108 S^5
$K_{sp^2}[A^{3+}]^2[x^{2-}]^3$ $A_2X_3 2 \rightleftharpoons A^{3+} + 3X^{2-}$
$= (25)^2 (35)^3$
$= 108\ S^5$

36. **c)** Bothe (a) and (b)
For color of HIn $\therefore \frac{[HIn]}{[In]} \geq 10$

37. **c)** $NH_3 + NH_3 \rightleftharpoons NH_2^- + NH_4^+$
$2NH_3 \rightleftharpoons NH_2^- + NH_4^+$

38. **a)**


Conceptual

39. **c)** HCl with NH_4OH
Can't be used if base is weak

40. **c)** $\left[Hg_2^{2+}\right]\left[NO_3^-\right]^2$
$Hg_2(NO_3)_2 \rightleftharpoons Hg_2^{2+} + 2NO_3^-$

41. **b)** 1×10^{-9} M
$K_{sp} = s^2 = 10^{-10}$
$\Rightarrow s = 10^{-9}$
$BaSO_4 \rightleftharpoons B_4^{2+} + SO_4^{2-}$
$0.1 + s$ s
≈ 0.1
$K_{sp} = \left(10^{-5}\right)^2 = 0.1\times s$
$\Rightarrow s = 10^{-9}$

42. **c)** Both (a) and (b)
On adding HCl, OH^- will be consumed so solubility ↑ on adding NaOH, Solubility ↑ as $AlCOHl_3$ is amphoteric

43. **a)** 100 mL of 4×10^{-3} M $BaCl_2$ + 300 mL of 6.0×10^{-4} M Na_2SO_4
$Q_{sp} > K_{sp}$

44. **a)** HX
Solution of strong Base of weak acid
$p^x = 7 + \frac{1}{2}[\log C + p^{ka}]$
As p^x ↑, p^{ka} ↑ so ka ↓

45. **c)** $\left(\frac{y+z}{2}\right)$
amphiprotic → $HPO_2^{2-} = \frac{p^{ka_2} + p^{Ka_3}}{2}$

46. **a)** After addition of 100 mL of KOH
Solution is completely titrated at 100 mL so p^H is max. at 100 mL

47. **d)** I, II, III
Degree of hydrolysis & p^H of amphiprotic and salt of SA & SB is independent of Conc.

48. **b)** $\log\frac{2}{3}$

$CH_3COOH + NaOH \longrightarrow CH_3COONa$

10	4	0
6	0	4

$p^H = p^{ka} + \log\frac{4}{6} \Rightarrow p^H - p^{ka} = \log\frac{2}{3}$

49. **d)** 9.73

$NHa^+ + NaOH \longrightarrow NHaOH$

10	7.5	0
2.5	0	7.5

$p^{OH} = p^{kb} + Wg\frac{2.5}{7.5} = 4.74 + \log\frac{1}{3}$

$= 4.74 - \log 3$

$p^H = 14 - (4.74 - \log 3)$

$= 9.73$

50. **a)** 1.8×10^{-5} M

$Ca(CH_3COO)_2 = 0.005$

$\Rightarrow [CH_3COO^-] = 0.01$

$p^H = p^{ka} = \log\left(\frac{0.01}{0.01}\right) = -\log(1.8\times10^{-5}) + 0$

$\Rightarrow [H^+] = 1.8\times10^{-5}$

51. **d)** 7.02

$[OH^-] = 10^{-8} + 10^{-7} = 1.1\times10^{-7}$

$p^{OH} = 7 - \log 1.1$

$p^H = 7 + \log 1.1$

52. **b)** Basic

p^H of water $= \frac{p^{kw}}{2} = 5$

so solution is basic as $p^H > p^H$ of water

53. **d)** 100 mL of pH = 2 solution is mixed with 100 mL of pH = 12

Neutralisation

54. **d)** 1.0%

$[OH^-] = 10^{-14}$ as $p^{OH} = 4$

$10^{-4} = C\alpha = 0.01\times\alpha$

$\alpha = 10^{-2}$

% = 1

55. **d)** 1.0×10^{6}

Reverse of ionization so $K = \frac{1}{ka}$

56. **d)** 0.1 M, < 0.1 M

Second stage is weak

57. **a)** (A), (D)

conceptual

58. **d)** All of these

All are stronger than H_3O^+ so there is no equilibrium

59. **b)** NH_3

In Stronger base than water

60. **c)** O_2^{2-} (Peroxide ion)

$HO_2^- \xrightarrow{-H^+} O_2^{2-}$

acid C.B

61. **c)** $\frac{5}{7}$

Ellipse passing through O, (0, 0) and having foci P(3, 3) and Q(-4, 4),

Then $e = \frac{PQ}{OP + OQ}$

$= \frac{\sqrt{50}}{3\sqrt{2} + 4\sqrt{2}}$

$= \frac{5}{7}$

62. **b)** $\frac{10}{\sqrt{6}}$

Here $a^2 + 2 > a^2 + 1$

$\Rightarrow a^2 + 1 = (a^2 + 2)(1 - e^2)$

$\Rightarrow a^2 + 1 = (a^2 + 2)\frac{5}{6}$

$\Rightarrow 6a^2 + 6 = 5a^2 + 10$

$\Rightarrow a^2 = 10 - 6 = 4$

$\Rightarrow a = \pm 2$

Latus rectum $= \frac{2(a^2 + 1)}{\sqrt{a^2 + 2}} = \frac{2 \times 5}{\sqrt{6}} = \frac{10}{\sqrt{6}}$

63. **a)** $1 : \sqrt{2}$

Given that

$\frac{\text{distance between foci}}{\text{distance between two directrix}} = \frac{3}{2}$

$\Rightarrow \frac{2ae}{2\frac{a}{e}} = \frac{3}{2}$

$\Rightarrow e^2 = \frac{3}{2}$

$\Rightarrow 1 + \frac{b^2}{a^2} = \frac{3}{2}$

$\Rightarrow \frac{b}{a} = \frac{1}{\sqrt{2}}$

64. **a)** $2xy = c^2$

Let the given straight line be axis of coordinates and let the equation of the variable line be

$\frac{x}{a} + \frac{y}{b} = 1$

This line cuts the coordinate axis at the point A(a, 0) and B(0, b).

Therefore, the area of ΔAOB is

$\frac{1}{2}ab = c^2$

If (α, β) be the co-ordinates of middle point of AB, then $\alpha = \frac{a}{2}$ and $\beta = \frac{b}{2}$

Eliminating a and b we get

$4\alpha\beta = 2c^2$

$\therefore\ 2\alpha\beta = c^2$

Hence, the locus of (α, β) is $2xy = c^2$

65. **b)** $\frac{20}{3}$

$(5x - 10)^2 + (5y + 15)^2 = \frac{(3x - 4y + 7)^2}{4}$

$\Rightarrow (x - 2)^2 + (y + 3)^2 = \left(\frac{1}{2}\frac{3x - 4y - 7}{5}\right)^2$

$\Rightarrow \sqrt{(x - 2)^2 + (y + 3)^2} = \frac{1}{2}\frac{|3x - 4y - 7|}{5}$ is an ellipse, whose focus is (2, -3), directrix $3x - 4y + 7 = 0$, and eccentricity is $\frac{1}{2}$.

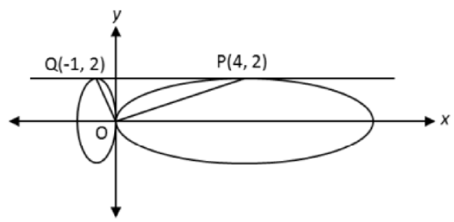
Length of Perpendicular from focus to directrix is $\frac{|3 \times 2 - 4\,(-3) + 7|}{5} = 5$

$\Rightarrow \frac{a}{e} - ae = 5$

$\Rightarrow 2a - \frac{a}{2} = 5$

$\Rightarrow a = \frac{10}{3}$

66. **d)** $\frac{\pi}{2}$



$\frac{(x - 4)^2}{25} + \frac{y^2}{4} = 1$ and $(x + 1)^2 + \frac{y^2}{4} = 1$

Clearly, $m_{OP} \times m_{OQ} = -1$

$\Rightarrow$ OP and OQ are perpendicular to each other.

67. **b)** 2

Since $\frac{e}{2}$ and $\frac{e'}{2}$ are eccentricities of a hyperbola and its conjugate hyperbola,

$$\frac{4}{e^2} + \frac{4}{e'^2} = 1$$

$$\Rightarrow 4 = \frac{e^2 e'^2}{e'^2 + e'^2}$$

This line passing through the points (e, 0) and (0, e') is $e'x + ey - ee' = 0$.

It is tangent to the circle $x^2 + y^2 = r^2$.

Hence, $\frac{ee'}{\sqrt{e^2 + e'^2}} = r$

$$\Rightarrow r^2 = \frac{e^2 e'^2}{e^2 + e'^2} = 4$$

$$\Rightarrow r = 2$$

68. **c)** $\frac{2}{\sqrt{3}}$

We have

$$\frac{2b^2}{a} = 8$$

and $2b = \frac{1}{2}(2ae)$

$$\therefore \frac{2}{ae}\left(\frac{ae}{2}\right)^2 = 8$$

$$\Rightarrow ae^2 = 16$$

Also, $2\frac{b^2}{a} = 8$

$$\Rightarrow b^2 = 4a$$

$$\Rightarrow a^2(e^2 - 1) = 4a$$

$$\Rightarrow ae^2 - a = 4$$

Form (1) and (2), we have

$$\Rightarrow 16 - \frac{16}{e^2} = 4$$

$$\Rightarrow \frac{16}{e^2} = 12$$

$$\Rightarrow e = \frac{2}{\sqrt{3}}$$

69. **a)** $d_1 : d_2$

Tangent at P($a\cos\alpha$, $b\sin\alpha$) is

$$\frac{x}{a}\cos\alpha + \frac{y}{b}\sin\alpha = 1 \qquad (1)$$

Distance of focus S(ae, 0) from this tangent is

$$d_1 = \frac{|e\cos\alpha - 1|}{\sqrt{\frac{\cos^2\alpha}{a^2} + \frac{\sin^2\alpha}{b^2}}}$$

$$= \frac{1 - e\cos\alpha}{\sqrt{\frac{\cos^2\alpha}{a^2} + \frac{\sin^2\alpha}{b^2}}}$$

Distance of focus S'($-ae$, 0) from this line

$$d_2 = \frac{1 + e\cos\alpha}{\sqrt{\frac{\cos^2\alpha}{a^2} + \frac{\sin^2\alpha}{b^2}}}$$

$$\Rightarrow \frac{d_1}{d_2} = \frac{1 - e\cos\alpha}{1 + e\cos\alpha}$$

Now SP = $a - ae\cos\alpha$ and S'P = $a + ae\cos\alpha$

70. **a)** 15π

Let P(x, y) be the position of the man at any time. Let S(4, 0) and S'(-4, 0) be the fixed flag, post, with C as the origin.

Since SP + S'P = 10 m i.e., a constant, the locus of P is an ellipse with S and S' as foci

$$\Rightarrow ae = 4, \text{ and } 2a = 10$$

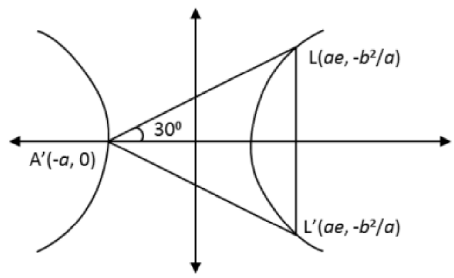
$$\Rightarrow e = \frac{4}{5}$$

Now $b^2 = a^2(1 - e^2)$

$$\Rightarrow b^2 = 25\left(1 - \frac{16}{25}\right) = 9$$

$$\Rightarrow b = 3$$

71. **d)** $\frac{\sqrt{3} + 1}{\sqrt{3}}$



$$\tan 30^0 = \frac{\frac{b^2}{a}}{a + ae}$$

$$\Rightarrow \frac{1 + e}{\sqrt{3}} = e^2 - 1$$

$$\Rightarrow e - 1 = \frac{1}{\sqrt{3}}$$

$$\Rightarrow e = \frac{\sqrt{3} + 1}{\sqrt{3}}$$

72. **b)** $\frac{\pi}{4}$

$$\frac{x^2}{5} - \frac{y^2}{5\cos^2\alpha} = 1,$$

We have

$$e_1^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{5\cos^2\alpha}{5}$$

$$= 1 + \cos^2\alpha$$

For the ellipse

$$\frac{x^2}{25\cos^2\alpha} + \frac{y^2}{25} = 1.$$

We have

$$e_1^2 = 1 - \frac{25\cos^2\alpha}{25} = \sin^2\alpha$$

Given that $e_1 = \sqrt{3}e_2$. So

$$e_1^2 = 3e_2^2$$

$$\Rightarrow 1 + \cos^2\alpha = 3\sin^2\alpha$$

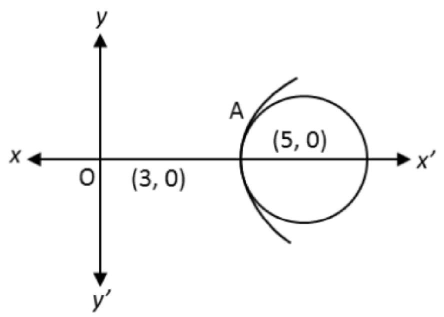
$$\Rightarrow 2 = 4\sin^2\alpha$$

$$\Rightarrow \sin\alpha = \frac{1}{\sqrt{2}}$$

73. **b)** 2

Given hyperbola is

$$\frac{x^2}{9} - \frac{y^2}{16} = 1 \quad (1)$$



$$\Rightarrow e^2 = 1 + \frac{16}{9} = \frac{25}{9}$$

$$\Rightarrow e = \frac{5}{3}$$

Hence, its foci are $(\pm 5, 0)$.

The equation of the circle with (5, 0) as centre is $(x - 5)^2 + y^2 = r^2$ (2)

Solving (1) and (2), we have

$$16x^2 - 9[r^2 - (x - 5)^2] = 144$$

or $25x^2 - 90x - 9r^2 + 81 = 0$

Since the circle touches the hyperbola, above equation must have equal roots. Hence,

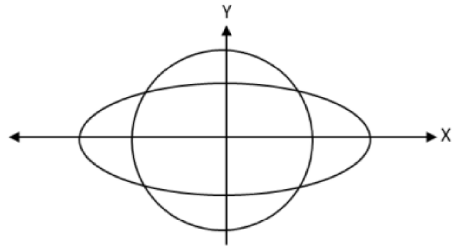
$$90^2 - 4(25)(81 - 9r^2) = 0$$

$$\Rightarrow 9 - (9 - r^2) = 0$$

$\Rightarrow r = 0$, which is not possible.

Hence, the circle cannot touch at two points. It can only be tangent at the vertex. Hence, $r = 5 - 3 = 2$.

74. **b)** $e \in \left(1/\sqrt{2}, 1\right)$



Radius of the circle having SS′ as diameter is $r = ae$. If it cuts an ellipse, then $r > b$

$$\Rightarrow ae > b$$

$$\Rightarrow e^2 > \frac{b^2}{a^2}$$

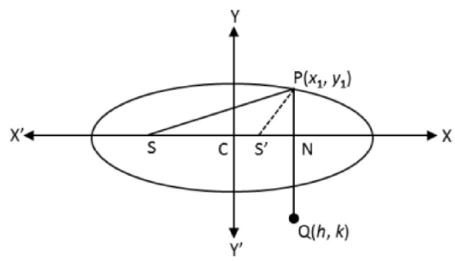
$$\Rightarrow e^2 > 1 - e^2$$

$$\Rightarrow e^2 > \frac{1}{2}$$

$$\Rightarrow e > \frac{1}{\sqrt{2}}$$

$$\Rightarrow e \in \left(\frac{1}{\sqrt{2}}, 1\right)$$

75. **b)** $3x + 5y + 25 = 0$



$a^2 = 25$ and $b^2 = 16$

$$\Rightarrow e = \sqrt{1 - \frac{16}{25}}$$

$$= \frac{3}{5}$$

Let point Q be (h, k), where $k < 0$.

Given that $k = SP = a + ex_1$, where $P(x_1, y_1)$ lies on the ellipse

$\Rightarrow |k| = a + eh$ (as $x_1 = h$)

$$\Rightarrow -y = a + ex$$

$$\Rightarrow 3x + 5y + 25 = 0$$

76. d) Ellipse

Let the vertex A be ($a\cos\theta$, $b\sin\theta$).

Since AC and BC touch the hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$, BC is the chord of contact. Its equation is

$$\frac{x\cos\theta}{a} - \frac{y\sin\theta}{b} = -1$$

or $-\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$

or $\frac{x\cos(\pi - \theta)}{a} + \frac{y\sin(\pi - \theta)}{b} = 1$

which is the equation of the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $(a\cos(\pi - \theta), b\sin(\pi - \theta))$. Hence, BC touches the given ellipse.

77. b) $\sqrt{2}$

Equation of tangent at (x_1, y_1) is

$$\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$$

It passing through (0, $-b$). So,

$$0 + \frac{y_1}{b} = 1 \Rightarrow y_1 = b$$

Equation of normal is

$$\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2e^2,$$

which passes through $(2a\sqrt{2}, 0)$. Hence,

$$\frac{a^2 2a\sqrt{2}}{x_1} = a^2e^2$$

$$\Rightarrow x_1 = \frac{2a\sqrt{2}}{e^2}$$

Now (x_1, y_1) lies on the hyperbola

$$\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} = 1$$

$$\Rightarrow \frac{8}{e^4} - 1 = 1$$

$$\Rightarrow e^2 = 2$$

78. a) $2a^2$

Any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ having Slope m is

$$y = mx + \sqrt{a^2m^2 + b^2}$$

Points on the minor axis are (0, ae), (0, $-ae$).

Sum of the squares of the perpendicular on the tangent from (0, ae) and (0, $-ae$)

$$= \left[\frac{\sqrt{a^2m^2 + b^2} - ae}{\sqrt{m^2 + 1}}\right]^2 + \left[\frac{\sqrt{a^2m^2 + b^2} + ae}{\sqrt{m^2 + 1}}\right]^2$$

$$= \frac{2(a^2m^2 + b^2 + a^2e^2)}{m^2 + 1}$$

$$= \frac{2(a^2m^2 + a^2 - a^2e^2 + a^2e^2)}{m^2 + 1}$$

$$= \frac{2a^2(m^2 + 1)}{m^2 + 1} = 2a^2$$

79. c) (0, 0)

Equation of the tangent to the ellipse at P($5\cos\theta$, $4\sin\theta$) is

$$\frac{x\cos\theta}{5} + \frac{y\sin\theta}{4} = 1$$

It meets the line $x = 0$ at Q (0, $4\operatorname{cosec}\theta$).

Image of Q in the line $y = x$ is R ($4\operatorname{cosec}\theta$, 0).

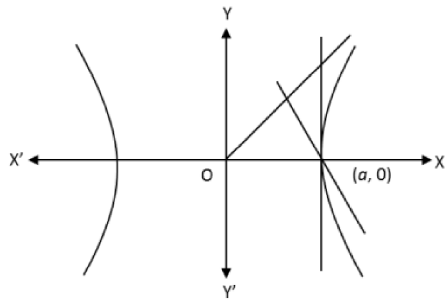
Therefore, equation of the circle is

$x(x - 4\operatorname{cosec}\theta) + y(y - \operatorname{cosec}\theta) = 0$

i.e., $x^2 + y^2 - 4(x + y)\operatorname{cosec}\theta = 0$

Therefore, each member of the family passes through the intersection of $x^2 + y^2 = 0$ and $x + y = 0$, i.e., the point (0, 0).

80. b) $\lambda \in (0, \infty)$



The line $y + \lambda(x - a) = 0$ will intersect the portion of the asymptote in the first quadrant only if its slope is negative. Hence,

$\Rightarrow -\lambda < 0$

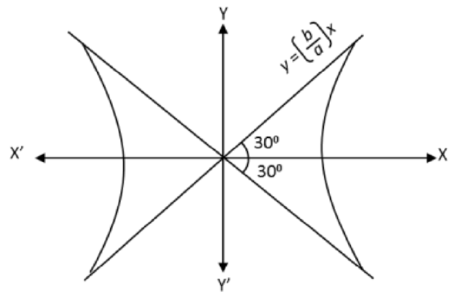
$\therefore \lambda \in (0, \infty)$

81. d) $x^2 + y^2 = 18$

Product of perpendiculars drawn from foci upon any of its tangents = 9

$\Rightarrow b^2 = 9$

Also, $\frac{b}{a} = \tan 30^0 = \frac{1}{\sqrt{3}}$



$\therefore a^2 = 27$

Therefore, required locus is the director circle of the hyperbola which is by given

$x^2 + y^2 = 27 - 9$ or $x^2 + y^2 = 18$

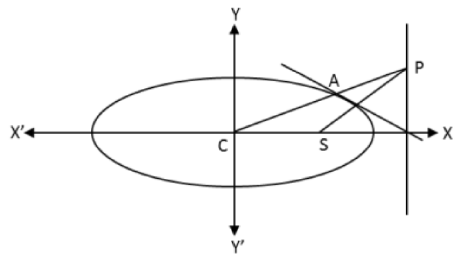
If $\frac{b}{a} = \tan 60^0$, then

$a^2 = \frac{b^2}{3} = \frac{9}{3} = 3$.

Hence, the required locus is

$x^2 + y^2 = 3 - 9 = -6$, which is not possible.

82. b) 2



Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and

Let $A \equiv (a \cos \theta, b \sin \theta)$

Equation of AC will be $y = \frac{b}{a} \tan\theta\, x$.

Solving with $x = \frac{a}{e}$, we get

$P \equiv \left(\frac{a}{e}, \frac{b}{e} \tan \theta\right)$

Slope of tangent at A is $-\frac{b}{a \tan \theta}$

Slope of PS $= \frac{\frac{b}{e} \tan \theta}{\frac{a}{e} - ae} = \frac{b \tan \theta}{a(1 - e^2)}$

$= \frac{a}{b} \tan \theta$

So $\alpha = \frac{\pi}{2}$.

83. a) 4

A tangent of slope 2 to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

is $y = 2x \pm \sqrt{4a^2 + b^2}$ (1)

this is normal to the circle $x^2 + y^2 + 4x + 1 = 0$

$\Rightarrow$ Eq. (1) passes through (-2, 0)

$\Rightarrow 0 = -4 \pm \sqrt{4a^2 + b^2}$

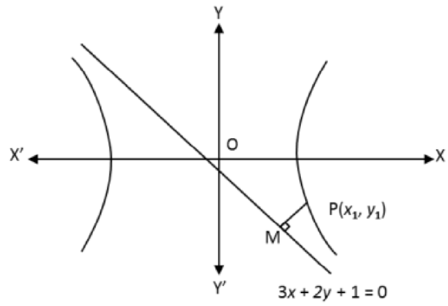
$\Rightarrow 4a^2 + b^2 = 16$

Using A.M. $\geq$ G.M., we get

$\frac{4a^2 + b^2}{2} \geq \sqrt{4a^2 b^2}$

$\Rightarrow ab \leq 4$

84. c) (6, 3)



Point P is nearest to the given line if tangent at P is parallel to the given line.

Now slope of tangent at $P(x_1, y_1)$ is

$\left(\frac{dx}{dx}\right)_{(x_1, y_1)} = \frac{18y_1}{24x_1} = \frac{3}{4}\frac{y_1}{x_1}$, which must be equal to $-\frac{3}{2}$

$\Rightarrow \frac{3}{4}\frac{y_1}{x_1} = -\frac{3}{2}1$

$\Rightarrow y_1 = -2x_1$ (1)

Also (x_1, y_1) lies on the curve. Hence,

$\frac{x_1^2}{24} - \frac{y_1^2}{18} = 1$

Solving (1) and (2) we get two points (6, -3) and (-6, 3) of which (6, -3) is nearest.

85. a) zero

Tangent to $\frac{x^2}{9} - \frac{y^2}{4} = 1$ at $P(3 \sec \theta, 2 \tan \theta)$

is $\frac{x}{3}\sec \theta - \frac{y}{2} \tan \theta = 1$

This is perpendicular to

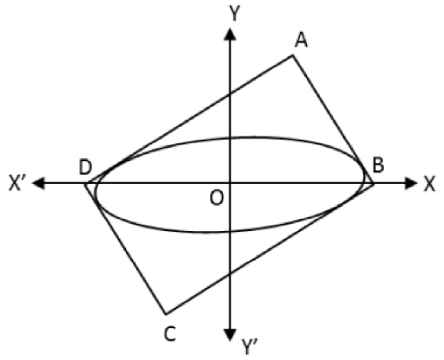
$5x + 2y - 10 = 0$

$\Rightarrow \frac{2\sec\theta}{3\tan\theta} = \frac{2}{5}$

$\Rightarrow \sin\theta = \frac{5}{3}$ which is not possible.

Hence, there is no such tangent.

86. d) no such a exists



Since side of the square are tangent and perpendicular to each other, so the vertices lie on director circle $\Rightarrow x^2 + y^2 = (a^2 - 7) + (13 - 5a)$

$= a^2$ ($\sqrt{2}\,a$ is side of the square)

$\Rightarrow (a^2 - 7) + (13 - 5a) = a^2$

$\Rightarrow a = \frac{6}{5}$

But for an ellipse to exist $a^2 - 7 > 0$ and $13 - 5a > 0$

$\Rightarrow a \in (-\infty, -\sqrt{7})$

Hence, $a \neq \frac{6}{5}$

Hence, no such a exists.

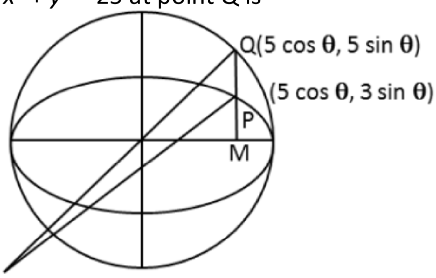
87. c) $x^2 + y^2 = 64$

Equation of normal to the ellipse at P is

$5x \sec\theta - 3y \operatorname{cosec}\theta = 16$ (1)

Equation of normal to the circle

$x^2 + y^2 = 25$ at point Q is



$y = x \tan\theta$ (2)

Eliminating θ from (1) and (2), we get

$x^2 + y^2 = 64$

88. d) $(x^2 + y^2)^2 = 4xy$

Let the foot of perpendicular from O(0, 0) to tangent to hyperbola is P(h, k). Slope of OP = k/h. Then equation of tangent to hyperbola is

$y - k = -\frac{h}{k}(x - h)$

or $hx + ky = h^2 + k^2$

Solving it with $xy = 1$, we have

$hx + \frac{k}{x} = h^2 + k^2$

or $hx^2 - (h^2 + k^2)x + k = 0$

this equation must have real and equal roots. Hence, D = 0

$\Rightarrow (h^2 + k^2)^2 - 4hk = 0$

$\Rightarrow (x^2 + y^2)^2 = 4xy$

89. b) $\frac{\pi}{2}$

Let directrix be $x = a/e$ and focus be S $(ae, 0)$.

Let P($a \sec\theta$, $b \tan\theta$) be any point on the curve. Equation of tangent at P is

$\frac{x\sec\theta}{a} + \frac{y\tan\theta}{b} = 1.$

Let F be the intersection point of tangent of directrix.

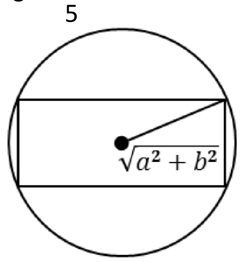
Then $F = \left(\frac{a}{e}, \frac{b(\sec\theta - e)}{e\tan\theta}\right)$

$\Rightarrow m_{SF} = \frac{b(\sec\theta - e)}{-e\tan\theta(a^2 - 1)},$

And $m_{PS} = \frac{b\tan\theta}{a(\sec\theta - e)}$

$\Rightarrow m_{SF} \times m_{PS} = -1$

90. d) $8\frac{a^2 + b^2}{5}$



Since mutually perpendicular tangents can be drawn from vertices of rectangle. So all the vertices of rectangle should lie on director circle $x^2 + y^2 = a^2 + b^2$.

Let breadth = $2l$ and length = $4l$, then

$l^2 + (2l)^2 = a^2 + b^2$

$\Rightarrow l^2 = \frac{a^2 + b^2}{5}$

$\Rightarrow$ Area $= 4l \times 2l = 8\frac{a^2 + b^2}{5}$.